Refutation of modal logics that bound the circumference of transitive frames

Abstract: We evaluate six seminal equations, none of which is tautologous. (The author mistakenly labels the Löb axiom as a "fact" as proved by another author.) Therefore modal logics bounding the circumference of transitive frames is refuted and becomes another *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Goldblatt, R. (2019). Modal logics that bound the circumference of transitive frames. arxiv.org/pdf/1905.11617.pdf

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[Definition of strict implication] is □(ϕ→ψ) (2.0.1)

LET p, q: ϕ, ψ.

(p>q)>#(p>q) ; NTNN NTNN NTNN NTNN (2.0.2)

Remark 2.1: Eq. 2.1, not shown here, is missing a leading left parentheses.

This is the Löb axiom, referred to as a "fact": □(□p→p)→□p (2.3.1)

#(#p>p)>#p ; CTCT CTCT CTCT CTCT (2.3.2)

Remark 2.3.2: Eq. 2.3.2 is *not* tautologous, and hence not factual.

To indicate the nature of the axioms C_n, we indicate first that C_0 is equivalent over all frames to the formula C_0 is equivalent over all frames to the formula

◊p→◊(p∧¬◊p), (C_0.1)

%p>%(p&~%p) ; TCTC TCTC TCTC TCTC (C_0.2)

which is itself equivalent to the Löb axiom (2.3.1): (C_0.1)=(2.3.1)

(%p>%(p&~%p))=(#(#p>p)>#p) ;
CCCC CCCC CCCC CCCC (C$_0$.2)=(2.3.2)

Remark C$_0$: Eqs. C$_0$.2 and 2.3.2 are *not* tautologous and *no*t equivalent as claimed.

Remark 2.4, 2.5, 5.1: The subsequent various combinations of C$_1$ with Eqs. 2.4, 2.5, 5.1, and others form self-evident tautologies, ignored as trivial.

6 Extensions of K4C$_n$; Linearity

K4.3 is the smallest normal extension of K4 that includes the scheme $(\phi\wedge\phi\rightarrow\psi)\vee(\psi\wedge\psi\rightarrow\phi)$. (6.1.1)

#((p&#p)>q)+#((q&#q)>p) ; NNNN NNNN NNNN NNNN (6.1.2)

Remark 6.1.2: Eq. 6.1.2 is *no*t tautologous, but rather a *truthity*.

The canonical frame of any normal extension of K4.3 is weakly connected. If a transitive weakly connected frame is *point-generated*, i.e. $W=\{x\}\cup\{y\in W:xRy\}$ for some point $x\in W$, then the frame is *connected*: it satisfies

$$\forall y\forall z(yRz\vee y=z\vee zRy). \quad (6.2.1)$$

LET p, q, r, s: x, y, R, z

((#q&(r&#s))+#q)=(#s+(#s&(r&#q))) ;
TTCC TTCC CCTT CCTT (6.2.2)

Such a connected frame can be viewed as a linearly ordered set of clusters.

Remark 6.2.2: Eq. 6.2.2 is *not* tautologous and hence *not* a linearly ordered set of clusters as claimed.

The six equations evaluated above are *not* tautologous, hence refuting the conjecture that modal logics bound the circumference of transitive frames. The author makes a serious mistake in labeling the Löb axiom as a fact, relying on Segerberg.