

# Refutation of Church's thesis as a consistency property to fulfill the minimalist foundation

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**Abstract:** Church's thesis (CT) is *not* tautologous as an essential consistency property to fulfill the requirement of the intensional level of a constructive foundation proposed of the minimalist foundation (MF) for constructive mathematics. Therefore, this relegates CT and MF to a *non* tautologous fragment of the universal logic VL4.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬; + Or, ∨, ∪, ∪; - Not Or; & And, ∧, ∩, ∩, ∩; \ Not And;  
 > Imply, greater than, →, ⇒, ⇨, >, ⊃, ≻; < Not Imply, less than, ∈, <, ⊂, ⊆, ⊆, ⊆, ⊆;  
 = Equivalent, ≡, :=, ⇔, ↔, ≅, ≈, ≅; @ Not Equivalent, ≠;  
 % possibility, for one or some, ∃, ∃, M; # necessity, for every or all, ∀, □, L;  
 (z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;  
 (%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;  
 ~(y < x) (x ≤ y), (x ⊆ y), (x ⊆ y); (A=B) (A~B).  
 Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Maietti, M.E.; Maschio, S.; Rathjen, M. (2019). [arxiv.org/pdf/1905.11966.pdf](https://arxiv.org/pdf/1905.11966.pdf)  
 A realizability semantics for inductive formal topologies, Church's thesis and axiom of choice.

Church's thesis (CT) ... states that from any total relation on natural numbers we can extract a (code of a) recursive function by using the Kleene predicate T and the extracting function U

$$(CT) (\forall x \in \mathbb{N})(\exists y \in \mathbb{N})R(x,y) \rightarrow (\exists e \in \mathbb{N})(\forall x \in \mathbb{N})(\exists z \in \mathbb{N})(T(e,x,z) \wedge R(x,U(z))) \quad (1.1)$$

LET r, s, t, u, w, x, y, z:  
 R, N, t, U, e, x, y, z.

$$\begin{aligned} & (((\#x < s) \& (\%y < s)) \& (r \& (x \& y))) > \\ & ((\%w < s) \& ((\#x < s) \& (\%z < s))) \& ((t \& (w \& (x \& z))) \& ((r \& x) \& (u \& z))) ; \\ & \quad \text{TTTT TTTT TTTT TTTT (48)} \\ & \quad \text{TTTT CCCC TTTT TTTT (16)} \\ & \quad \text{TTTT TTTT TTTT TTTT (48)} \\ & \quad \text{TTTT CCCC TTTT TTTT ( 8)} \\ & \quad \text{TTTT CCCC TTTT TTTT ( 3) } \times 2 \\ & \quad \text{TTTT TTTT TTTT TTTT ( 1) } \end{aligned} \quad (1.2)$$

Such a consistency property is essential to fulfill the requirement of the intensional level of a constructive foundation proposed [toward a minimalist foundation for constructive mathematics].

Eq. 1.2 as rendered is *not* tautologous, to refute Church's thesis as an essential consistency property to fulfill the requirement of the intensional level of a constructive foundation proposed of the minimalist foundation (MF) for constructive mathematics.