Refutation of Gobbay's separation theorem

Abstract: The separation theorem of Gobbay takes eight basic cases and four cases for disjunction. None is tautologous. In fact, three groups of the basic cases share unique truth table result values, and one group of the disjunctive cases shares the same truth table result values. This refutes the theorem and adds it as another *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Gabbay%27s_separation_theorem [from footnote source below]

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formula until there are no nestings of $\mathcal{U}^+$ within $\mathcal{S}^-$ and vice versa. For $\mathcal{U}^+$ within $\mathcal{S}^-$, there are eight basic cases to handle. Let φ and ψ stand for arbitrary formulae and letters a and b, etc., denote propositional atoms. The eight cases are:-

1.	$\varphi\mathcal{S}^-(\psi\wedge a\mathcal{U}^+b)$	2.	$\varphi\mathcal{S}^-(\psi\wedge\neg(a\mathcal{U}^+b))$
3.	$(\varphi\vee a\mathcal{U}^+b)\mathcal{S}^-\psi$	4.	$(\varphi\vee\neg(a\mathcal{U}^+b))\mathcal{S}^-\psi$
5.	$(\varphi\vee a\mathcal{U}^+b)\mathcal{S}^-(\psi\wedge a\mathcal{U}^+b)$	6.	$(\varphi\vee\neg(a\mathcal{U}^+b))\mathcal{S}^-(\psi\wedge a\mathcal{U}^+b)$
7.	$(\varphi\vee a\mathcal{U}^+b)\mathcal{S}^-(\psi\wedge\neg(a\mathcal{U}^+b))$	8.	$(\varphi\vee\neg(a\mathcal{U}^+b))\mathcal{S}^-(\psi\wedge\neg(a\mathcal{U}^+b))$

Other nested $\mathcal{U}^+$ forms reduce to one of the 8 schema for atomic $\mathcal{U}^+$ formula. For example, consider $\varphi\mathcal{S}^-(a\mathcal{U}^+(p\mathcal{U}^+q))$, however, this can be viewed as a formula of shape 1 above. Replace the sub-formula $p\mathcal{U}^+q$ by pq say, and note that the formula ψ in 1 is `true`.

For each of the above shapes, one can provide an equivalent formula of form $E_1\vee E_2\vee E_3$ where each E_i is a boolean combination of pure past, present and pure future formulae. An inductive proof can then establish that separation can occur for all formulae.

In the following we establish the first of the above eliminations. Let $E\overset{def}{=}\varphi\mathcal{S}^-(\psi\wedge a\mathcal{U}^+b)$. We can write E as the disjunction of $E_1E_2E_3$ such that the E_i contain no nested $\mathcal{U}^+$, in fact in a separated form,

$$\models E\Leftrightarrow E_1\vee E_2\vee E_3$$

In order to construct the formulae E_i, consider a model for $\varphi\mathcal{S}^-(\psi\wedge a\mathcal{U}^+b)$.

$$E_1 \;:\; y<n \;:\; \varphi\mathcal{S}^-(b\wedge\varphi\wedge(\varphi\wedge a)\mathcal{S}^-\psi)$$
$$E_2 \;:\; y=n \;:\; b\wedge(\varphi\wedge a)\mathcal{S}^-\psi$$
$$E_3 \;:\; y>n \;:\; a\mathcal{U}^+b\wedge a\wedge(\varphi\wedge a)\mathcal{S}^-\psi$$

In mathematical logic and computer science, Gabbay's separation theorem, named after Dov Gabbay, states that any arbitrary temporal logic formula can be rewritten in a logically equivalent "past → future" form. I.e. the future becomes what must be satisfied.[1]

[1] Fisher, M.; Gabbay, D.; Vila, L. Eds. (2005). Handbook of temporal reasoning in artificial intelligence. Foundations of artificial intelligence. 1. Elsevier. [See image above.]

LET p, q, s, u, x, y: φ, ψ, S^-, U^+, a, b.

```
(p&s)&(q& (x&(u&y))) ;     FFFF FFFF FFFT FFFT(48)
                           FFFF FFFF FFFF FFFF( 2)}x4
                           FFFF FFFF FFFT FFFT( 2)}     (1.2)

(p&s)&(q&~(x&(u&y))) ;     FFFF FFFF FFFT FFFT(48)
                           FFFF FFFF FFFT FFFT( 2)}x4
                           FFFF FFFF FFFF FFFF( 2)}     (2.2)

(p& (x&(u&y)))&(s&q) ;     FFFF FFFF FFFT FFFT(48)
                           FFFF FFFF FFFF FFFF( 2)}x4
                           FFFF FFFF FFFT FFFT( 2)}     (3.2)

(p&~(x&(u&y)))&(s&q) ;     FFFF FFFF FFFT FFFT(48)
                           FFFF FFFF FFFT FFFT( 2)}x4
                           FFFF FFFF FFFF FFFF( 2)}     (4.2)

(p& (x&(u&y)))&(s&(q& (x&(u&y)))) ;
                           FFFF FFFF FFFT FFFT(48)
                           FFFF FFFF FFFF FFFF( 2)}x4
                           FFFF FFFF FFFT FFFT( 2)}     (5.2)

(p&~(x&(u&y)))&(s&(q& (x&(u&y)))) ;
                           FFFF FFFF FFFF FFFF          (6.2)

(p& (x&(u&y)))&(s&(q&~(x&(u&y)))) ;
                           FFFF FFFF FFFF FFFF          (7.2)

(p&~(x&(u&y)))&(s&(q&~(x&(u&y)))) ;
                           FFFF FFFF FFFT FFFT(48)
                           FFFF FFFF FFFT FFFT( 2)}x4
                           FFFF FFFF FFFF FFFF( 2)}     (8.2)
```

Remark 1.2-8.2: Eqs. 1.2-8.2 are *not* tautologous. In fact, the following groupings have identical truth table result values with abbreviated differences:

2.2, 4.2, 8.2:	FFFF, FFFT, FFFF
1.2, 3.2, 5.2:	FFFT, FFFF, FFFT
6.2, 7.2:	FFFF

This refutes the eight *different* basic cases to handle in Gabbay's separation result.

E: def [see image above] (E0.1)

(p&s)&((q&x)&(u&y)) ; **FFFF FFFF FFFF FFFF** (E0.2)

E_1: y<n (E1.1)

(p&s)&((y&p)&((p&x)&(s&q))) ;

```
FFFF FFFF FFFF FFFF(48)
FFFF FFFF FFFT FFFT(16)
```
(E1.2)

E_2: y=n (E2.1)

y&((p&x)&(s&q)) ;

```
FFFF FFFF FFFF FFFF(48)
FFFF FFFF FFFT FFFT(16)
```
(E2.2)

E_3: y>n (E3.1)

(x&(u&y))&(x&((p&x)&(s&q))) ;

```
FFFF FFFF FFFF FFFF(48)
FFFF FFFF FFFF FFFF( 2)}x4
FFFF FFFF FFFT FFFT( 2)}
```
(E3.2)

$E = E_1 + E_2 + E_3$: Disjunctions (E4.1)

((p&s)&((q&x)&(u&y)))=(((p&s)&((y&p)&((p&x)&(s&q))))+
((y&((p&x)&(s&q)))+((x&(u&y))&(x&((p&x)&(s&q)))))) ;

```
TTTT TTTT TTTT TTTT(48)
TTTT TTTT TTTF TTTF( 2)}x4
TTTT TTTT TTTT TTTT( 2)}
```
(E4.2)

Remark E1-E4: Eqs. E1.2-E4.2 are *not* tautologous. In fact, the following grouping has identical truth table result values with abbreviated differences:

E1.2, E2.2: **FFFF, FFF**T

This refutes the disjunction equations of the model for Gobbay's separation theorem.