Refutation of logically-consistent hypothesis testing and the hexagon of oppositions

Abstract: Definitions for □H and ¬ ◊H are supposed to be equivalent for a classical mapping of agnostic hypothesis tests. While each definition reduces to a theorem in the conjecture, they are *not* tautologous. This refutes that agnostic hypothesis tests are proved to be logically consistent. Hence the characterization of credal modalities in agnostic hypothesis tests cannot be mapped to the hexagon of oppositions to explain the logical relations between these modalities. Therefore the 11 definitions tested form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

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arxiv.org/pdf/1905.07662.pdf rbstern@gmail.com

Abstract: Although logical consistency is desirable in scientific research, standard statistical hypothesis tests are typically logically inconsistent. In order to address this issue, previous work introduced agnostic hypothesis tests and proved that they can be logically consistent while retaining statistical optimality properties. This paper characterizes the credal modalities in agnostic hypothesis tests and uses the hexagon of oppositions to explain the logical relations between these modalities.

Table 1: Modalities of agnostic hypothesis tests

Remark 1: We evaluate Tab. 1 beginning with Eq. 3.1 because it is the only atomic definition without the delta or nabla injections.

LET p, H; delta Δ; nabla ∇.

Modality	**Name**	**Equivalence**	**Interpretation**	
□H	Necessity (A)	ΔH∧◊H	H is accepted.	(1.1)

(#p+~(%p&~#p))&%p ; **F**N**F**N **F**N**F**N **F**N**F**N **F**N**F**N (1.2)

#p=((#p+~(%p&~#p))&%p) ; TTTT TTTT TTTT TTTT (1.3)

¬ ◊H	Impossibility (E)	ΔH∧¬ □H	H is rejected.	(2.1)

(#p+~(%p&~#p))&~#p) ; NFNF NFNF NFNF NFNF (2.2)

~%p=((#p+~(%p&~#p))&~#p) ; TTTT TTTT TTTT TTTT (2.3)

∇H Contingency (Y) $\Diamond H \wedge \neg \Box H$ H is not decided. (3.1)

∇H: %p&~#p ; CCCC CCCC CCCC CCCC (3.2)

$\Diamond H$ Possibility (I) $\Box H \vee \nabla H$ H is not rejected. (4.1)

#p&(%p&~#p) ; FFFF FFFF FFFF FFFF (4.2)

%p=(#p&(%p&~#p)) ; NFNF NFNF NFNF NFNF (4.3)

$\neg \Box H$ Non-necessity (O) $\neg \Diamond H \vee \nabla H$ H is not accepted. (5.1)

~%p&(%p&~#p) ; FFFF FFFF FFFF FFFF (5.2)

~#p=(~%p&(%p&~#p)) ; FNFN FNFN FNFN FNFN (5.3)

ΔH Non-contingency (U) $\Box H \vee \neg \nabla H$ H is decided. (6.1)

ΔH: #p+~(%p&~#p) ; NNNN NNNN NNNN NNNN (6.2)

Remark 1-2: Eqs. 1.3 and 2.3 as rendered result in theorems, so we test the modalities as equivalences: $\Box H = \neg \Diamond H$. (7.1)

#p=~%p ; CCCC CCCC CCCC CCCC (7.2)

Eq. 7.2 is *not* tautologous. This refutes that agnostic hypothesis tests are proved as logically consistent. Therefore the characterization of credal modalities in agnostic hypothesis tests cannot map to the hexagon of oppositions to explain the logical relations between these modalities.