Electron Proton Escape Velocity from Electrical Attraction Produces a Dark Matter Candidate

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I. INTRODUCTION

The escape velocity between two opposite electrical charges can be treated in a similar fashion as the escape velocity of two gravitational masses. For a given positive and negative charge, one can ask at what radial distance would a particle need to travel at the speed of light in order to escape the electrical attraction towards the other charged mass. In an electron-proton system this distance is found to be 5.62×10^{-15} meters, which is several times larger than the proton radius. It is proposed that once inside this radius, where it would have to travel the speed of light to escape the electrical attraction of the proton, it can never move to that radius or beyond toward the outside world. This new bound composite particle would be stable, electrically neutral, have no chemical properties, and a mass in the range of 1.73×10^{-26} kg. It could have also been produced in copious amounts in the early universe when it was extremely hot and dense, in which case it could be a dark matter candidate.

II. ESCAPE VELOCITY: GRAVITIONAL vs ELECTRICAL ATTRACTION

When the kinetic energy and gravitational potential energy sum is zero, the escape velocity can be calculated. The radial distance can be determined for which the escape velocity would be equal to the speed of light [1]:

$$\tfrac{1}{2} mv^2 + (-GMm / r) = 0 \qquad (1)$$

$$r = 2GM / c^2 \qquad (2)$$

This can also be calculated using the electric potential energy for two opposite and attractive charged masses instead of the gravitational potential energy. Setting the velocity equal to the speed of light yields a radial distance at which the electron could never escape from if it was at or inside this radius relative to the proton's center of charge:

$$\tfrac{1}{2} mv^2 + (-kQq / r) = 0 \qquad (3)$$

$$r = 2kQq / mc^2 \qquad (4)$$

Substituting in the mass of the electron and solving for the radius at which this occurs yields:

$$r = 5.62 \times 10^{-15} \text{ meters} \qquad (5)$$

This is the radius at which the electron would have to travel at the speed of light in order to escape the electrical attraction between it and the proton. Given that the electron has mass, it can only travel at less than the speed of light. This distance is 6.4 times larger than the radius of a proton (0.88×10^{-15} meters). If an electron is able to be close enough to a proton that it cannot escape it's electrical attraction at a radial distance of 5.62×10^{-15} meters, then a new type of composite particle emerges. This composite particle would be electrically neutral to the macroscopic world. The details of this paper will only concern a system consisting of a single electron and single proton.

III. ENERGY OF CONFINED ELECTRON

The uncertainty principle dictates a minimum kinetic energy for a particle confined in a 3-dimensional "box". An approximation for the minimum (n = 1) kinetic energy of an electron confined within the length scale calculated above (twice the radius) in a 3D box is therefore [2]:

$$KE = (n_x^2 + n_y^2 + n_z^2)\, \pi^2\, \hbar^2 / 2mL^2 \tag{6}$$

$$KE = 1.4 \times 10^{-9} \text{ joules} \tag{7}$$

Using $E = mc^2$ we can convert this to a mass equivalent for the newly confined electron:

$$\text{Mass equivalent} = 1.4 \times 10^{-9} \text{ joules} / c^2 = 1.56 \times 10^{-26} \text{ kg} \tag{8}$$

Given that the proton mass is 1.67×10^{-27} kg, it would be expected that the mass of this new bound composite particle would be in the range of 1.73×10^{-26} kg.

IV. EXPERIMENTAL EVIDENCE

It could also be possible to test this hypothesis experimentally if an electron and a proton were brought within the radial distance determined in (5). The HERA particle accelerator was the last major electron-proton collider, and has been shut down since 2007. Electrons reached a maximum energy of 27.5 GeV [3], which should be high enough to create this new composite particle discussed above. The minimum kinetic energy calculated in (7) for the bound electron is 8.74 GeV, so a further analysis of HERA electron-proton collision data might reveal this dark matter candidate.

V. EARLY UNIVERSE AND DARK MATTER

When the early universe was extremely hot and dense it could have made electrons and protons come into very close distances of each other. If a significant number of electrons were made to be inside the radial distance calculated in (5), they would not readily be able to escape the electrical attraction to a respective proton. This electron-proton configuration would therefore be an ideal massive, stable and electrically neutral dark matter candidate.

By using the mass of this bound electron-proton system found in section III, a number density can be calculated if it is indeed the dark matter. The Milky Way galaxy is approximately 3×10^{42} kg, of which 2.7×10^{42} kg is thought to be comprised of a spherically uniform distribution of dark matter [4]. If the hypothetical particle outlined in this paper is indeed the dark matter, the number density of these particles in the Milky Way galaxy would be:

$$n = (2.7 \times 10^{42} \text{ kg} / 1.73 \times 10^{-26} \text{ kg}) / [(4/3)\pi\, (1.22 \times 10^{21} \text{m})^3] = 2.05 \times 10^4 / \text{m}^3 \tag{9}$$

VI. CONCLUSION

This analysis has been done predominantly with classical physics. Additional research using a quantum relativistic theory is required for a complete analysis. Given that the length scales are very small and velocities approach the speed of light, a full quantum relativistic investigation should provide additional insights into this model. Classical physics was used in this paper based on a result from gravitational physics in which the Schwarzschild radius obtained from the General Theory of Relativity is the same distance as the radius at which the classical escape velocity from a mass is equal to the speed of light [1].

A classical analysis has nonetheless provided essential insights into the nature of an electron-proton system experiencing electrical attraction. A very close analogy to gravity has been drawn, and an equation for when the escape velocity is equal to the speed of light has been outlined. The result is similar to a gravitational black hole, but very different in many ways. The radius at which the electron would have to travel the speed of light to escape it's electrical attraction to the proton was found to be 6.4 times larger than the radius of a proton. It was then speculated that this bound system could be a viable dark matter candidate and a mass was estimated, as well as an expected number density in the Milky Way galaxy.

References

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[3] Collider Physics at HERA. M. Klein, R. Yoshida, arXiv:0805.3334 (2008)

[4] Evidence for an Intermediate-Mass Milky Way from Gaia DR2 Halo Globular Cluster Motions. L. Watkins et al., arxiv:1804.11348 (2019)