

Refutation of forcing to change large cardinal strength

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Abstract: The seminal theorem of the dissertation states a cardinal is greatly inaccessible if and only if it is Mahlo. Three non trivial equations of the proof are *not* tautologous, thereby refuting theorems derived therefrom. These conjectures form a *non* tautologous fragment of the universal logic $\forall\mathcal{L}4$.

We assume the method and apparatus of Meth8/ $\forall\mathcal{L}4$ with Tautology as the designated proof value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET \sim Not, \neg ; + Or, \vee , \cup , \sqcup ; - Not Or; & And, \wedge , \cap , \sqcap , ; ; \ Not And;
> Imply, greater than, \rightarrow , \Rightarrow , \mapsto , $>$, \supset , \succ ; < Not Imply, less than, \in , $<$, \subset , \prec , \neq , \ll , \lesssim ;
= Equivalent, \equiv , $:=$, \Leftrightarrow , \leftrightarrow , $\hat{=}$, \approx , \simeq ; @ Not Equivalent, \neq ;
% possibility, for one or some, \exists , \diamond , **M**; # necessity, for every or all, \forall , \square , **L**;
($z=z$) **T** as tautology, **T**, ordinal 3; ($z@z$) **F** as contradiction, \emptyset , Null, \perp , zero;
($\%z>\#z$) **N** as non-contingency, Δ , ordinal 1; ($\%z<\#z$) **C** as contingency, ∇ , ordinal 2;
 $\sim(y < x)$ ($x \leq y$), ($x \subseteq y$); ($A=B$) ($A\sim B$).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Carmony, E. (2015). Force to change large cardinal strength.
arxiv.org/pdf/1506.03432.pdf ecarmody@gradcenter.cuny.edu

Abstract: This dissertation includes many theorems which show how to change large cardinal properties with forcing. I consider in detail the degrees of inaccessible cardinals (an analogue of the classical degrees of Mahlo cardinals) and provide new large cardinal definitions for degrees of inaccessible cardinals extending the hyper-inaccessible hierarchy. I showed that for every cardinal κ , and ordinal α , if κ is α -inaccesssible, then there is a P forcing that κ which preserves that α -inaccessible but destorys [sic] that κ is $(\alpha+1)$ -inaccessible. I also consider Mahlo cardinals and degrees of Mahlo cardinals. I showed that for every cardinal κ , and ordinal α , there is a notion of forcing P such that κ is still α -Mahlo in the extension, but κ is no longer $(\alpha+1)$ -Mahlo. I also show that a cardinal κ which is Mahlo in the ground model can have every possible inaccessible degree in the forcing extension, but no longer be Mahlo there. The thesis includes a collection of results which give forcing notions which change large cardinal strength from weakly compact to weakly measurable, including some earlier work by others that fit this theme. I consider in detail measurable cardinals and Mitchell rank. I show how to change a class of measurable cardinals by forcing to an extension where all measurable cardinals above some fixed ordinal α have Mitchell rank below α . Finally, I consider supercompact cardinals, and a few theorems about strongly compact cardinals. Here, I show how to change the Mitchell rank for supercompactness for a class of cardinals.

Theorem 10. *A cardinal κ is greatly inaccessible if and only if κ is Mahlo.*

Remark 10: The first step of the proof as 10.1.1 is trivial and ignored here.

Proof. . . . Next, if $A \in F$, and $A \subseteq B$, then there is a club $C \in F$ such that $C \cap I \subseteq A \subseteq B$, thus $B \in F$, by construction, since F is the filter generated by sets of this form. (10.2.1)

$((\langle p \langle t \rangle \rangle \& \sim \langle q \langle p \rangle \rangle) \langle \langle r \langle t \rangle \rangle \rangle \langle \sim q \langle \sim (p \langle r \& u \rangle) \rangle \rangle) \langle \langle q \langle t \rangle \rangle \rangle ;$

$$\begin{array}{l}
\mathbf{FFTT} \mathbf{FFTT} \mathbf{FFTT} \mathbf{FFTT}, \\
\mathbf{FFFF} \mathbf{FFFF} \mathbf{FFFF} \mathbf{FFFF}, \\
\mathbf{FFTT} \mathbf{FTTT} \mathbf{FFTT} \mathbf{FTTT}, \\
\mathbf{FFFF} \mathbf{FFFF} \mathbf{FFFF} \mathbf{FFFF}
\end{array} \tag{10.2.2}$$

Third, if A and B are elements of F, then there are clubs C and D such that $C \cap I \subseteq A$, and $D \cap I \subseteq B$, so $A \cap B$ contains $(C \cap D) \cap I$. (10.3.1)

$$\begin{array}{l}
(((p\&q)\<t)\>((\sim p\<(\%r\&u))\&\sim(q\<(\%s\&u))))\>((p\&q)\>((r\&s)\&u)) ; \\
\mathbf{TTTT} \mathbf{TTTT} \mathbf{TTTT} \mathbf{TTTT}, \\
\mathbf{TTTF} \mathbf{TTTF} \mathbf{TTTF} \mathbf{TTTT}
\end{array} \tag{10.3.2}$$

This is of the form which generated the filter, thus $A \cap B \in F$. (10.4.1)

$$\begin{array}{l}
((((p\&q)\<t)\>((\sim p\<(\%r\&u))\&\sim(q\<(\%s\&u))))\>((p\&q)\>((r\&s)\&u))\>((p\&q)\<t)) ; \\
\mathbf{FFFFT} \mathbf{FFFFT} \mathbf{FFFFT} \mathbf{FFFFT} (3) , \\
\mathbf{FFFFT} \mathbf{FFFFT} \mathbf{FFFFT} \mathbf{FFFF} (1)
\end{array} \tag{10.4.2}$$

Eqs. 10.2.2-10.4.2 as rendered are *not* tautologous. Since Theorem 10 is seminal to the dissertation, theorems derived therefrom are refuted.