Refutation of rooted hypersequent calculus for modal logic S5

Abstract: We evaluate two example equations as *not* tautologous, thereby refuting the rooted hypersequent calculus for modal propositional logic S5. The sequent calculus forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

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Abstract: We present a rooted hypersequent calculus for modal propositional logic S5. We show that all rules of this calculus are invertible and that the rules of weakening, contraction, and cut are admissible. Soundness and completeness are established as well.

3 Rooted Hypersequent R_{S5}: Our calculus is based on finite multisets, i.e. on sets counting multiplicities of elements. We use certain categories of letters, possibly with subscripts or primed, as metavariables for certain syntactical categories (locally different conventions may be introduced) ...

Example 3.3. The following sequents are derivable in RS5. 1. ⇒(r∧p)→(q→(◊(p∧q)∧◊r)) (3.3.1.1)

(r&p)>(q>#(%(p&q)&%r)) ; `TTTT TTTN TTTT TTTN` (3.3.1.2)

5 Structural properties: In this section, we prove the admissibility of weakening and contraction rules, and also some properties of R_{S5}, which are used to prove the admissibility of cut rule.

5.2 Invertibility: In this subsection, first we introduce a normal form called Quasi Normal Form, which is used to prove the admissibility of the contraction and cut rules. Then we show that the structural and modal rules are invertible.

Example 5.9. ... (¬□(A→B)∨p∨◊C) ∧ (¬q) ∧ (◊A∨¬◊(A∧B)∨¬r) is in CQNF (5.9.1)

((~(#(x>y)&(p&%z))=(p=p))&~q)&((#%x+~(%(x&y)=(p=p)))+~r) ;
`TT`**`FF`** `TT`**`FF`** `TT`**`FF`** `TT`**`FF`**`(64),` `TC`**`FF`** `TC`**`FF`** `TC`**`FF`** `TC`**`FF`**`(16),`
`TT`**`FF`** `TT`**`FF`** `TT`**`FF`** `TT`**`FF`**`(16),` `TC`**`FF`** `TC`**`FF`** `TC`**`F`**`F` `TC`**`FF`**`(32)` (5.9.2)

Eqs. 3.3.1.2 and 5.9.2 as rendered are *not* tautologous, thereby refuting the rooted hypersequent calculus for modal propositional logic S5.