**Refutation of Pascal's wager**



**Abstract:** The antecedent and consequent of the thought experiment of Pascal's wager are *not* tautologous. However, to determine gain by one wager or the other is tautologous. This refutes the conjecture of Pascal's wager as ultimately not allowing reason to determine faith. In other words, the "existence of God is possible to prove by human reason". What follows furthermore is that the existence of God is more profitable from this thought experiment. Therefore the conjecture forms a tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,· ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⇻ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↚, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≙, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~( y < x) ( x ≤ y), ( x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Pascal%27s_wager

"The wager uses the following logic (excerpts from Pensées, part III, §233):

> God is, or God is not. Reason cannot decide between the two alternatives. A Game is being played... where heads or tails will turn up. You must wager (it is not optional). Let us weigh the gain and the loss in wagering that God is. Let us estimate these two chances. If you gain, you gain all; if you lose, you lose nothing." (1.0)

We write Eq. 1.0 as:

> Antecedent: ((God is, or God is not) implies (either (if God is, then wager gains) or (if God is not, then wager breaks even)) (1.1.1)
>
> LET p, q: God, gain
>
> (p+~p)>((p>(q>(s@s)))&(~p>(q=(s@s)))) ;
> TT**FF** TT**FF** TT**FF** TT**FF** (1.1.2)
>
> **Remark 1.1.2:** Eq. 1.1.2 can be weakened by inserting modal operators as
>
> #(p+~p)>%((p>(q>(s@s)))&(~p>(q=(s@s)))) ;
> TTCC TTCC TTCC TTCC (1.1.3)
>
> Consequent: implies ((if God is not, then wager breaks even) is more profitable than (if God is, then wager gains)). (1.2.1)

(~p>(q=(s@s)))>(p>(q>(s@s))) ;
TTT**F** TTT**F** TTT**F** TTT**F** (1.2.2)

"Pascal begins by painting a situation where both the existence and non-existence of God are impossible to prove by human reason." (2.0)

We write Eq. 2.0 as consequent Eq. 1.1.1 implies antecedent Eq. 1.2.1: (2.1)

((p+~p)>((p>(q>(s@s)))&(~p>(q=(s@s)))))>((~p>(q=(s@s)))>(p>(q>(s@s)))) ;
TTTT TTTT TTTT TTTT (2.2)

**Remark 2.2:** If the antecedent is chosen as the weakened modal Eq. 1.1.3, the result is different from Eq. 2.2 and is *not* tautologous:

(#(p+~p)>%((p>(q>(s@s)))&(~p>(q=(s@s)))))>((~p>(q=(s@s)))>(p>(q>(s@s)))) ;
TTTN TTTN TTTN TTTN (2.3)

The antecedent Eq. 1.1.2 of Pascal's conjecture and the consequent Eq. 1.2.2 are *not* tautologous. However, to determine gain by one wager or the other as in Eq. 2.2 results in a theorem to do just that. This refutes the conjecture of Pascal's wager as ultimately not allowing reason to determine faith. In other words, the "existence of God is possible to prove by human reason". What follows is that existence of God is more profitable from the thought experiment.