Refutation of BF calculus (and square root of negation)

Abstract: We evaluate the eight defining equations of the Spencer-Brown system. None is tautologous. This refutes the subsequent primary arithmetic renamed as BF calculus. We previously refuted the Dunn-Belnap 4-valued bilattice as *not* bivalent and thus non tautologous, so to draw in refinements and extensions by others and apply BF to it compounds the mistakes. Further producing a square root operation on negative 1 is also *not* tautologous. Spencer-Brown and BF systems subsequently form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Kauffman, L.H.; Collings, A.M. (2019). The BF calculus and the square root of negation. arxiv.org/pdf/1905.12891.pdf kauffman@uic.edu, otter@mac.com

Comment: If the "otter" email prefix above implies use of the Prover9(nee Otter) proof assistant by the authors, we show elsewhere that assistant is *not* bivalent.

II. Laws of form

A. *Distinction*

Laws of Form by Spencer-Brown [..], and the Primary Algebra (PA) it describes, is based on the idea of *distinction*, represented by the dividing of a space into two regions, one *marked* , the second *unmarked* . In *Laws of Form*, the mark ⃧ indicates the *marked state*, and the empty value " " [or Not(⃧)] indicates the *unmarked state*. The step of representing a value by an empty space, by the lack of a sign, is motivated by a key idea: doing so permits the mark ⃧ to act both as the name of a value and as an operation.

The Mark as an Operation

I⃧= O, O⃧= I, I inside circle, and O outside circle

Fig. 1. Representing a Distinction between Inside (I) and Outside (O)

Consider Figure 1, in which we have drawn a closed circle, creating a distinction between inside, I, and outside O. We regard the mark ⏋ as an operator that takes I to O and O to I. Then we observe the following:

I⌉= O,   O⌉= I,   (1.1.1, 1.2.1)

LET p, q, r, s: I, O, r, ⌉ or ⏋.

(p&s)=q ;   TT**FF** TT**FF** T**FF**T T**FF**T   (1.1.2)
(q&s)=p ;   T**F**T**F** T**F**T**F** T**FF**T T**FF**T   (1.2.2)

I⌉⏋ = O⌉= I,   O⌉⏋ = I⌉= O,   (2.1.1, 2.2.1)

(((p&s)&s)=(q&s))=p ;   **F**T**F**T **F**T**F**T **FF**TT **FF**TT   (2.1.2)
(((q&s)&s)=(p&s))=q ;   **FF**TT **FF**T**T** **F**T**F**T **F**T**F**T   (2.2.2)

so for any state X we have X⌉⏋ = X.   (2.3.1)

Remark 2.3.1: Eq. 2.3.1 is a trivial tautology, for which also see below at 5.1.1.

The conceptual shift is to designate the inside to be unmarked (literally to have no symbol), so that

I = “ ” .   (2.4.1)

p=~s ;   **F**T**F**T **F**T**F**T T**F**T**F** T**F**T**F**   (2.4.2)

Then from (1) we obtain

⏋ = O,   O⌉ = ,   (3.1.1)

(((p&s)=q)&((q&s)=p))>((s=q)&((q&s)=~s)) ;
FTTT **F**TTT **F**TT**F** **F**TT**F**   (3.1.2)

which means we have equated the mark ⏋ with the outside O.   (3.2.1)

((((p&s)=q)&((q&s)=p))>((s=q)&((q&s)=~s)))>(s=q) ;
TT**FF** TT**FF** T**F**TT T**F**TT   (3.2.2)

From (2), we obtain

⌉⏋ = “ ”   (4.1.1)

(((((p&s)&s)=(q&s))=p)&((((q&s)&s)=(p&s))=q))>((s&s)=~s);
TTT**F** TTT**F** TTT**F** TTT**F**   (4.1.2)

By identifying the value of the outside with the result of crossing from the unmarked inside, Spencer-Brown has introduced a multiplicity meanings to the mark. The statement ⏋=⏋ can be interpreted on the left side to mean “cross from the inside” and on the right as “the name of the outside”.

The mark itself can be seen to divide its surrounding space into an inside and an outside. When we write ⏋ =⏋ , the two marks are positioned mutually outside each other, and we can choose to interpret either mark as a name that refers to the outside of the other. We may also interpret two such juxtaposed marks to indicate successive naming of the state indicated by the mark. In either case we can take as an instance of the principle that to repeat a name can be identified with a single calling of the name:

$$\urcorner \urcorner = \urcorner . \tag{5.1.1}$$

Remark 5.1.1: Eq. 5.1.1 is a trivial tautology.

At this point we have a single sign $\urcorner$ representing both the operation of crossing the boundary of a distinction and representing the name of the outside of that distinction. Furthermore, since the mark itself can be seen to make a distinction in its own space, the mark can be regarded as referent to itself and to the (outer side) of the distinction that it makes. The two equations (4) and (5) represent these aspects of understanding a distinction and the signs that can represent this distinction. We will now see that the two equations and a natural formalism for expressions in the mark become a formal system that can be seen as an 'arithmetic' for Boolean algebra.

B. The Primary Arithmetic
On the basis of these considerations, Spencer-Brown defines a very simple calculus, which he calls the *Primary Arithmetic*. …

We evaluated the eight defining equations of the Spencer-Brown system. None is tautologous. This refutes the subsequent primary arithmetic renamed as BF calculus. We previously refuted the Dunn-Belnap 4-valued bilattice as *not* bivalent and thus non tautologous, so to draw in refinements and extensions on it by by others and apply BF to it compounds the mistakes. By further producing a square root operation on negative 1 is also *not* tautologous.