Confirmation of VL4 as complete

© Copyright 2019 by Colin James III  All rights reserved.

Abstract: Logic VL4 is defined as a bivalent classical logic that maps quantifiers to modalities as a tautology making VL4 complete. Paraconsistent, non bivalent, vector logics are defined as *non* tautologous fragments of VL4 as a universal logic.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, \( F \) as contradiction, \( N \) as truthity (non-contingency), and \( C \) as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

Logic VL4, for variant MŁₜₜ, is a bivalent classical logic that maps quantifiers to modalities: the existential quantifier is equivalent to the modal operator of possibility; and the universal quantifier is equivalent to the modal operator of necessity. This definition is expressed in words as:

\[
\text{The possibility of } p \text{ implying the necessity of } p \text{ implies}
\]
\[
\text{the possibility of } q \text{ implying the necessity of } q. \quad (1.1)
\]

\[\begin{align*}
((%p>#p)>((%q>#q)); \\
TTTT TTTT TTTT TTTT
\end{align*}\]  

(1.2)

Eq. 1.2 as rendered invokes the equivalence of the quantifiers to modal operators to map the logical value of non contingency \( N \) or truthity to imply the logical value of non contingency \( N \) or truthity. Eq. 1.2 results in \( T \) or tautology as self proving and complete.

Paraconsistent, non-bivalent, vector logics are expressed in words as:

\[
\text{The possibility of } p \text{ implying the necessity of } p \text{ implies}
\]
\[
\text{the possibility of } q \text{ *not* implying the necessity of } q. \quad (1.1)
\]

\[\begin{align*}
(%p>#p)>(%q<#q) ; \\
cccc cccc cccc cccc
\end{align*}\]  

(2.2)

Eq. 2.2 invokes the logical value of non contingency \( N \) or truthity to imply the logical value of contingency \( C \) or falsity. Eq. 2.2 results in \( C \) or falsity as *not* tautologous.

VL4 classifies conjectures as a tautologous or *not* tautologous result, with the latter to include the contradictory result. This qualifies VL4 as a universal logic because it maps known logics, some of which as *non* tautologous fragments of VL4, another indication that VL4 is complete.