The Relationship of the Fine Structure Constant and Pi

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May 19, 2019

Abstract: In this paper, the fine structure constant is derived from a geometric ratio of surface areas, as a result of vibrations in a lattice with a body-centered cubic arrangement.

Introduction

The fine structure constant (α) is a mysterious constant in physics with relations to many fundamental physical constants, including relating the square of the Planck charge (q_P) to the square of the elementary charge (e_e). It also relates the electron's classical radius (r_e) with the Bohr radius (a_0) by the square of the fine structure constant. It is a curious number, often referred to in its inverse form as the number 137, although it's not an integer. To six digits, the fine structure constant is 0.007297 and the inverse is 137.036 [1]. Like another famous constant in mathematics, pi (π), it is a dimensionless constant with no units.

In fact, there are more similarities between α and π than just a number with never-ending digits and a dimensionless value. Both describe a geometric ratio. Whereas π is the ratio of a circle's circumference to diameter, α can be shown to be the ratio of geometries that include circular properties. Thus, α can be derived from π.

Geometry of Body Centered Cubic Unit Cell

In *The Relationship of Mass and Charge* by Yee and Gardi [2], it was found that charge could be related to wave amplitude, redefining the units of Coulomb charge to be units of distance (amplitude). In the paper, the mechanism for charge was described as vibrations of *granules*, filling the empty space between particles such as a proton and electron, but without describing the structure of such granules.

Here in this paper, the structure of these granules that fill space is proposed and related to the fine structure constant. It also must account for the inverse square law associated with forces like gravity and the electric force. A body-centered cubic (bcc) structure fits the criteria. Fig. 1 illustrates a unit cell of granules in a bcc structure. Colored in blue is a granule in motion (from left-to-right in the illustration). As it moves, it affects four granules, which then these four proceed in motion, each affecting another quadrant of four granules.

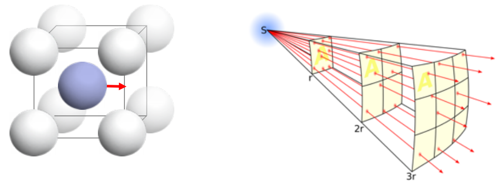

Fig. 1 – Body centered cubic unit cell (left) and inverse square relationship (right)

If each granule returns to its original position like a spring-mass system, then its oscillation can be represented as a sine wave and its displacement as wave amplitude. Amplitude will decline from the source as it transfers its energy to a greater number of granules. Its energy or force is the square of this distance – known as the inverse square law.

From the inverse square law, intensity (I) is related to source power (P_0) and surface area (S) [3]:

$$I = \frac{P_0}{S} \tag{1}$$

Intensity is a measurement of power per unit area with SI units of watts per square meter. Thus, the power to be measured at a point in space (P_1) can be related to the intensity measured at this point by dividing by an arbitrary unit area (x^2 – which will later cancel in the derivation).

$$I = \frac{P_1}{x^2} \tag{2}$$

Substituting Eq. 2 into Eq. 1 allows it to be rearranged to define a ratio of power at a point in space (P_1) to source power (P_0).

$$\frac{P_1}{x^2} = \frac{P_0}{S} \tag{3}$$

$$\frac{P_1}{P_0} = \frac{x^2}{S} \tag{4}$$

Intensity is proportional to the square of amplitude (A) [4]. And although the variables for intensity and power – including density and frequency – may not be known, assuming they are constant at both measured points, these variables will cancel in a ratio. This leaves the square of the amplitude from the power equation, which is why intensity is proportional to the square of amplitude ($I \approx A^2$).

$$\frac{A_1^2}{A_0^2} = \frac{x^2}{S} \tag{5}$$

Surface Area of Granule Motion

The surface area (S) in Eq. 5 can be solved for in a bcc unit cell, where the center granule moves along an x-axis. Fig. 2 shows the vibration of the center granule at four times – at the peak of displacement traveling one direction (π/2) and then returning to equilibrium (π). Then, the peak of displacement traveling in the opposite direction (3π/2) and returning to equilibrium again (2π). In the figure, the motion of eight granules at the vertices of the cube spread such that the effect on other granules is spherical. The vibrational motion of the center granule creates a cylindrical shape after a complete return to equilibrium at 2π.

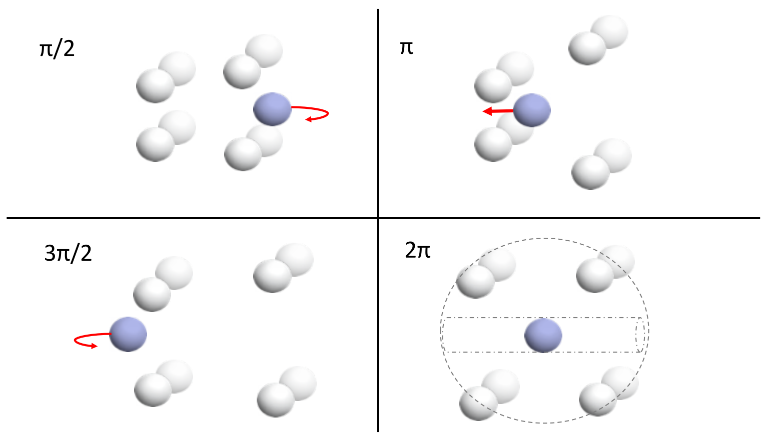

Fig. 2 – Vibration of center granule in body centered cubic unit cell

In 1971, E. D. Reilly, Jr. presented a mathematical relationship for the inverse of α, without explaining the reason [5]. Using the bcc unit structure, the geometry of the motion of each granule can derive Reilly's relationship (later in Eq. 14). Since Eq. 5 is a ratio of wave amplitude, and one amplitude occurs at π, the surface areas of the sphere and cylinder are taken at π. It will still be spherical, but the cylinder will be half. For example, if the radius of the circle is x (r=x), then it is half the circumference of the circle, which is πx. This distance also becomes the length (l=πx) of the cylinder, and in turn, becomes the radius of the sphere. This is illustrated in Fig. 3.

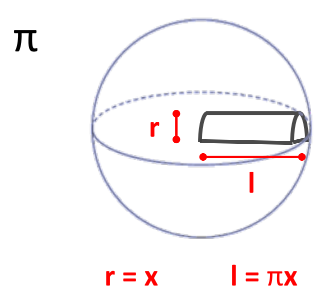

Fig. 3 – Surface area of sphere and half cylinder at π

The surface areas of the sphere with radius of l, and half cylinder with radius of r and length l, are added together in Eq. 6 (note there are two half circles on both ends of the cylinder and it is open on the bottom). The same variable set for unit area of intensity (I) from Eq. 2 is used here to make it proportional – x. Therefore, r=x and l=πx.

$$S = 4\pi l^2 + \left(\pi r l + \frac{1}{2}\pi r^2 + \frac{1}{2}\pi r^2\right) \quad \textbf{(6)}$$

$$r = x \quad \textbf{(7)}$$

$$l = \pi x \tag{8}$$

Substitute Eqs. 7 and 8 into Eq. 6 and simplify:

$$S = 4\pi(\pi x)^2 + \left(\pi x(\pi x) + \frac{1}{2}\pi x^2 + \frac{1}{2}\pi x^2\right) \tag{9}$$

$$S = 4\pi^3 x^2 + \pi^2 x^2 + \pi x^2 \tag{10}$$

Substitute Eq. 10 into Eq. 5 and simplify:

$$\frac{A_1^2}{A_0^2} = \frac{x^2}{4\pi^3 x^2 + \pi^2 x^2 + \pi x^2} \tag{11}$$

$$\frac{A_1^2}{A_0^2} = \frac{1}{4\pi^3 + \pi^2 + \pi} \tag{12}$$

The ratio of amplitudes from Eq. 12 is the fine structure constant (α). It is 0.007297, or in inverse format it is 137.036, matching the CODATA value to this level of digits.

$$\alpha = \frac{1}{4\pi^3 + \pi^2 + \pi} = 0.007297 \tag{13}$$

$$\frac{1}{\alpha} = 4\pi^3 + \pi^2 + \pi = 137.036 \tag{14}$$

Solving for Wave Amplitude

The fine structure constant is a ratio of geometric surface areas. It can be used to calculate wave amplitude as it transitions from one geometry to another – specifically spherical to a one-dimensional vibration. A potential analogy is the repeated process of blowing a small amount of air into a balloon and releasing the same amount. The vibration of air molecules affected at the spherical surface of the balloon may be calculated by knowing the vibration of the air molecules at the one-dimensional inlet of the balloon, or vice versa.

Thus, the fine structure constant becomes a proportionality constant between the squares of amplitudes (A) as seen in the substitution of Eq. 13 into Eq. 12, then rearranged to solve for A_1. As charge is wave amplitude as found in *The Relationship of Mass and Charge* paper, the elementary charge (e_e) can be related to the Planck charge (q_P) by substituting these values for the amplitude variables A_1 and A_0 respectively. This yields one of the known derivations of the fine structure constant in Eq. 17.

$$A_1^2 = A_0^2 \alpha \qquad \textbf{(15)}$$

$$e_e^2 = q_P^2 \alpha \qquad \textbf{(16)}$$

$$\alpha = \frac{e_e^2}{q_P} \qquad \textbf{(17)}$$

Finally, the fine structure constant derived exclusively in terms of pi can be tested by calculating the elementary charge. When the CODATA value of 1.8756 x 10^{-18} is used for the Planck charge, the resulting calculation for the elementary charge matches the known value of 1.602 x 10^{-19}.

$$e_e = \frac{q_P}{\sqrt{4\pi^3 + \pi^2 + \pi}} = 1.602 \cdot 10^{-19} \qquad \textbf{(18)}$$

Conclusion

The fine structure constant can be derived in terms of pi due to a ratio of geometric shapes, possibly the result of the motion of something that fills empty space.

References

1 Mohr, P., Newell, D. and Taylor, B., 2014. CODATA Recommended Values of the Fundamental Physical Constants, *Rev. Mod. Phys. 88*, 035009.

2 Yee, J., Gardi, L., 2019. The Relationship of Mass and Charge. *ResearchGate*. Online: https://www.researchgate.net/publication/332083618.

3 Wikipedia (accessed May 2019). Inverse Square Law*, Online*: https://en.wikipedia.org/wiki/Inverse-square_law.

4 The Physics Hypertextbook (accessed May 2019). Intensity*, Online*: https://physics.info/intensity/.

5 Roskies R., et. al, 1971. A new pastime–calculating alpha to one part in a million. *Phys. Today*, 24 9.