Refutation of reverse mathematics and nets

Abstract: We evaluate four definitions for reverse mathematics (3) and nets (1). None is tautologous. This refutes reverse mathematics and nets. Therefore these definitions form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Sanders, S. (2019). Nets and reverse mathematics, a pilot study.
arxiv.org/pdf/1905.04058.pdf sasander@me.com

Abstract. Nets are generalisations of sequences involving possibly *uncountable* index sets … More recently, nets are central to the development of *domain theory* ... This paper deals with the Reverse Mathematics study of basic theorems about nets. ...

2.1. **Reverse Mathematics.** … we introduce the collection of all finite types T, defined by the two clauses: (i) 0 ∈ T and (ii) If σ, τ ∈ T then (σ → τ) ∈ T, where 0 is the type of natural numbers, and σ → τ is the type of mappings from objects of type σ to objects of type τ.

(2.1.1.1)

LET p, q, r, s: T, τ, r, σ

((p@p)<p)&(((s&q)<p)>((s>q)<p)) ; **FFFF FFFF FFFF FFFF** (2.1.1.2)

In this way, 1 ≡ 0 → 0 is the type of functions from numbers to numbers, and where n + 1 ≡ n → 0.

(2.1.2.1)

((p+(%p>#p))=(p>(p@p)) &((%p>#p)=((p@p)>(p@p))) ;
N**F**N**F** N**F**N**F** N**F**N**F** N**F**N**F** (2.1.2.2)

Remark 2.1.2.2: If in Eq. 2.1.2.1 the one and zero are taken as tautology and contradiction, then the result is strengthened as the same:

((p= p)=((p@p)>(p@p)))&((p+(p= p))=(p>(p@p))) ;
T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (2.1.2.3)

2.3. **Introducing nets.** We introduce the notion of net and associated concepts. We first consider the following standard definition ...

Definition 2.7. [Nets] A set $D \neq \emptyset$ with a binary relation '$\preccurlyeq$' is directed if

(a) The relation is transitive, i.e. $(\forall x, y, z \in D)([x \preccurlyeq y \wedge y \preccurlyeq z] \rightarrow x \preccurlyeq z)$.
(b) For $x, y \in D$, there is $z \in D$ such that $x \preccurlyeq z \wedge y \preccurlyeq z$.
(c) The relation $\preccurlyeq$ is reflexive, i.e. $(\forall x \in D)(x \preccurlyeq x)$ (2.7.1)

(p@(p@p))>((((((#q&#r)&#s)<p)&((~(r<q)&~(s<r))>~(s<q)))&(((q&r)<p)>((s<p)>(~(s<q)&
~(s<r)))))&((#q<p)&~(q<q))) ; T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (2.7.2)

Eqs. 2.1.2.2 and 2.7.2 as rendered are *not* tautologous. This refutes reverse mathematics and nets.