A classical explanation for the anomalous magnetic moment of the electron?

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Abstract: Critics of the *Zitterbewegung* model often ask what *predictions* come out of the model. The answer to this question is quite simple: in order to gain credibility, the model would need to explain the anomalous magnetic moment as measured in experiments. If it can do this, then it should probably be recognized as a valid and alternative interpretation of quantum mechanics. This paper explores the geometry of the model in very much depth and, as such, lays the foundations for such explanation.

Keywords: *Zitterbewegung*, mass-energy equivalence, wavefunction interpretations, realist interpretation of quantum mechanics.

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Introduction

There are various varieties of the *Zitterbewegung* model. In our previous paper[1], we presented the simplest of simple models that, in our humble opinion, is consistent with the interpretation. It is probably useful to repeat the basics. We took Einstein's mass-energy equivalence relation (E = m·c^2) and, interpreting *c* as the tangential velocity of the naked charge (the toroidal photon, as Burinskii refers to it[2]), substituted *c* for *a*·ω: the tangential velocity equals the radius times the angular frequency. We then can then use the Planck-Einstein relation (E = ħ·ω) to find the Compton radius:

$$a = \frac{c}{\omega} = \frac{c \cdot \hbar}{\mathrm{m} \cdot c^2} = \frac{\hbar}{\mathrm{m} \cdot c} = \frac{\lambda_C}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

The idea here is that one rotation – one *cycle* of the electron in its *Zitterbewegung* – packs the electron's energy (E = E = m·c^2) and – importantly – it also packs one unit of physical action (*S* = *h*). This idea may not be very familiar but it is quite simple: just re-write the Planck-Einstein relation as *h* = E·*f* = E/T. The cycle time T = *h*/E is equal to:

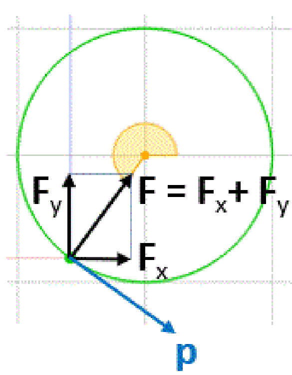
$$\mathrm{T} = \frac{h}{\mathrm{E}} \approx \frac{6.626 \times 10^{-34}\ \mathrm{J \cdot s}}{8.187 \times 10^{-14}\ \mathrm{J}} \approx 0.8 \times 10^{-20}\ \mathrm{s}$$

Hence, this cycle time T is the time it takes for the *zbw* charge (or the naked charge, if you prefer that term) to go around the loop (λ_C) at the extreme velocity we assume it has (*v* = *c*):

$$\mathrm{T} = \lambda_C/c = (h/\mathrm{m}c)\cdot(1/c) = h/\mathrm{E}$$

Figure 1 illustrates the model. We have a centripetal force (F) holding our *zbw* charge (the naked charge, which has zero rest mass) in its circular orbit around some center.

Figure 1: The *Zitterbewegung* model of an electron



[1] See: Jean Louis Van Belle, *The electron as a harmonic electromagnetic oscillator*, 1 June 2019, http://vixra.org/abs/1905.0521.

[2] See Alexander Burinskii's 2008, 2016 and 2017 publications.

Because the naked charge goes around at the speed of light (or *almost* the speed of light, as we will argue later), it acquires some mass which we'll denote as m_γ. We use the γ subscript here because it is just like a photon, which also acquires relativistic mass because of its extreme velocity. The only thing is that our *zbw* charge also has electric charge (all of the charge of the electron, in fact), which a photon doesn't have, of course! The point is: the zbw charge will also have some non-zero momentum $p = m_\gamma v = \gamma m_0 v = \gamma m_0 c$, even if m_0 (the rest mass of the naked charge) is zero.

Now, the angular momentum of the electron is equal to ħ/2 or some value very close to it.[3] We also know that angular momentum should be equal to the length of the lever arm (a) and the momentum $p = m_\gamma c$, so p is equal to $p = L/a$. It is useful to note that this formula – just like the others – is relativistically correct, so one should not cry wolf here. Hence, we get the following result:

1. $L = \hbar/2 \Leftrightarrow p = L/a = (\hbar/2)/a = (\hbar/2)\cdot mc/\hbar = mc/2$
2. $p = m_\gamma c$

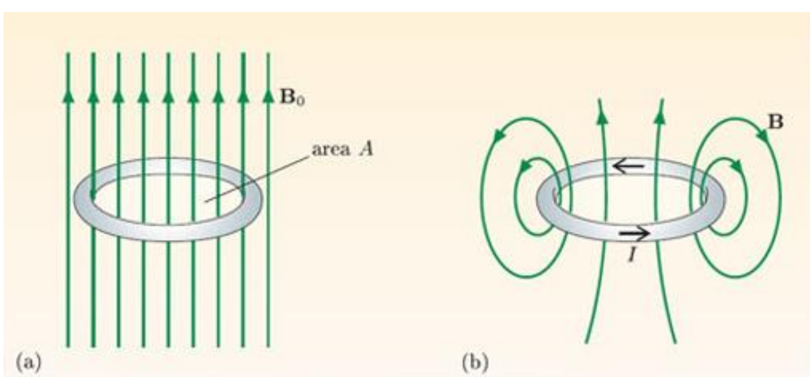
$\Rightarrow m_\gamma c = mc/2 \Leftrightarrow m_\gamma = m/2$

This is the grand result we expected to find: the *effective* mass of the pointlike charge – as it whizzes around the center of the two-dimensional oscillation that makes up our electron – is half of the (rest) mass of the electron. We interpreted this result in terms of a mathematical *equivalence* between the rotational motion and a two-dimensional oscillation—one perpendicular to (and, therefore, independent from) the other, each packing half of the total energy of the electron:

$$E_x = E_x = m_\gamma \cdot a^2 \cdot \omega^2 = m_\gamma \cdot c^2 = m \cdot a^2 \cdot \omega^2/2 = m \cdot c^2/2$$

Notation can be confusing here. E_x and E_y are often used to refer to the x- and y-component of the electric field vector (***E***), but that is not the case here: E_x and E_y is the *energy* (E) associated with the oscillation in the x- and y-direction respectively. To be precise, our model analyzes the electron pretty much like a perpetual current in a superconducting ring, as illustrated in Figure 2. Hence, the field is magnetic, rather than electric (in this particular reference frame, that is).

Figure 2: A perpetual current in a superconducting ring[4]



[3] The anomalous magnetic moment or – to be precise – the anomalous g-ratio suggest angular momentum or magnetic moment, or both, are slightly off.

[4] Source: Open University, Superconductivity, https://www.open.edu/openlearn/science-maths-technology/engineering-and-technology/engineering/superconductivity/content-section-2.2#.

This explains Hestenes' interpretation of the *zbw* model of an electron, which is equivalent to the oscillator model, and which he summarizes as follows:

> "The electron is nature's most fundamental superconducting current loop. Electron spin designates the orientation of the loop in space. The electron loop is a superconducting LC circuit. The mass of the electron is the energy in the electron's electromagnetic field."[5]

You may think this interpretation has a problem because we do not have any *real* material ring or wire in free space to hold and our guide our charge. However, the more advanced calculations of Hestenes (1990, 2008) and Burinskii (2008, 2016 and 2017) show that the scale and the magnitude of the force and the other variables don't require any wiring: Nature has tuned this LC circuit perfectly well.

We explored many interesting properties and implications of this model in the mentioned paper (we mentioned, most notably, that this allows for a realist interpretation of the wavefunction) but we won't repeat these here. We will want to focus on the intriguing possibility that the rest mass of our toroidal photon (the naked charge) may be *almost* zero rather than zero, and that its velocity may be *almost* the speed of light, but not exactly.

Before we do so, we would just like to make one small point on the energy density inside the loop. We do so because we said little or nothing about that in our previous analysis. Let us use the *metaphor* of that superconducting ring to say a few words about it here. Figure 2(a) above shows a uniform magnetic field going through that ring made of superconducting material. The idea then is that we then cool the ring below the critical temperature and switch off the field. Lenz's law – Faraday's law of induction, really – then tells us the *change* in the magnetic field (so that's us flipping the *kill* switch, basically) will induce an electromotive force. Hence, we get an *induced* electric current, and its direction and magnitude will be such that the magnetic flux it generates will compensate for the flux change: the induced current in the superconducting circuit will just maintain the flux through the ring at the same value. However, while the flux will the same, you should note that the field looks different now: in Figure 2(a), we have a *uniform* magnetic field within the ring – the field in our apparatus, really – while in Figure 2 (b) we have a field that's produced by the current flowing in the ring now. The new field gives us the same flux, but the field density is now much larger close to the ring, and the field density at the center is rather weak, even if the total flux has the same value.

Why is this important? It is important because we will probably want to know, at some point in the analysis, where the (field) energy is actually located. Why? Our $m_\gamma = m/2$ formula establishes an equivalence between:

1. The moment of inertia of a point mass m_γ at a distance $r = a$ from the axis of rotation: $I = m_\gamma \cdot a^2$.
2. The moment of inertia of a *disk* with radius r and mass m: $I = m \cdot a^2/2$.

Hence, we must show that – somehow – the energy (or *mass*[6]) effective mass of the electron will be spread over the disk. *If* we assume its energy – and, therefore, its mass – is spread uniformly over the whole disk[7], *then* we can use the 1/2 form factor for the moment of inertia (I). Hence, we conceptually

[5] Email from Dr. David Hestenes to the author dated 17 March 2019.

[6] Einstein's mass-energy equivalence relation – written as $E/m = c^2$ here – tells us that energy and mass are linearly proportional, and that the constant of proportionality is equal to c^2.

[7] This is a very essential point. It is also very deep and philosophical. We say the energy is in the motion, but it's also in the oscillation. It is difficult to capture this in a mathematical formula. In fact, we think this is the key paradox in the model.

distinguish the moment of inertia of the pointlike charge (I_γ) and the moment of inertia of our electron (I_γ), and we write:

$$(1)\ \mathrm{L} = I_\gamma \cdot \omega = \mathrm{m}_\gamma a^2 \cdot \frac{c}{a} = \frac{\mathrm{m}}{2} \cdot \frac{\hbar^2}{\mathrm{m}^2 c^2} \cdot \frac{\mathrm{m}c^2}{\hbar} = \frac{\hbar}{2}$$

$$(2)\ \mathrm{L} = I_\mathrm{e} \cdot \omega = \frac{\mathrm{m}a^2}{2} \cdot \frac{c}{a} = \frac{\mathrm{m}}{2} \cdot \frac{\hbar^2}{\mathrm{m}^2 c^2} \cdot \frac{\mathrm{m}c^2}{\hbar} = \frac{\hbar}{2}$$

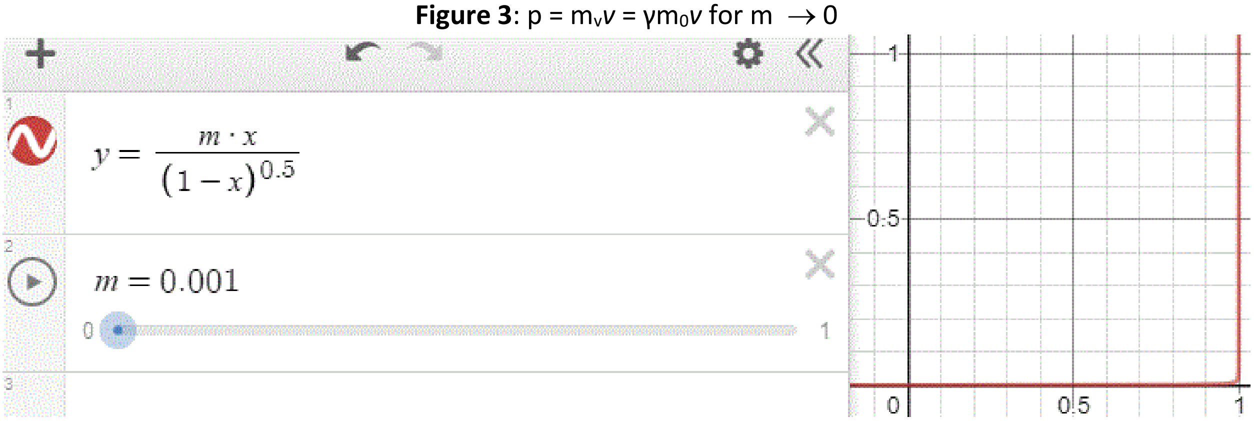
You may think this is rather obvious, but it isn't. It is a very deep and philosophical point. The energy is in the motion, but there is also energy in the magnetic field and we should, therefore, show how the magnetic energy is spread uniformly over the whole disk to validate the second of the two equations above. We haven't had the time to delve in this matter. The magnetic field becomes *weaker* as *r* goes to 0, and we know the energy *density* is proportional to the *square* of the magnetic field. Hence, if the magnetic field drops off as we move from the current ring to the center, we'd expect energy and, therefore, mass densities to *decrease* exponentially. This is a paradox which, hopefully, will not be too difficult to solve. We hope it's not a spoiler!

Let's move to the main topic of this paper.

The rest mass of the *zbw* charge

The *Zitterbewegung* model of an electron – or most *flavors* of that model, at least – assume the rest mass of the pointlike charge is zero. So why would we assume it would actually have some *very tiny* mass. The reason is the following: the $\mathrm{p} = \mathrm{m}_\gamma v = \gamma \mathrm{m}_0 v = \gamma \mathrm{m}_0 c$ involves the product of zero (m_0) and infinity (γ for $v = c$). Such product doesn't make sense – not mathematically, and not physically. To illustrate the issue, we used an online graphing tool (desmos.com) to illustrate what happens with the p = $\mathrm{m}_v v = \gamma \mathrm{m}_0 v$ function for m = 0.001 and v/c ranging between 0 and 1.

Figure 3: $\mathrm{p} = \mathrm{m}_v v = \gamma \mathrm{m}_0 v$ for $\mathrm{m} \to 0$



It is quite enlightening: p is (very close to) zero for v/c going from 0 to 1 but then becomes infinity at v/c = 1 itself. This is, obviously, not a regular function: we don't have a unique value for it at v/c = 1. What can we say about this? We think a particle that has some momentum should have some non-zero rest mass. Let us go through the math.

At first sight, the $\mathrm{m}_\gamma = \gamma \mathrm{m}_0 = \mathrm{m}/2$ is just like an $x \cdot y = \mathrm{k}$ relation: we have two variables (γ and m_0), and their product is some constant (m/2), so they are inversely proportional to each other. However, the relationship is, obviously, much more complicated. To be precise, the variables are not m_0 and γ but γ and *v*, or *v*/*c*. In fact, if we think of β = *v*/*c* as the variable, we may want to think of the other variable as some ratio between 0 and 1 too, so we can write it as m_0/m and re-write the equation accordingly. However, that doesn't help all that much. Let us try something else: if *v* is not equal to *c*, then it's actually the *radius* of that circular orbit that's going to change: $v = r \cdot \omega = r \cdot E/\hbar = r \cdot E/\hbar = r \cdot m \cdot c^2/\hbar$. Hence, we can write the relation as:

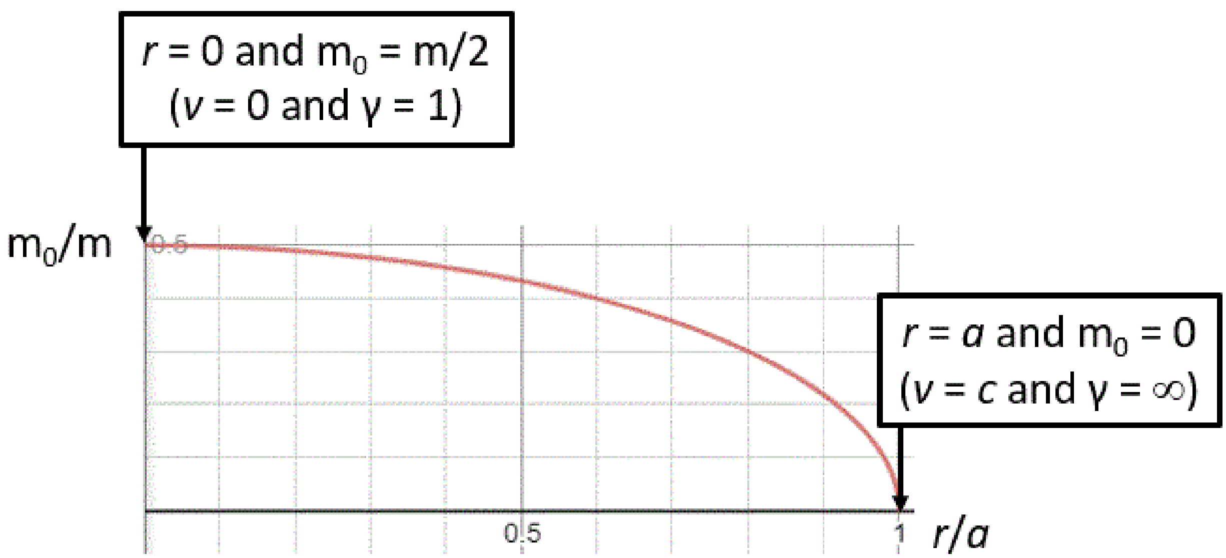
$$\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1-\frac{r^2 \cdot m^2 \cdot c^4}{c^2 \cdot \hbar^2}}} = \frac{m_0}{\sqrt{1-\frac{r^2}{a^2}}} = \frac{m}{2}$$

That's interesting because we can rewrite this as:

$$\frac{m_0}{\sqrt{1-\frac{r^2}{a^2}}} = \frac{m}{2} \Leftrightarrow 2 \cdot \left(\frac{m_0}{m}\right) = \sqrt{1-\left(\frac{r}{a}\right)^2}$$

This is a function that makes us think of the $y^2 = 1 - x^2$ relation for a circle except for the 1/2 factor, but then we should note that the m_0/m ratio will effectively vary between 0 and 1/2, as opposed to *r*/*a*, which will – just like the x in the $x^2 + y^2 = 1$ relation – vary between 0 and 1. We get the following graph:

Figure 4: The m_0/m ratio as a function of the *r*/*a* ratio



However, this nice graph still doesn't give us a good *second* fundamental relation that would solve the problem: what's the *actual* m_0/m ratio? Is it zero ($m_0 = 0$), 1/2 (($m_0 = m/2$) or some value in-between?

We will let this matter rest for a while (this sounds a bit funny in this context) and first explore why it would depend on the *r*/*a* ratio.

The dependence of the anomalous magnetic moment on the *zbw* radius

It is easy to show why the anomalous magnetic moment would depend on the *Zitterbewegung* radius. If we denote this radius by *r* (which may or may not be equal to $a = ħ/mc$), then the formula for the angular momentum becomes:

$$\mathrm{L} = I_{\mathrm{e}} \cdot \omega = \frac{\mathrm{m}r^2}{2} \cdot \frac{v}{r} = I_{\gamma} \cdot \omega = \mathrm{m}_{\gamma} r^2 \cdot \frac{v}{r} = \mathrm{m}_{\gamma} \cdot r \cdot v = \frac{\mathrm{m} \cdot r \cdot v}{2}$$

The m is, once again, the *rest* mass of the electron[8], so the formula is just the one we mentioned already. However, we substituted *c* for *v* and *a* for *r*. The idea here is that the angular frequency ω remains the same ($\omega = E/ħ = v/r$) because the rest mass (or rest energy) of the electron is what it is and, therefore, the radius *r* and *v* may be different from *a* and *c* but they are still related through the tangential velocity formula: $v = r\cdot\omega = r\cdot E/ħ = r\cdot m\cdot c^2/ħ$. Note that I_e and I_γ denote the moment of inertia of the electron and the *zbw* charge respectively.

To calculate the anomalous magnetic moment – which is actually an anomalous g-ratio[9] - we need the electric current $I = q_e\cdot\omega$. The current does *not* depend on *v* or *r*: qe is just the (naked) charge, and ω is the same angular frequency $\omega = E/ħ = v/r$. As mentioned, we assume *v* and *r* may vary but their *ratio* remains the same. The magnetic moment is equal to the current times the area of the loop and is, therefore, equal to:

$$\mu = \mathrm{I} \cdot \pi r^2 = \mathrm{q_e} \frac{\mathrm{m}c^2}{h} \cdot \pi r^2 = \mathrm{q_e} c \frac{\pi r^2}{2\pi a} = \mathrm{q_e} c \frac{r^2}{2a}$$

We substituted m*c*/h for $\lambda_c = 2\pi\cdot a$ in the formula above. For $a = r$, the formula simplifies to the one you know:

$$\mu = \mathrm{q_e} c \frac{\pi a^2}{2\pi a} = \frac{\mathrm{q_e} c}{2} \frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}} \hbar$$

However, we don't simplify here. Let us have a look at the formula for the g-ratio:

$$g_r = \frac{\mu_r}{\mathrm{L}_r} = \frac{\mathrm{I} \cdot \pi r^2}{\mathrm{m}_{\gamma} \cdot r \cdot v} = \frac{\mathrm{I} \cdot \pi \cdot \mathrm{r}}{\mathrm{m}_{\gamma} \cdot v}$$

What can we do with this? Nothing much. However, note that we introduced a subscript (g_r) to distinguish the *actual* value for *g* from its theoretical value, which we get from equating *r* to *a* and *v* to *c*:

$$g = \frac{\mu}{\mathrm{L}} = \frac{\mathrm{I} \cdot \pi a^2}{\mathrm{m}_{\gamma} \cdot a \cdot c} = \frac{\mathrm{q_e} \cdot c \cdot \frac{a^2}{2a}}{\mathrm{m} \cdot a \cdot c/2} = \frac{\mathrm{q_e}}{\mathrm{m}}$$

You will say this doesn't look like the *g*-factor for the pure spin moment, and you are right. The convention is to write the g-factor as a multiple of $q_e/2m$, so it is a pure number:

[8] We could write it with a subscript ($m = m_e$) but, for the sake of keeping the notation as simple as possible, we refrained from that.

[9] The gyromagnetic ratio is the ratio of the magnetic moment and the angular momentum. As mentioned, the anomalous magnetic moment is actually a misnomer. First, it is *not* a magnetic moment: it is the g-ratio. Second, as we try to show here, it may actually not be anomalous at all!

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g} = 2$$

We think this convention obscures the matter[10], so we'll just stick with our ratio – which is a real gyromagnetic ratio instead of some number – and let's see what happens. The anomaly is usually defined as the *difference* between real gyromagnetic ratio and the theoretical value ($g_r - g$) but we'll also write it as a ratio:

$$\frac{g_r}{g} = \frac{\dfrac{\mathrm{q_e} c \dfrac{r^2}{2a}}{\mathrm{m} \cdot r \cdot v/2}}{\dfrac{\mathrm{q_e} c \dfrac{a^2}{2a}}{\mathrm{m} \cdot a \cdot c/2}} = \frac{r^2}{a^2}\frac{a}{r} = \frac{r}{a}$$

This is a wonderful result: the anomaly is just the ratio between the *actual* or *effective* Zitterbewegung radius and its theoretical value. We can write it very simply:

$$g_r = (r/a)\cdot g$$

We know Schwinger's first-order value for the anomaly is $\alpha/2\pi \approx 0.00116141$. We also know and we know – *from experiments that measure this g-ratio* – that this first-order correction explains 99.85% of the anomaly. The second-, third-, or n^{th}-order corrections that one gets only need to explain 0.15%.

The α in the formula is the fine-structure constant ($\alpha \approx 1/137$), and it also relates the Compton radius to the Thomson radius. The Thomson radius is the classical electron radius: $r_e = \alpha \cdot a \approx a/137 \approx 2.818 \times 10^{-15}$ m. We get this radius from *elastic* scattering experiments. They are referred to as elastic because the photon seems to bounce off some hard *core*: there is no interference. In contrast, Compton scattering is usually explained by some electron-photon interference. It involves high-energy photons (the light is X- or gamma-rays) whose energy will be briefly absorbed before the electron comes back to its equilibrium situation by emitting another photon. The wavelength of the emitted photon will be longer. The photon has, therefore, less energy, and the difference in the energy of the incoming and the outgoing photon gives the electron some linear momentum.

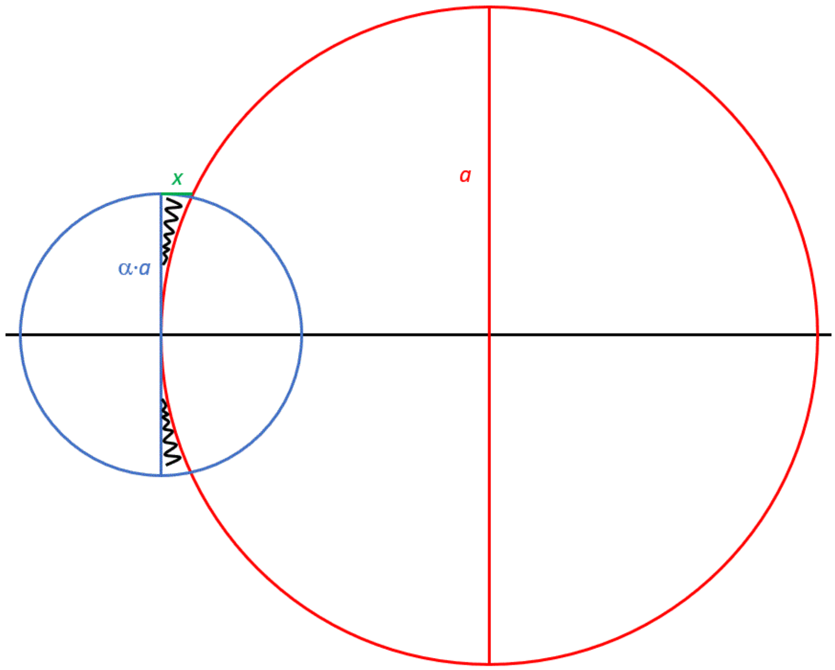
This picture is fully consistent with the *Zitterbewegung* model of an electron: the hard core is just the pointlike charge itself. It is, effectively, pointlike (10^{-15} m is the *femto*meter scale) but, as we can see, pointlike does not mean dimensionless. So what is going on here, and how can we explain Schwinger's $\alpha/2\pi$ factor for the anomaly?

A classical explanation for the anomaly?

Figure 5 is not to scale but illustrates the geometry of the situation. We think of the naked charge as a charged sphere with radius $\alpha \cdot a$ moving in a circular orbit with radius $r = a$.

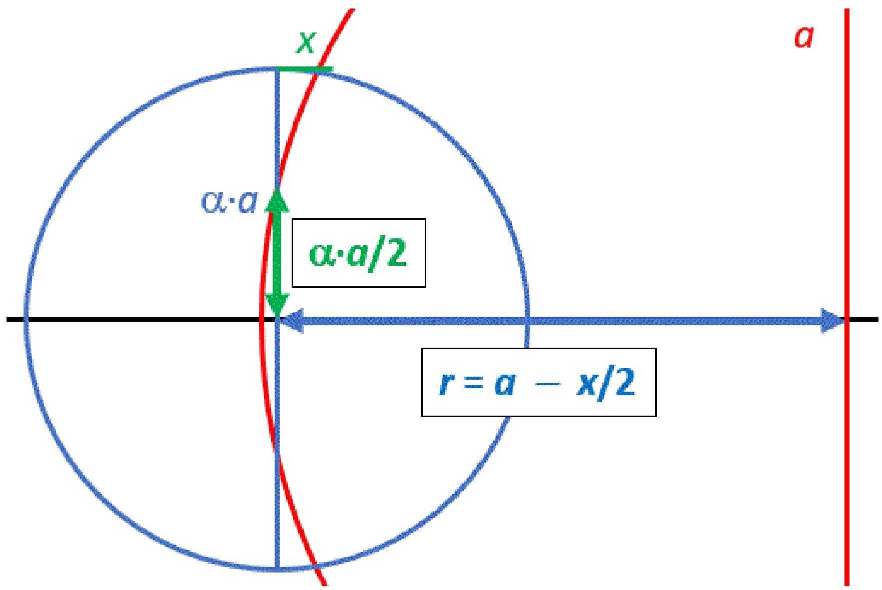
[10] See: Jean Louis Van Belle, *The Not-So-Anomalous Magnetic Moment*, 21 December 2018 (http://vixra.org/abs/1812.0233).

Figure 5: Geometry of *zbw* charge and electron (1)



The points in the two triangular areas will move at a velocity *v* which is slightly higher than $c = a \cdot \omega$. Hence, the effective *center* of charge is slightly changed. If we want the charged sphere – on average and as a whole – to move around the center at the speed of light (c), then we have to reduce the effective *zbw* radius somewhat. This correction is approximated by the distance $x/2$ in Figure 6.

Figure 6: Geometry of *zbw* charge and electron (2)



All that remains to be done is to prove that the correction is equal (or not) to $\alpha/2\pi$. That should not be so difficult using the formula for the length of an arc ($L = \theta \cdot r$) but, as yet, we have not been able to figure this out.

References

This paper discusses general principles in physics only. Hence, references were mostly limited to references to general physics textbooks. For ease of reference – and because most readers will be familiar with it – we often opted to refer to:

1. Feynman's *Lectures* on Physics (http://www.feynmanlectures.caltech.edu). References for this source are per volume, per chapter and per section. For example, Feynman III-19-3 refers to Volume III, Chapter 19, Section 3.

One should also mention the rather delightful set of Alix Mautner Lectures, although we are not so impressed with their transcription by Ralph Leighton:

2. Richard Feynman, *The Strange Theory of Light and Matter*, Princeton University Press, 1985

Specific references – in particular those to the mainstream literature in regard to Schrödinger's *Zitterbewegung* – were mentioned in the footnotes. We should single out the various publications of David Hestenes and Francesco Celani:

3. David Hestenes, Found. Physics., Vol. 20, No. 10, (1990) 1213–1232, *The Zitterbewegung Interpretation of Quantum Mechanics*, http://geocalc.clas.asu.edu/pdf/ZBW_I_QM.pdf.
4. David Hestenes, 19 February 2008, *Zitterbewegung in Quantum Mechanics – a research program*, https://arxiv.org/pdf/0802.2728.pdf.
5. Francesco Celani et al., *The Electron and Occam's Razor*, November 2017, https://www.researchgate.net/publication/320274514_The_Electron_and_Occam's_Razor.

We would like to mention the work of Stefano Frabboni, Reggio Emilia, Gian Carlo Gazzadi, and Giulio Pozzi, as reported on the phys.org site (https://phys.org/news/2011-01-which-way-detector-mystery-double-slit.html). In addition, it is always useful to read an original:

6. Paul A.M. Dirac, 12 December 1933, Nobel Lecture, *Theory of Electrons and Positrons*, https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf

We should, perhaps, also mention the following critical appraisal of the quantum-mechanical framework:

7. *How to understand quantum mechanics* (2017) from John P. Ralston, Professor of Physics and Astronomy at the University of Texas.

It is one of a very rare number of exceptional books that address the honest questions of amateur physicists and philosophers upfront. We love the self-criticism: "Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research." (p. 1-10)

Finally, I would like to thank Prof. Dr. Alex Burinskii for taking me seriously. He is in a different realm – and he has made it clear that my writings are extremely simplistic and probably serve pedagogic purposes only. However, his confirmation that I am not making any *fundamental* mistakes while trying to understand the fundamentals, have kept me going on this. We refer to his publications (Burinskii, 2008, 2016, 2017) in the body of our paper itself.