Refutation of bounded and Σ_1 formulas in PA

Abstract: A fundamental proposition for bounded and Σ_1 formulas in PA is *not* tautologous. While the author states that the informal notes are full of errors, this fundamental mistake causes the entire section about Rosser's form of Gödel's theorems to collapse. Therefore the proposition is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Moschovakis, Y.N. (2014). Lecture notes in logic.
math.ucla.edu/~ynm/lectures/lnl.pdf ynm@ucla.edu

Proposition 4C.12. *Suppose T is an extension of* PA*, in the language of* PA.

(1) *The class of T-Σ_1 formulas includes all prime formulas and is closed under the positive propositional connectives* & *and* ∨*, bounded quantification of both kinds, and unbounded existential quantification.*

... to show for the proof of (1) that the class of *T*-Σ_1 formulas is closed under universal bounded quantification, it is enough to show that for any extended formula $\varphi(x,y,z)$,

$$T \vdash (\forall x \leq y)(\exists z)\varphi(x,y,z) \leftrightarrow (\exists w)(\forall x \leq y)(\exists z \leq w)\varphi(x,y,z); \quad (4.12.1)$$

LET p, w, x, y, z: *φ*, w, *x*, *y*, *z*.

(~(y>#x)&(p&((x&y)&%z)))=((~(y>#x)&~(%w<%z))&(p&((x&y)&z))) ;
TTTT TTTT TTTT TTTT(48),
TNTN TNTN TNTN TNTN(16),
TTTT TTTT TTTT TTTT(48) (4.12.2)

the equivalence expresses an obvious fact about numbers, which can be easily proved by induction on *y*|and this induction can certainly be formalized in PA.

Eq. 4.12.2 as rendered is *not* tautologous. While the author states that the informal notes are full of errors, this fundamental mistake causes the entire section about Rosser's form of Gödel's theorems to collapse.