Refutation of the Schaefer theorem for the P, NP problem (undecided)

Abstract: We evaluate the Schaefer theorem for the P, NP problem by two examples for Graph-SAT(Ψ). Neither example is tautologous; while claimed to be different, they result in the same truth table values. (The injection of NP-intermediate does not describe our result.) This refutes NP-complete (and P, NP, NP-hard). We also evaluate the P, NP problem as based on P≤NP with the same result. Therefore P, NP, NP-complete, NP-hard, NP-intermediate form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , ·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Manuel Bodirsky, M.; Pinsker, M. (2019). Schaefer's theorem for graphs. cs.umd.edu/users/gasarch/TOPICS/ramsey/schaefergraph.pdf

Graph-SAT(Ψ)

… As an example, let Ψ be the set that just contains the formula

$$(E(x,y)\wedge\neg E(y,z)\wedge\neg E(x,z))\vee$$
$$(\neg E(x,y)\wedge E(y,z)\wedge\neg E(x,z))\vee$$
$$(\neg E(x,y)\wedge\neg E(y,z)\wedge E(x,z)). \quad (1.1)$$

LET p, q, r: *E* (*x,y*), *E* (*y,z*), *E* (*x,z*)

(((p&~q)&~r)+((~p& q)&~r))+((~p&~q)& r) ;
FTT**F** T**FFF** **F**TT**F** T**FFF** (1.2)

Remark 1.2: Eq. 1.2 as rendered is *not* tautologous. This means the claim that 1.1 is P, NP, NP-Complete (NPC), or NP-Hard (NPH) is refuted. We note that the injection of NP-Intermediate (NPI) does not describe "*not* tautologous". What follows is that the P, NP problem cannot be decided as *not* tautologous. (In this regard, see Remark 6.)

Then Graph-SAT(Ψ) is the problem of deciding whether there exists a graph such that certain prescribed subsets of its vertex set of cardinality at most three induce subgraphs with exactly one edge. This problem is NP-complete (the curious reader can check this by means of our classification in Theorem 17). There are also many interesting tractable Graph-SAT problems, for instance when Ψ consists of the formulas

$$
\begin{aligned}
&x{\neq}y \vee y{=}z \text{ and}\\
&(\ \ E(x,y)\wedge\neg E(y,z)\wedge\neg E(x,z))\vee\\
&(\neg E(x,y)\wedge\ \ E(y,z)\wedge\neg E(x,z))\vee\\
&(\neg E(x,y)\wedge\neg E(y,z)\wedge\ \ E(x,z))\vee\\
&(\ \ E(x,y)\wedge\ \ E(y,z)\wedge\ \ E(x,z)). \qquad (2.1)
\end{aligned}
$$

((p@q)+(q+r)) & (((((p&~q)&~r)+((~p& q)&~r))+((~p&~q)& r))+((p& q)& r)) ;
FTTF TFFF FTTF TFFF (2.2)

Remark 2.2: Eq. 2.2 is *not* tautologous and is equivalent by truth table value result to Eq. 1.2. This means 1.2 and 2.2 are not NP-complete as claimed.

It is obvious that the problem Graph-SAT(Ψ) is for all Ψ contained in NP. (3.0)

Remark 3.0: Eq. 2.2 refutes conclusion 3.0.

The goal of this paper is to prove the following dichotomy result.

Theorem 1. *For all* Ψ*, the problem* Graph-SAT(Ψ) *is either NP-complete or in P*. (4.0)

Remark 4.0: Eq. 4.0 cannot be asserted because 3.0 is refuted by 2.2.

One of the main contributions of the paper is the general method of combining concepts from universal algebra and model theory, which allows us to use deep results from Ramsey theory to obtain the classification result. (5.0)

Remark 5.0: We show elsewhere that Ramsey's theorem is *not* tautologous.

Remark 6.0: As an example, see en.wikipedia.org/wiki/P_versus_NP_problem .

Clearly, P $\leq$ NP (per the link above). (6.0.1)

LET p, q: P, NP.

~(q<p)=(p=p) ; FTTF FTTF FTTF FTTF (6.0.2)

We ask: If (1.1), then (p=q)? (6.1.1)

~(q<p)>(p=q) ; TFTT TFTT TFTT TFTT (6.1.2)

We ask: If (1.1), then NOT(p=q)? (6.2.1)

~(q<p)>~(p=q) ; FTTF FTTF FTTF FTTF (6.2.2)

Eqs. 6.0.2, 6.1.2, and 6.2.2 are *not* tautologous (but *not* contradictory either). This means the P, NP problem as based on P $\leq$ NP can not be decided.