Refutation of the Barwise compactness theorem via sublanguage L_A

Abstract: A definition with variant to establish a sublanguage in support of the Barwise compactness theorem is *not* tautologous. By extension the theorem is also refuted. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Bell, J.L. (2016). 5. Sublanguages of $L(\omega_1,\omega)$ and the Barwise compactness theorem. plato.stanford.edu/entries/logic-infinitary/#5. jbell@uwo.ca

We say that L_A is a sublanguage of $L(\omega_1,\omega)$ if the following conditions are satisfied:

(ii.) if $\varphi, \psi \in L_A$, then $\varphi \wedge \psi \in L_A$ and $\neg\varphi \in L_A$ (5.ii.1)

((p&s)<q)>((p&(s<q))&(~p<q)) ;
TTTT TTTT TT**F**T TT**F**T (5.ii.2)

Remark 5.ii.2: Eq. 5.ii.1 could be interpreted and rendered in part as $(\varphi\wedge\psi)\in L_A$ for

((p&s)<q)>(((p&s)<q)&(~p<q)) ;
TTTT TTTT T**F**TT T**F**TT (5.ii.3)

Eq. 5.ii.2 (with 5.ii.3 as rendered) is *not* tautologous. The purpose of Eq. 5.ii.1 was in the first place to support a proof of the Barwise compactness theorem, herewith refuted by extension.