

Confirmation of VL4 as complete

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Abstract: Logic VL4 is defined as a bivalent classical logic that maps quantifiers to modalities as a tautology making VL4 complete. Paraconsistent, non bivalent, vector logics are defined as *non* tautologous fragments of VL4 as a universal logic.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated proof value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬; + Or, ∨, ∪, ⊔; - Not Or; & And, ∧, ∩, ⊓, ; \ Not And;
 > Imply, greater than, →, ⇒, ⇨, >, ⊃, ≻; < Not Imply, less than, ∈, <, ⊂, ⊆, ≠, ≪, ≲;
 = Equivalent, ≡, :=, ⇔, ↔, ≐, ≈, ≃; @ Not Equivalent, ≠;
 % possibility, for one or some, ∃, ∂, M; # necessity, for every or all, ∀, □, L;
 (z=z) **T** as tautology, **T**, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
 (%z>#z) **N** as non-contingency, Δ, ordinal 1; (%z<#z) **C** as contingency, ∇, ordinal 2;
 ~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
 Note for clarity, we usually distribute quantifiers onto each designated variable.

Logic VL4, for variant ML4, is a bivalent classical logic that maps quantifiers to modalities: the existential quantifier is equivalent to the modal operator of possibility; and the universal quantifier is equivalent to the modal operator of necessity. This definition is expressed in words as:

The possibility of p implying the necessity of p implies
 the possibility of q implying the necessity of q. (1.1)

$$((\%p>\#p)>(\%q>\#q)); \quad \text{TTTT TTTT TTTT TTTT} \quad (1.2)$$

Eq. 1.2 as rendered invokes the equivalence of the quantifiers to modal operators to map the logical value of non contingency **N** or truthity to imply the logical value of non contingency **N** or truthity. Eq. 1.2 results in **T** or tautology as self proving and complete.

Paraconsistent, non-bivalent, vector logics are expressed in words as:

The possibility of p implying the necessity of p implies
 the possibility of q *not* implying the necessity of q. (1.1)

$$(\%p>\#p)>(\%q<\#q); \quad \text{CCCC CCCC CCCC CCCC} \quad (2.2)$$

Eq. 2.2 invokes the logical value of non contingency **N** or truthity to imply the logical value of contingency **C** or falsity. Eq. 2.2 results in **C** or falsity as *not* tautologous.

VL4 classifies conjectures as a tautologous or *not* tautologous result, with the latter to include the contradictory result. This qualifies VL4 as a universal logic because it maps known logics, some of which as *non* tautologous fragments of VL4, another indication that VL4 is complete.