Refutation of condition/decision duality and the internal logic of extensive restriction categories

Abstract: The equations for condition/decision duality are *no*t tautologous, hence refuting what follows as internal logic of extensive restriction categories. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

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Condition/decision duality and the internal logic of extensive restriction categories.

Abstract: … While categorical treatments of flowchart languages are abundant, none of them provide a treatment of this dual nature of predicates. In the present paper, we argue that extensive restriction categories are precisely categories that capture such a condition/decision duality, by means of morphisms which, coincidentally, are also called decisions. Further, we show that having these categorical decisions amounts to having an internal logic: Analogous to how subobjects of an object in a topos form a Heyting algebra, we show that decisions on an object in an extensive restriction category form a De Morgan quasilattice ...

4 The internal logic of extensive restriction categories

4.1 Kleene's three valued logics and De Morgan quasilattices

As for Boolean algebras, one can derive a partial order on De Morgan quasilattices by

p ≼ q iff p ∧ q = p, (4.1.1.1)

((p&q)=p)>~(q<p) ; TT**F**T TT**F**T TT**F**T TT**F**T (4.1.1.2)

and another one by

p ⊑ q iff p ⋁ q = q. (4.1.2.1)

((p+q)=q)>~(q<p) ; TT**F**T TT**F**T TT**F**T TT**F**T (4.1.2.2)

Unlike as for Boolean algebras, however, these do not coincide, though they are anti-isomorphic, as it follows from the De Morgan laws that

$p \preccurlyeq q$ iff $\neg q \sqsubseteq \neg p$. (4.1.3.1)

((((p+q)=q)>~(q<p))>(((p&q)=p)>~(q<p)))&
~((((p&q)=p)>~(q<p))>(((p+q)=q)>~(q<p))) ;

FFFF FFFF FFFF FFFF (4.1.3.2)

Eqs. 4.1.1.2 and 4.1.2.2 are *no*t tautologous; and 4.1.3.2 is contradictory because of the *iff* in 4.1.3.1. This refutes condition/decision duality and hence what follows as internal logic of extensive restriction categories.