Refutation of Bourbaki's fixed point theorem and the axiom of choice

Abstract: We evaluate Moroianu's and the Tarski-Bourbaki fixed point theorem and axiom of choice (AC). Two versions of the theorem and then seven theorems and corollary which follow are also *not* tautologous. Therefore these conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Zarouali-Darkaoui, M. (2019). On the Bourbaki's fixed point theorem and the axiom of choice. arxiv.org/abs/1905.09782 mohssin.zarouali@gmail.com

Lemma 2 (Tarski–Bourbaki). Let E be a set, S⊂P(E), and ϕ:S→E a map such that ϕ(X)∉X for all X∈S. Therefore there is a unique subset M of E that can be well-ordered satisfying 1) for all x∈M:S_x∈S and ϕ(S_x)=x; 2) M∉S. (2.1.1)

Remark 2.1.1: We map Eq. 2.1.1 with a conjunctive consequent of 1) and 2).

LET q, r, p, s, x: E, M, ϕ, S, X

```
(p=((s>q)>(((#x<x)>(x<s))>~((p&x)<x))))>(((#x<r)=(((s&x)<x)&((p&(s&x))=x)))&~(r<s)) ;
                                 TTTT TFTF TTTT TTTT(16)
                                 TCTC TFTF TCTC TTTT(16)       (2.1.2)
```

Remark 2.1.3: If we map the consequent to a weakened condition of 1) implies 2), then:

```
(p=((s>q)>(((#x<x)>(x<s))>~((p&x)<x))))>(((#x<r)=(((s&x)<x)&((p&(s&x))=x)))>~(r<s)) ;
                                 TTTT TFTF TTTT TTTT           (2.1.3)
```

Eqs. 2.1.2 or 2.1.3 are *not* tautologous, to refute Moroianu's and the Tarski-Bourbaki fixed point theorem and axiom of choice (AC). The seven theorems and corollary which follow are also *not* tautologous.