Refutation of Tarski's geometric axioms and betweenness

Abstract: Of 13 equations evaluated, five are tautologous and eight are *non* tautologous. This refutes Tarski's geometric axioms and betweenness which form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Tarski%27s_axioms

[Alfred] Tarski's … axiom set for the substantial fragment of Euclidean geometry ... is formulable in first-order logic with identity, and requiring no set theory ([from] 1959) (i.e., that part of Euclidean geometry that is formulable as an elementary theory).

Congruence axioms

Reflexivity of congruence xy≡yx (1.1)

Remark 1.1: xy is not read point x is less than y as equivalent to point y is greater than x but rather x&y is equivalent to y&x.

LET p, q, r, s, t, u, v, w, x, y, z: a, b, d, e, u, v, w, x, y, z.

(x&y)=(y&x) ; TTTT TTTT TTTT TTTT (1.2)

Identity of congruence xy≡zz→x=y (2.1)

((x&y)=(z&z))>(x=y) ; TTTT TTTT TTTT TTTT(16)
FFFF FFFF FFFF FFFF(32)
TTTT TTTT TTTT TTTT(80) (2.2)

Transivity of congruence (xy≡zu∧xy≡vw)→zu≡vw (3.1)

(((x&y)=(z&u))&((x&y)=(v&w)))>((z&u)=(v&w)) ;
TTTT TTTT TTTT TTTT (3.2)

Betweenness axioms

Identity of betweenness Bxyx→x=y (4.1)

((y>x)&(x>y))>(x=y) ;

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TTTT TTTT TTTT TTTT
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(4.2)

Axiom of Pasch (Bxuz∧Byvz)→∃a(Buay∧Bvax) (5.1)

(((u>x)&(z>u))&((v>y)&(z>v)))>(((%p>u)&(y>%p))&((%p>v)&(x>%p))) ;

```
NFNF NFNF NFNF NFNF( 2)}x2
TTTT TTTT TTTT TTTT( 6)}
FFFF FFFF FFFF FFFF( 4)}x2
TTTT TTTT TTTT TTTT( 4)}
FFFF FFFF FFFF FFFF( 2)}x4
TTTT TTTT TTTT TTTT( 2)}
FFFF FFFF FFFF FFFF( 6)}x2
CTCT CTCT CTCT CTCT( 2)}
TTTT TTTT TTTT TTTT(48)
TTTT TTTT TTTT TTTT( 6)}x2
CTCT CTCT CTCT CTCT( 2)}
```
(5.2)

Axiom schema of continuity

∃a∀x∀y[(φ (x)∧ψ(y))→Baxy] → ∃b∀x∀y[(φ (x)∧ψ(y))→Bxby] (11.1)

LET u, v: φ, ψ.

(((u&#x)&(v&#y))>((#x>%p)&(#y>#x)))>(((u&#x)&(v&#y))>((%q>#x)&(#y>%q))) ;

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TTTT TTTT TTTT TTTT(54)
TCTT TCTT TCTT TCTT( 2)
TTTT TTTT TTTT TTTT( 6)
TCTT TCTT TCTT TCTT( 2)
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(11.2)

Lower dimension ∃a∃b∃c[¬Babc∧¬Bbca∧¬Bcab] (6.1)

(~((%q>%p)&(%r>%q))&~((%r>%q)&(%p>%r))&~((%p>%r)&(%q>%p)) ;

```
FFFF FFFF FFFF FFFF
```
(6.2)

Congruence and betweenness

Upper dimension (xu≡xy∧yu≡yv∧zu≡zv∧u≠v) → (Bxyz∨Byzx∨Bzxy) (7.1)

((((x&u)=(x&v))&((y&u)=(y&v)))&(((z&u)=(z&v))&(u@v)))>
((((y>x)&(z>y))&((z>y)&(x>z)))&((x>z)&(y>x))) ;

```
TTTT TTTT TTTT TTTT
```
(7.2)

Axiom of Euclid

A: (Bxyw∧xy≡yw)∧(Bxuv∧xu≡uv)∧(Byuz∧yu≡zu))→yz≡vw (8.1.1)

((((((y>x)&(w>y))&(x&y))=(y&w))&((((u>x)&(v>u))&(x&u))=(u&v)))&
((((u>y)&(z>u))&(y&u))=(z&u)))>((y&z)=(v&w)) ;

```
TTTT TTTT TTTT TTTT(12)
FFFF FFFF FFFF FFFF( 2)
TTTT TTTT TTTT TTTT(14)
FFFF FFFF FFFF FFFF( 4)
TTTT TTTT TTTT TTTT(28)
FFFF FFFF FFFF FFFF( 2)
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TTTT TTTT TTTT TTTT(14)
FFFF FFFF FFFF FFFF(2)
TTTT TTTT TTTT TTTT(14)
FFFF FFFF FFFF FFFF(2)
TTTT TTTT TTTT TTTT(2)
FFFF FFFF FFFF FFFF(6)
TTTT TTTT TTTT TTTT(18)
FFFF FFFF FFFF FFFF(2)
TTTT TTTT TTTT TTTT(6) (8.1.2)

B: Bxyz∨Byzx∨Bxzy∨∃a(xa≡ya∧xa≡za) (8.2.1)

((((y>x)&(z>y))&((z>y)&(x>z)))&((x>z)&(y>x)))&(((x&%p)=(y&%p))&((x&%p)=(z&%p))) ;
TTTT TTTT TTTT TTTT(16)
FFFF FFFF FFFF FFFF(96)
TTTT TTTT TTTT TTTT(16) (8.2.2)

C: (Bxuv∧Byuz∧x≠u)→∃a∃b(Bxya≠Bxzb∧Bavb) (8.3.1)

((((u>x)&(v>u))&((u>y)&(z>u)))&(x@u))>
((((y>x)&(%p>y))&((z>x)&(%q>z)))&((v>%p)&(%q>v))) ;
TTTT TTTT TTTT TTTT(10)
TTTT TTTT TTTT TTTT(6) }x2
NNFF NNFF NNFF NNFF(2) }
TTTT TTTT TTTT TTTT(6) }
NNFF NNFF NNFF NNFF(2) }
TTTT TTTT TTTT TTTT(16) }
TTTT TTTT TTTT TTTT(54) (8.3.2)

Five segment (x≠y∧Bxyz∧Bx'y'z'∧xy≡x'y'∧yz≡y'z'∧xu≡x'u'∧yu≡y'u')→zu≡z'u'
(9.1)

LET p, q, r, t, u, x, y, z:
x', y', z', u', u, x, y, z

(((x@y)&(((y>x)&(z>y))&((q>p)&(r>q))))&
((((x&y)=(p&q))&((y&z)=(q&r)))&(((x&u)=(p&t))&((y&u)=(q&t)))))>((z&u)=(r&t)) ;
TTTT TTTT TTTT TTTT (9.2)

Segment construction ∃z[Bxyz∧yz≡ab](10.1)

LET p, q: a, b

((y>x)&(#z>y))&((y&#z)=(p&q)) ;
TTT**F** TTT**F** TTT**F** TTT**F**(32)
FFFF FFFF FFFF FFFF(16)
TTT**F** TTT**F** TTT**F** TTT**F**(16)
CCC**F** CCC**F** CCC**F** CCC**F**(32)
FFFF FFFF FFFF FFFF(16)
CCCN CCCN CCCN CCCN(16) (10.2)

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