Refutation of counterpart theory via modal logic

Abstract: Of seven examples for counterpart theory, none is tautologous. In fact, a translation is *not* tautologous in the counterpart model *or* in QMT, but rather shares the same truth table result. Two definitions of intensionality are also *not* tautologous and hence refuted. Therefore counterpart theory forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

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3. Uncontested principles of modal logic

While Lewis (1968: 123) proposed to test various contested principles of quantified modal logic (QML) by checking whether his translation validated them, he did not test the translation itself by checking whether it validated the most basic uncontested principles of QML—or even propositional modal logic. I will now carry out that test by examining four uncontested principles (axiom schemas) of propositional modal logic, in order of increasing weakness in Kripke semantics:

Intensionality: $\Box(\varphi \leftrightarrow \psi) \rightarrow (\chi \rightarrow \chi[\varphi/\psi])$, where $\chi[\varphi/\psi]$ is the formula (if any) that results from replacing all free occurrences of ψ in χ, with free occurrences of φ. (3.1.1.1)

LET p, q, r, s, t, u, v, w, x, y, z
A, b, C, G, I, u, v, w, x, y, z

#(p=q)>(r>(r&(p\q))) ; `TTTT TTCT TTTT TTCT` (3.1.1.2)

Remark 3.1.1.2: Eq. 3.1.1.2 as rendered is *not* tautologous as it should be if intensionality is a theorem.

We should not expect Intensionality to hold in every language, but, by definition, it does hold in any language with no hyperintensional operators, and QML is such a language. (3.1.1.3)

Remark 3.1.1.3: As we show above, and elsewhere (Refutation of realizability semantics for QML), intensionality does not hold in QML such as in VŁ4.

The model is not a model of:
∃w(∀x(Ixw↔Ax)∧(∀v∀x(Cxbv→(Gx→∃x(Ixv∧Gx)))→(Gb→∃x(Ixw∧Gz))) (3.2.1.1)

((((t&#x)&%w)=(p&#x)) & (((r&#x)&(q&#v))>((s&#x)>((t&(%x&#v))&(s&%x)))))>
((s&#q)>((t&(#x&%w))&(s&z))) ;

```
TTTT TTTT TTCC TTCC(16),
TTTT TTTT TTCT TTCT( 4), TTTT TTTT TTCT TTTT( 1),
TTTT TTTT TTCT TTCT( 1), TTTT TTTT TTCT TTTT( 1),
TTTT TTTT TTCT TTCT( 2), TTTT TTTT TTTT TTTT( 1),
TTTT TTTT TTCT TTCT( 1), TTTT TTTT TTTT TTTT( 1),
TTTT TTTT TTCT TTTT( 1), TTTT TTTT TTTT TTTT( 1),
TTTT TTTT TTCT TTTT( 1), TTTT TTTT TTTT TTTT( 1)
```
(3.2.1.2)

which is the translation of the following T-instance □(Gb→∃xGx)→(Gb→∃xGx)(3.2.2.1)

#((s&q)>(s&%x))>((s&q)>(s&%x)) ;`TTTT TTTT TTTT TTTT` (3.2.2.2)

Remark 3.2.2.2: Eq. 3.2.1.2 as a counterpart theorem is *not* tautologous. However the T-instance it maps in 3.2.2.2 is tautologous.

The model is also not a model of: ∀w∀x∀y(Cxyabw → (Fx∧Gy))→∃w∃x∃y(Cxyabw∧(Fx∧Gy)), (3.2.3.1)

LET p, q, r, s, t, u, v, w, x, y, z
a, b, C, G, I, F, v, w, x, y, z

((((r&#x)&(#y&p))&((p&q)&#w))>((u&#x)&(s&#y)))>
((((r&%x)&(%y&p))&((p&q)&%w))&((u&%x)&(s&%y))) ;

```
FFFF FFFF FFFF FFFF(2), FFFF FFFF FFFF FFFC(2)} x14
FFFF FFFN FFFF FFFN(2), FFFF FFFN FFFF FFFT(2)} x 2
```
(3.2.3.2)

which is the translation of the D-instance □(Fa∧Gb)→◊(Fa∧Gb). (3.2.4.1)

#((u&y)&(s&q))>%((u&y)&(s&q)) ;`TTTT TTTT TTTT TTTT` (3.2.4.2)

Remark 3.2.4.2: Eq. 3.2.3.2 as a counterpart theorem is *not* tautologous. However the D-instance it maps in 3.2.4.2 is tautologous.

Nor is it a model of: ∀w∀x(Cxaw→(Fx→∃y(Iyw∧Fy)))→(∀w∀x(Cxaw→Fx)→∀w∃x(Ixw∧Fx)) (3.2.5.1)

(((r&#x)&(p&#w))>((u&#x)>(((t&%y)&#w)&(u&y))))>
(((r&#x)&((p&#w)>(u&#x)))>(((t&%x)&#w)&(u&%x))) ;

```
TTTT TTTT TTTT TTTT(16),
TTTT CCCC TTTT CCCC( 8),
TTTT CTCT TTTT CTCT( 3),}x 2
TTTT TTTT TTTT TTTT( 1) }
```
(3.2.5.2)

which is the translation of the K-instance □(Fa→∃xFx)→(□Fa→□∃xFx)). (3.2.6.1)

#((u&p)>(u&%x))>((#u&p)>(#u&%x)) ;

```
TTTT TTTT TTTT TTTT
```
(3.2.6.2)

Remark 3.2.6.2: Eq. 3.2.5.1 as a counterpart theorem is *not* tautologous. However the K-instance it maps in 3.2.6.2 is tautologous.

Finally, while the translation of □(¬ ∃xFx↔(Fa∧¬Fa)) (3.2.7.1)

#(~(u&%x)=((u&p)&~(u&p))) = (p=p) ;

```
FFFF FFFF FFFF FFFF(16),
FFFF FFFF FFFF FFFF( 2),}x 4
NNNN NNNN NNNN NNNN( 2) }
```
(3.2.7.2)

is true in the model, the translation of □(¬ ∃xFx→⊥) is not. (3.2.8.1)

#(~(u&%x)>(p@p)) = (p=p) ;

```
FFFF FFFF FFFF FFFF(16),
FFFF FFFF FFFF FFFF( 2),}x 4
NNNN NNNN NNNN NNNN( 2) }
```
(3.2.8.2)

Remark 3.2.8.2: In fact, Eqs. 3.2.8.2 is equivalent to 3.2.7.2 which the text denies.

The model, then, is not a model of the translation of the Intensionality instance
□((Fa∧¬Fa)↔⊥)→(□(¬ ∃xFx↔(Fa∧¬Fa))→□(¬ ∃xFx→⊥)). (3.2.9.1)

#((((u&p)&~(u&p))=(p@p))>(#(~(u&x)=((u&p)&~(u&p)))>#(~(u&x)>(p@p)))) = (p=p) ;

```
NNNN NNNN NNNN NNNN
```
(3.2.9.2)

Remark 3.2.9.2: Eq. 3.2.9.2 as rendered is not a model of the intensionality instance because it returns N non-contingency (truthity) and not T tautology.

Of seven examples for counterpart theory, none is tautologous. In fact, a translation is *not* tautologous in the counter point model *or* in QMT, but rather shares the same truth table result. Two definitions of intensionality are also *not* tautologous and hence refuted.