

## Refutation of Jaccard index

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**Abstract:** The definition of the Jaccard index is *not* tautologous, hence refuting it with derivations and forming a *non* tautologous fragment of the universal logic VL4.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated proof value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬; + Or, ∨, ∪, ⊔; - Not Or; & And, ∧, ∩, ⊓, ; \ Not And;  
 > Imply, greater than, →, ⇒, ⇨, >, ⊃, ≻; < Not Imply, less than, ∈, <, ⊂, ⊆, ≠, ≪, ≤;  
 = Equivalent, ≡, :=, ⇔, ↔, ≐, ≈, ≃; @ Not Equivalent, ≠;  
 % possibility, for one or some, ∃, ∅, M; # necessity, for every or all, ∀, □, L;  
 (z=z) **T** as tautology, **T**, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;  
 (%z>#z) **N** as non-contingency, Δ, ordinal 1; (%z<#z) **C** as contingency, ∇, ordinal 2;  
 ~(y < x) (x ≤ y), (A=B) (A~B).

Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Jaccard\_index

The **Jaccard index**, also known as **Intersection over Union** and the **Jaccard similarity coefficient** (originally given the French name *coefficient de communauté* by Paul Jaccard), is a statistic used for gauging the similarity and diversity of sample sets. The Jaccard coefficient measures similarity between finite sample sets, and is defined as the size of the intersection divided by the size of the union of the sample sets:

$$J(A,B) = \frac{|A \cap B|}{|A \cup B|} = \frac{|A \cap B|}{(|A| + |B| - |A \cap B|)}. \quad (\text{If } A \text{ and } B \text{ are both empty, we define } J(A,B)=1.)$$

$$0 \leq J(A,B) \leq 1. \quad (1.1)$$

$$\sim(((A \& B) \setminus (A + B)) = ((A \& B) \setminus ((A + B) - (A \& B)))) < (C @ C) \&$$

$$\sim((\%C > \#C) < (((A \& B) \setminus (A + B)) = ((A \& B) \setminus ((A + B) - (A \& B)))));$$

$$\mathbf{FFFF \ FCFC \ FFFF \ FCFC} \quad (1.2)$$

**Remark 1.2:** Without the relational limits in Eq. 1.1, the formula alone is not tautologous:

$$((A \& B) \setminus (A + B)) = ((A \& B) \setminus ((A + B) - (A \& B)));$$

$$\mathbf{TTTT \ TNTN \ TTCC \ TNCF} \quad (1.3)$$

Eq. 1.2 is *not* tautologous, and nearly contradictory, refuting the Jaccard index with its derivations.