

## Refutation of Lusin's separation theorem

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**Abstract:** “In descriptive set theory and mathematical logic, **Lusin's separation theorem** states that if  $A$  and  $B$  are disjoint analytic subsets of Polish space, then there is a Borel set  $C$  in the space such that  $A \subseteq C$  and  $B \cap C = \emptyset$ .” We evaluate two renditions of that equation, both *non* tautologous, refuting it. Therefore, the separation theorem of Lusin forms a *non* tautologous fragment of the universal logic  $\forall\mathcal{L}4$ .

We assume the method and apparatus of Meth8/ $\forall\mathcal{L}4$  with Tautology as the designated proof value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET  $\sim$  Not,  $\neg$ ; + Or,  $\vee$ ,  $\cup$ ,  $\sqcup$ ; - Not Or; & And,  $\wedge$ ,  $\cap$ ,  $\square$ ,  $;$ ; \ Not And;  
 $>$  Imply, greater than,  $\rightarrow$ ,  $\Rightarrow$ ,  $\mapsto$ ,  $>$ ,  $\supset$ ,  $\succ$ ;  $<$  Not Imply, less than,  $\in$ ,  $<$ ,  $\subset$ ,  $\prec$ ,  $\preceq$ ,  $\leq$ ;  
 $=$  Equivalent,  $\equiv$ ,  $:=$ ,  $\Leftrightarrow$ ,  $\leftrightarrow$ ,  $\hat{=}$ ,  $\approx$ ,  $\simeq$ ; @ Not Equivalent,  $\neq$ ;  
 $\%$  possibility, for one or some,  $\exists$ ,  $\diamond$ , **M**; # necessity, for every or all,  $\forall$ ,  $\square$ , **L**;  
 $(z=z)$  **T** as tautology,  $\top$ , ordinal 3;  $(z@z)$  **F** as contradiction,  $\emptyset$ , Null,  $\perp$ , zero;  
 $(\%z\>\#z)$  **N** as non-contingency,  $\Delta$ , ordinal 1;  $(\%z\<\#z)$  **C** as contingency,  $\nabla$ , ordinal 2;  
 $\sim(y < x)$  ( $x \leq y$ ), ( $x \subseteq y$ ), ( $x \sqsubseteq y$ );  $(A=B)$   $(A\sim B)$ .  
 Note for clarity, we usually distribute quantifiers onto each designated variable.

From: [en.wikipedia.org/wiki/Lusin%27s\\_separation\\_theorem](https://en.wikipedia.org/wiki/Lusin%27s_separation_theorem)

In descriptive set theory and mathematical logic, **Lusin's separation theorem** states that if  $A$  and  $B$  are disjoint analytic subsets of Polish space, then there is a Borel set  $C$  in the space such that

$$A \subseteq C \text{ and } B \cap C = \emptyset.[..] \tag{1.1}$$

LET  $p, q, r, s:$   $A, B, C, D$

$$(\sim(r < p) \& (q \& r)) = (s @ s); \quad \text{TTTT TTTF TTTT TTTF} \tag{1.2}$$

**Remark 1.1:** If Eq. 1.1 is rendered in theorem variables, then

$$(\sim(C < A) \& (B \& C)) = (D @ D); \quad \begin{array}{l} \text{TTTT TTTT TTTT TTTT,} \\ \text{TTTT TNTN TTTT TNTN,} \\ \text{TTTT TTTT TTCC TTCC,} \\ \text{TTTT TNTN TTCC TNCF} \end{array} \tag{1.3}$$

Eqs. 1.2 and 1.3 are *not* tautologous, thereby refuting Lusin's separation theorem.