Refutation of lattice effect algebra

Abstract: A seminal definition of lattice effect algebra is *not* tautologous. This refutes lattice effect and lattice pseudoeffect algebras along with the chain effect of quasiresiduation. The conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

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In order to axiomatize quantum logic effects in a Hilbert space, Foulis and Bennett [1994] introduced the so-called effect algebras. … The aim of the present paper is to introduce the more general concept of quasiresiduation and to show that lattice effect algebras and lattice pseudoeffect algebras satisfy this concept.

Definition 1. An effect algebra is a partial algebra E=(E,+,′,0,1) of type (2,1,0,0) where (E,′,0,1) is an algebra and + is a partial operation satisfying the following conditions for all x, y, z ∈ E:
(E1) x + y is defined if and only if so is y + x and in this case x + y = y + x,
(E2) (x + y) + z is defined if and only if so is x + (y + z) and in this case (x + y) + z = x + (y + z),
(E3) x ′ is the unique u ∈ E with x + u = 1,
(E4) if 1 + x is defined then x = 0. (1.1)

LET p, s: x, s

((%s>#s)+p)>(p=(s@s)) ; T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (1.2)

Eq. 1.2 is *not* tautologous. This refutes lattice effect and lattice pseudoeffect algebras along with the chain effect of quasiresiduation.