On an equality between space-time curvature and quantum entropy

A Preprint

Alexandre Harvey-Tremblay
Independent Researcher
aht@protonmail.ch

May 8, 2019

Abstract

We introduce the quantum statistical physics of space-time events. Quantum statistical physics connects a microscopic/quantum description to a macroscopic/bulk state by the use of statistical priors and under the principle of maximum entropy. As the main result, we show that the statistical distribution of events in space-time, quantified by entropy, is a microscopic equivalent to the bulk curvature of space-time.

Keywords Entropy · General Relativity · Statistical Physics

1 Introduction

What can be more well-suited to a statistical framework than a set of randomly selected events?

We state immediately that we will not be attempting to equate the entropy of existing quantum theories, for instance, QED or others, to the curvature of space-time.

We also state that we are not attributing an entropy to space (this is done by the Bekenstein-Hawking entropy and requires a horizon), but to space-time. In space-time (like in space) the role of the entropy will be to hide a region of it from the observer. In space-time, we will, in fact, call this hidden region the future, and it will be hidden from the present by an entropy.

Finally, we state that the Von Neumann entropy is inadequate for the description of the entropy of this ensemble, as it is not aware of measurement entropy and therefore only accounts for a subset of the total event entropy. We thus introduce a generalization of entropy, well defined for this ensemble, which we call space-time entropy.

Our goal will then be to prove that the statistical distribution of events in space-time, quantified by the entropy, equates the curvature of space-time. Regions of high curvature (the macroscopic/bulk description) represent regions of high event entropy (the microscopic/quantum description).

To prove our main result, we will introduce a quantum statistical ensemble with its own object of study; the space-time event. The partition function of the ensemble will be a full-fledged quantum partition function with features normally attributed to quantum systems: operators, quantum entropy, measurement basis, etc. These features come out because events are introduced into the partition function by the use of a non-commutative geometric basis.

We justify the applicability of the framework of statistical physics to an ensemble of events under a posited conservation relation between information, entropy, and geometry, and not via the usual law of conservation of energy. Consequently, we will not be using the Gibbs entropy to derive the ensemble, and instead, we will use the Shannon entropy. Energy will come into play, but only later when movement and action are added on top of the geometry and when the Einstein field equations are derived from this addition.

This paper assumes that the reader is familiar with geometric algebra, special and general relativity and with statistical physics. First, we summarize the prior work on the subject of entropy and gravity.

1.1 Entropy and space-time

An equivalence between quantum entropy and space-time has been (or the very least could have been) anticipated since probably the better part of four decades. The first hints were provided by the work of [Bekenstein(1973), Bekenstein(1974), Bekenstein(1980)] regarding the similarities between black holes and thermodynamics, culminating in the four laws of black hole thermodynamics. The temperature, originally introduced by analogy, was soon augmented to a real notion by [Hawking(1974)] with the discovery of the Hawking temperature derivable from quantum field theory on curved space-time. Another very significant result is that of Bekenstein-Hawking entropy, connecting the area of the surface of a horizon to be proportional to one fourth the number of elements with Planck area that can be fitted on the surface: $S = k_B c^3/(4\hbar G)A$. This last hint strongly suggests that quantum theory meets general relativity at the surface of black hole horizons.

Another key result is that of [Jacobson(1995)] who directly recovered the Einstein field equation as an equation of state of a suitable thermodynamic system. To justify the emergence of general relativity from entropy, Jacobson first postulated that the energy flowing out of horizons becomes hidden from observers. Next, he attributed the role of heat to this energy for the same reason that heat is energy that is inaccessible for work. In this case, its effects are felt, not as "warmth", but as gravity originating from the horizon. Finally, with the assumption that the heat is proportional to the area A of the system under some proportionality constant η, and some legwork, the Einstein field equations are eventually recovered. This is the essence of the argument presented by Ted Jacobson.

Later, [Verlinde(2011)] proposed an entropic derivation of the classical law of inertia and classical gravity. He compared the emergence of such laws to that of an entropic force, such as a polymer in a warm bath. Each law is emergent from the equation $TdS = Fdx$, under the appropriate temperature and a posited entropy relation. His proposal has encouraged a plurality of attempts to reformulate known laws of physics using the framework of statistical physics. [Visser(2011)] provides, in the introduction to his paper, a good summary of the literature on the subject. The ideas of Verlinde have been applied to loop quantum gravity ([Smolin(2010)]), the Coulomb force ([Wang(2010)]), Yang-Mills gauge fields ([Freund(2010)]), and cosmology ([Cai et al.(2010)Cai, Cao, and Ohta, Li and Wang(2010), Easson et al.(2011)Easson, Frampton, and Smoot]). Some criticism has, however, been voiced, notably by ([Hossenfelder(2010), Kobakhidze(2011a), Gao(2011), Hu(2011), Kobakhidze(2011b)]), as well as by Visser himself ([Visser(2011)]).

Finally, and even more recently, a connection between entanglement entropy and general relativity has been supported by multiple publications: [Swingle(2012), Van Raamsdonk(2010), Evenbly and Vidal(2011), Faulkner et al.(2014)Faulkner, Guica, Hartman, Myers, and Van Raamsdonk, Faulkner et al.(2017)Faulkner, Haehl, Hijano, Parrikar, Rabideau, and Van Raamsdonk, Czech et al.(2016)Czech, Lamprou, McCandlish, and Sully, Maldacena(1999), Swingle and Van Raamsdonk(2014), Maldacena and Susskind(2013), Sanches and Weinberg(2016), Susskind(2016), Cao et al.(2017)Cao, Carroll, and Michalakis, Bao et al.(2017b)Bao, Cao, Carroll, and McAllister, Bao et al.(2017a)Bao, Cao, Carroll, and Chatwin-Davies, Susskind(2017), Padmanabhan(2010), Jacobson(2016), Carroll and Remmen(2016)].

1.2 Recap: statistical physics

In statistical physics, we are interested in the distribution that maximizes the Boltzmann entropy,

$$S = -k_B \sum_{q\in\mathbb{Q}} \rho(q) \ln \rho(q) \tag{1}$$

subject to the fixed macroscopic quantities (the statistical priors). The solution is the Gibbs ensemble. Typical thermodynamic quantities are shown in Table 1.

Taking these quantities as examples, the partition function (Gibbs ensemble) becomes:

$$Z = \sum_{q\in\mathbb{Q}} e^{-\beta E(q) - \gamma V(q) - \delta N(q)} \tag{2}$$

The probability of occupation of a micro-state (Gibbs measure) is:

$$\rho(q) = \frac{1}{Z} e^{-\beta E(q) - \gamma V(q) - \delta N(q)} \tag{3}$$

Symbol	**Name**	**Units**	**Type**
$E(q)$	energy	[Joule]	extensive
$1/T = k_B \beta$	temperature	$1/$[Kelvin]	intensive
$\overline{E}$	average energy	[Joule]	macroscopic
$V(q)$	volume	[meter^3]	extensive
$p/T = k_B \gamma$	pressure	[Joule/(Kelvin-meter^3)]	intensive
$\overline{V}$	average volume	[meter^3]	macroscopic
$N(q)$	number of particles	[kg]	extensive
$-\mu/T = k_B \delta$	chemical potential	[Joule/(Kelvin-kg)]	intensive
$\overline{N}$	average number of particles	[kg]	macroscopic

Table 1: Typical thermodynamic quantities

The average values are:

$$\overline{E} = \sum_{q \in Q} \rho(q) E(q) \tag{4}$$

$$\overline{V} = \sum_{q \in Q} \rho(q) V(q) \tag{5}$$

$$\overline{N} = \sum_{q \in Q} \rho(q) N(q) \tag{6}$$

and the variance are for each quantity is:

$$\overline{(\Delta E)^2} = \frac{\partial^2 \ln Z}{\partial \beta^2} \tag{7}$$

$$\overline{(\Delta V)^2} = \frac{\partial^2 \ln Z}{\partial \gamma^2} \tag{8}$$

$$\overline{(\Delta N)^2} = \frac{\partial^2 \ln Z}{\partial \delta^2} \tag{9}$$

The entropy can be obtained from the partition function and is given by:

$$S = k_B \left(\ln Z + \beta \overline{E} + \gamma \overline{V} + \delta \overline{N} \right) \tag{10}$$

The laws of thermodynamics can be recovered by taking the following derivatives:

$$\left.\frac{\partial S}{\partial \overline{E}}\right|_{\overline{V},\overline{N}} = \frac{1}{T} \qquad \left.\frac{\partial S}{\partial \overline{V}}\right|_{\overline{E},\overline{N}} = \frac{p}{T} \qquad \left.\frac{\partial S}{\partial \overline{N}}\right|_{\overline{E},\overline{V}} = -\frac{\mu}{T} \tag{11}$$

and grouping them as follows:

$$d\overline{E} = TdS - pd\overline{V} + \mu d\overline{N} \tag{12}$$

This is the equation of the state of the system.

2 Definitions

Definition 1 (Set of events)**.** *We define a set of events $\mathbb{Q}$ as:*

$$\mathbb{Q} \subset \{q : q \in R^{n+m}\} \tag{13}$$

The quantity $n + m$ denotes the number of dimensions of the events. By convention, we pose that n denotes the number of time dimensions and m denotes the number of space dimensions. An event q is represented by an $(n + m)$-tuple:

$$q := (\underbrace{X_0, ...X_{n-1}}_{\text{time terms}}, \underbrace{X_n, ...X_{n+m-1}}_{\text{space terms}}) \tag{14}$$

where each member of the tuple is an element of $\mathbb{R}$.

Definition 2 (Algebra of events)**.** *In order of simplest to more general, we define an algebraic basis for events as follows:*

- *For Euclidean space, we use the basis $\sigma_x, \sigma_y, \sigma_z$ with the following properties:*

$$\sigma_x^2 = 1 \tag{15}$$
$$\sigma_y^2 = 1 \tag{16}$$
$$\sigma_z^2 = 1 \tag{17}$$
$$\sigma_x\sigma_y + \sigma_y\sigma_x = 0 \tag{18}$$
$$\sigma_x\sigma_z + \sigma_z\sigma_x = 0 \tag{19}$$
$$\sigma_y\sigma_z + \sigma_z\sigma_y = 0 \tag{20}$$

  One instance of the elements $\sigma_x, \sigma_y, \sigma_z$ are simply the well-known Pauli matrices. These elements correspond to a basis of the Geometric algebra $Cl_{0,3}(\mathbb{R})$. As an example, an event (point) in Euclidean space can be written algebraically using the basis as:

$$\vec{q} := x\sigma_x + y\sigma_y + z\sigma_z \tag{21}$$

- *For Lorentzian space-time (3+1), we use the basis $\gamma_x, \gamma_y, \gamma_z, \gamma_t$ with the following properties:*

$$\gamma_x^2 = -1 \tag{22}$$
$$\gamma_y^2 = -1 \tag{23}$$
$$\gamma_z^2 = -1 \tag{24}$$
$$\gamma_t^2 = 1 \tag{25}$$
$$\gamma_x\gamma_y + \gamma_y\gamma_x = 0 \tag{26}$$
$$\gamma_x\gamma_z + \gamma_z\gamma_x = 0 \tag{27}$$
$$\gamma_x\gamma_t + \gamma_t\gamma_x = 0 \tag{28}$$
$$\gamma_y\gamma_z + \gamma_x\gamma_y = 0 \tag{29}$$
$$\gamma_y\gamma_t + \gamma_t\gamma_y = 0 \tag{30}$$
$$\gamma_z\gamma_t + \gamma_t\gamma_z = 0 \tag{31}$$

  One instance of the elements $\gamma_x, \gamma_y, \gamma_z, \gamma_t$ are simply the well-known Dirac matrices. These elements correspond to a basis of the geometric algebra $Cl_{1,3}(\mathbb{R})$. As an example, an event in Lorentzian space can be written algebraically using the basis as:

$$\vec{q} := t\gamma_t + x\gamma_x + y\gamma_y + z\gamma_z \tag{32}$$

- *For a general ortho-normal basis, we use a geometric algebra $Cl_{n,m}(\mathbb{R})$. First, one defines a diagonal metric $\eta_{\mu\nu}$ with a signature. For instance, in $3+1$ space-time with signature $+---$, η is:*

$$\eta = \begin{array}{cc} \begin{array}{cccc} \mu{=}0 & \mu{=}1 & \mu{=}2 & \mu{=}3 \end{array} & \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} & \begin{array}{c} \nu{=}0 \\ \nu{=}1 \\ \nu{=}2 \\ \nu{=}3 \end{array} \end{array} \tag{33}$$

  Then $\forall \mu \in \{0, ..., m+n-1\} \wedge \forall \nu \in \{0, .., m+n-1\}$, the properties of the basis elements are:

$$\frac{1}{2}(e_\mu e_\nu + e_\nu e_\mu) = \eta_{\mu\nu} \tag{34}$$

- *Finally, for a general (not necessarily ortho-normal) basis, the properties of the elements of the basis of the algebra are:*

$$\frac{1}{2}(e_\mu e_\nu + e_\nu e_\mu) = g_{\mu\nu} \tag{35}$$

  where

$$g = \begin{array}{cc} \begin{array}{cccc} \mu{=}0 & \mu{=}1 & \mu{=}2 & \mu{=}3 \end{array} & \\ \begin{pmatrix} g_{00} & g_{10} & g_{20} & g_{30} \\ g_{01} & g_{11} & g_{21} & g_{31} \\ g_{02} & g_{12} & g_{22} & g_{32} \\ g_{03} & g_{13} & g_{23} & g_{33} \end{pmatrix} & \begin{array}{c} \nu{=}0 \\ \nu{=}1 \\ \nu{=}2 \\ \nu{=}3 \end{array} \end{array} \tag{36}$$

Then using a general basis, we write the $(m+n)$-tuple of an event q as an algebraic equation:

$$\vec{q} := \underbrace{X_0 e_0 + ... + X_{n-1} e_{n-1}}_{\text{time terms}} + \underbrace{X_n e_n + ... + X_{n+m-1} e_{n+m-1}}_{\text{space terms}} \tag{37}$$

Definition 3 (Interval)**.** *We give two examples, then the general case.*

- *Using the basis for $Cl_{0,3}(\mathbb{R})$ (a.k.a the Pauli algebra), let*

$$\vec{q}_1 := x_1\sigma_x + y_1\sigma_y + z_1\sigma_z \tag{38}$$
$$\vec{q}_2 := x_2\sigma_x + y_2\sigma_y + z_2\sigma_z \tag{39}$$

  Then, the interval between these events is:

$$\begin{aligned}
(\vec{q}_1 - \vec{q}_2)^2 &= ((x_1 - x_2)\sigma_x + (y_1 - y_2)\sigma_y + (z_1 - z_2)\sigma_z)^2 && (40)\\
&= ((\Delta x)\sigma_x + (\Delta y)\sigma_y + (\Delta z)\sigma_z)^2 && (41)\\
&= (\Delta x)\sigma_x(\Delta x)\sigma_x + (\Delta x)\sigma_x(\Delta y)\sigma_y + (\Delta x)\sigma_x(\Delta z)\sigma_z \\
&\quad + (\Delta y)\sigma_y(\Delta x)\sigma_x + (\Delta y)\sigma_y(\Delta y)\sigma_y + (\Delta y)\sigma_y(\Delta z)\sigma_z \\
&\quad + (\Delta z)\sigma_z(\Delta x)\sigma_x + (\Delta z)\sigma_z(\Delta y)\sigma_y + (\Delta z)\sigma_z(\Delta z)\sigma_z && (42)\\
&= (\Delta x)^2\sigma_x^2 + (\Delta y)^2\sigma_y^2 + (\Delta z)^2\sigma_z^2 \\
&\quad + \Delta x\Delta y(\sigma_x\sigma_y + \sigma_y\sigma_x) + \Delta x\Delta z(\sigma_x\sigma_z + \sigma_z\sigma_x) \\
&\quad + \Delta y\Delta z(\sigma_y\sigma_z + \sigma_z\sigma_y) && (43)\\
&= (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 && (44)
\end{aligned}$$

  where $\delta_{\mu\nu} = \frac{1}{2}(\sigma_\mu\sigma_\nu + \sigma_\nu\sigma_\mu)$.

- *Using the basis for $Cl_{1,3}(\mathbb{R})$ (a.k.a Dirac algebra), let*

$$\vec{q}_1 := t_1\gamma_t + x_1\gamma_x + y_1\gamma_y + z_1\gamma_z \tag{45}$$
$$\vec{q}_2 := t_2\gamma_t + x_2\gamma_x + y_2\gamma_y + z_2\gamma_z \tag{46}$$

Then, the interval between these events is:

$$\begin{aligned}
(\vec{q}_1 - \vec{q}_2)^2 &= ((t_1 - t_2)\gamma_t + (x_1 - x_2)\gamma_x + (y_1 - y_2)\gamma_y + (z_1 - z_2)\gamma_z)^2 && (47)\\
&= ((\Delta t)\gamma_t + (\Delta x)\gamma_x + (\Delta y)\gamma_y + (\Delta z)\gamma_z)^2 && (48)\\
&=(\Delta t)\gamma_t(\Delta t)\gamma_t + (\Delta t)\gamma_t(\Delta x)\gamma_x + (\Delta t)\gamma_t(\Delta y)\gamma_y + (\Delta t)\gamma_t(\Delta z)\gamma_z\\
&\quad + (\Delta x)\gamma_x(\Delta t)\gamma_t + (\Delta x)\gamma_x(\Delta x)\gamma_x + (\Delta x)\gamma_x(\Delta y)\gamma_y + (\Delta x)\gamma_x(\Delta z)\gamma_z\\
&\quad + (\Delta y)\gamma_y(\Delta t)\gamma_t + (\Delta y)\gamma_y(\Delta x)\gamma_x + (\Delta y)\gamma_y(\Delta y)\gamma_y + (\Delta y)\gamma_y(\Delta z)\gamma_z\\
&\quad + (\Delta z)\gamma_z(\Delta t)\gamma_t + (\Delta z)\gamma_z(\Delta x)\gamma_x + (\Delta z)\gamma_z(\Delta y)\gamma_y + (\Delta z)\gamma_z(\Delta z)\gamma_z && (49)\\
&=(\Delta t)^2\gamma_t^2 + (\Delta x)^2\gamma_x^2 + (\Delta y)^2\gamma_y^2 + (\Delta z)^2\gamma_z^2\\
&\quad + \Delta t\Delta x(\gamma_t\gamma_x + \gamma_t\gamma_x) + \Delta t\Delta y(\gamma_t\gamma_y + \gamma_y\gamma_t) + \Delta t\Delta z(\gamma_t\gamma_z + \gamma_z\gamma_t)\\
&\quad + \Delta x\Delta y(\gamma_x\gamma_y + \gamma_y\gamma_x) + \Delta x\Delta z(\gamma_x\gamma_z + \gamma_z\gamma_x)\\
&\quad + \Delta y\Delta z(\gamma_y\gamma_z + \gamma_z\gamma_y) && (50)\\
&=(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 && (51)
\end{aligned}$$

where $\delta_{\mu\nu} = \frac{1}{2}(\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu)$.

- *Likewise, using a general (not necessarily ortho-normal) basis, the vectors are:*

$$\vec{q}_1 := t_1e_t + x_1e_x + y_1e_y + z_1e_z \tag{52}$$
$$\vec{q}_2 := t_2e_t + x_2e_x + y_2e_y + z_2e_z \tag{53}$$

and the interval is defined using the familiar metric tensor g:

$$\begin{aligned}
(\vec{q}_1 - \vec{q}_2)^2 &= ((t_1 - t_2)e_t + (x_1 - x_2)e_x + (y_1 - y_2)e_y + (z_1 - z_2)e_z)^2 && (54)\\
&= ((\Delta t)e_t + (\Delta x)e_x + (\Delta y)e_y + (\Delta z)e_z)^2 && (55)\\
&=(\Delta t)e_t(\Delta t)e_t + (\Delta t)e_t(\Delta x)e_x + (\Delta t)e_t(\Delta y)e_y + (\Delta t)e_t(\Delta z)e_z\\
&\quad + (\Delta x)e_x(\Delta t)e_t + (\Delta x)e_x(\Delta x)e_x + (\Delta x)e_x(\Delta y)e_y + (\Delta x)e_x(\Delta z)e_z\\
&\quad + (\Delta y)e_y(\Delta t)e_t + (\Delta y)e_y(\Delta x)e_x + (\Delta y)e_y(\Delta y)e_y + (\Delta y)e_y(\Delta z)e_z\\
&\quad + (\Delta z)e_z(\Delta t)e_t + (\Delta z)e_z(\Delta x)e_x + (\Delta z)e_z(\Delta y)e_y + (\Delta z)e_z(\Delta z)e_z && (56)\\
&=(\Delta t)^2e_t^2 + (\Delta x)^2e_x^2 + (\Delta y)^2e_y^2 + (\Delta z)^2e_z^2\\
&\quad + \Delta t\Delta x(e_te_x + e_te_x) + \Delta t\Delta y(e_te_y + e_ye_t) + \Delta t\Delta z(e_te_z + e_ze_t)\\
&\quad + \Delta x\Delta y(e_xe_y + e_ye_x) + \Delta x\Delta z(e_xe_z + e_ze_x)\\
&\quad + \Delta y\Delta z(e_ye_z + e_ze_y) && (57)\\
&=\sum_{\mu\nu} g_{\mu\nu}\Delta X_\mu\Delta X_\nu && (58)
\end{aligned}$$

where $g_{\mu\nu} = \frac{1}{2}(e_\mu e_\nu + e_\nu e_\mu)$.

Definition 4 (Physical quantities of the partition function)**.** *We first introduce two physical quantities: 1) the entropic repetency $\tilde{k}$ (normally the repetency is represented by the symbol $\tilde{\nu}$, but we already use the symbol ν extensively for the indices of our basis; therefore, we opt to use $\tilde{k}$ here to eliminate confusion); and 2) the entropic frequency f. Specifically, $\tilde{k} = k/2\pi = 1/\lambda$, where k is the wave-number and λ is the wavelength.*

Symbol	Name	Units	Type
$x(q)$	space	[meter]	extensive
$\tilde{k}$	entropic repetency	[1/meter]	intensive
$\overline{x}$	thermal space	[meter]	macroscopic
$t(q)$	time	[second]	extensive
f	entropic frequency	[1/second]	intensive
$\overline{t}$	thermal time	[second]	macroscopic
$c := f/\tilde{k}$	entropic speed	[meter/second]	intensive

Table 2: The physical quantities of the ensemble

These quantities are the conjugated variables to a distance x and time t, respectively. By convention, we prefix the Lagrange multipliers with the word "entropic", and its averaged conjugated quantity will be prefixed with the word "thermal". $\tilde{k}$ and f are both intensive properties, whereas x and t are extensive. Indeed, a process taking 1 min followed by a process taking 2 min takes a total of 3 min (extensive). For the x quantity; walking 1 meter followed by walking 2 meter implies one has walked a total of 3 meter (extensive). Adding or removing clocks from a group of clocks ticking at a frequency f (say once per second) has no impact on the frequency of the other elements of the group (intensive). The same argument applies to the entropic repetency (intensive). The units of $\tilde{k}$ are m^{-1}, the units of x are the meters, the units of t are the seconds, and the units of f are s^{-1}. Finally, we define the quantity $c := f/\tilde{k}$. These quantities are summarized in Table 2.

Definition 5 (Ensemble of events)**.** *An ensemble of events $\mathbb{Q}$ is the probability distribution $\rho(q)$, which maximizes the Shannon entropy (in base e):*

$$S = -\sum_{q\in\mathbb{Q}} \rho(q) \ln \rho(q) \tag{59}$$

using the macroscopic priors defined in table 2 for events expressed in the algebra of events $Cl(n, m)$. Specifically, $\forall i \in \{0, ..., n+m-1\}$ the priors to the ensemble are:

$$\langle X_i e_i \rangle = \sum_{q\in\mathbb{Q}} X_i[q] e_i \rho(q) \tag{60}$$

The functions $X_i[q]$ are maps $X_i : \mathbb{Q} \to \mathbb{R}$ where $X_i[q]$ returns the value of the i^{th} element of the $(m+n)$-tuple of q.

3 Result

Theorem 1 (Partition function)**.** *The partition function of the ensemble of events is:*

$$Z = \sum_{q\in\mathbb{Q}} \exp\left(-f \underbrace{(X_0[q]e_0 + ... + X_{n-1}[q]e_{n-1})}_{\text{time terms}} - \tilde{k} \underbrace{(X_n[q]e_n + ... + X_{n+m-1}[q]e_{n+m-1})}_{\text{space terms}}\right) \tag{61}$$

where $\tilde{k}$ and f are Lagrange multipliers. The probability distribution $\rho(q)$ is:

$$\rho(q) = \frac{1}{Z} \exp\left(-f \underbrace{(X_0[q]e_0 + ... + X_{n-1}[q]e_{n-1})}_{\text{time terms}} - \tilde{k} \underbrace{(X_n[q]e_n + ... + X_{n+m-1}[q]e_{n+m-1})}_{\text{space terms}}\right) \tag{62}$$

Proof. One obtains the partition function Z with the usual method of the Lagrange multipliers.

1. The constraints are:

$$1 = \sum_{q \in \mathbb{Q}} \rho(q) \tag{63}$$

$$\langle X_i e_i \rangle = \sum_{q \in \mathbb{Q}} X_i[q] e_i \rho(q) \tag{64}$$

2. The Lagrange equation is:

$$\mathcal{L} = \left(- \sum_{q \in \mathbb{Q}} \rho(q) \ln \rho(q) \right) - \lambda \left(\sum_{q \in \mathbb{Q}} \rho(q) - 1 \right) - \sum_i \left(\lambda_i \sum_{q \in \mathbb{Q}} X_i[q] e_i \rho(q) - \langle X_i e_i \rangle \right) \tag{65}$$

   where λ and the set of λ_i are Lagrange multipliers.

3. Maximizing $\mathcal{L}$ with respect to $\rho(q)$ is done by taking its derivative and posing it equal to zero:

$$\frac{\partial \mathcal{L}}{\partial \rho(q)} = -\ln \rho(q) - 1 - \lambda - \sum_i (\lambda_i X_i[q] e_i) = 0 \tag{66}$$

4. Solving for $\rho(q)$ one obtains:

$$\rho(q) = \exp(-1 - \lambda) \exp \left(- \sum_i \lambda_i X_i[q] e_i \right) \tag{67}$$

5. From the constraint $1 = \sum_{q \in \mathbb{Q}} \rho(q)$, we can find the expression for $\exp(-1 - \lambda)$ as follows:

$$1 = \sum_{q \in \mathbb{Q}} \rho(q) \tag{68}$$

$$= \sum_{q \in \mathbb{Q}} \exp(-1 - \lambda) \exp \left(- \sum_i \lambda_i X_i[q] e_i \right) \tag{69}$$

$$\implies \exp(-1 - \lambda) = \frac{1}{\sum_{q \in \mathbb{Q}} \exp\left(- \sum_i \lambda_i X_i[q] e_i\right)} \tag{70}$$

6. We then define the inverse of the last term as the partition function:

$$Z := \sum_{q \in \mathbb{Q}} \exp \left(- \sum_i \lambda_i X_i[q] e_i \right) \tag{71}$$

7. Finally, we write $\rho(q)$ using Z. We obtain:

$$\rho(q) = \frac{1}{Z} \exp \left(- \sum_i \lambda_i X_i[q] e_i \right) \tag{72}$$

□

Theorem 2 (Geometric entropy). *The entropy of the ensemble of events is:*

$$\hat{S} = \ln z + \underbrace{f\langle X_0 e_0\rangle + ... + f\langle X_{n-1} e_{n-1}\rangle}_{\text{time terms}} + \underbrace{\tilde{k}\langle X_n e_n\rangle + ... + \tilde{k}\langle X_{n+m-1} e_{n+m-1}\rangle}_{\text{space terms}} \tag{73}$$

We interpret the entropy as the information one gains by knowing which event q was randomly selected from the set of events $\mathbb{Q}$ under the probability distribution $\rho(q)$.

Proof. The Shannon entropy in base e is:

$$S = -\sum_{q\in\mathbb{Q}} \rho(q) \ln \rho(q) \tag{74}$$

Replacing $\rho(q)$ in Equation 74 with $\rho(q)$ in Equation 62, one obtains:

$$S = -\sum_{q\in\mathbb{Q}} \frac{1}{Z} \exp\left(-f X_0[q] e_0 + ...\right) \ln \frac{1}{Z} \exp\left(-f X_0[q] e_0 + ...\right) \tag{75}$$

The terms in the exponential beyond the first one have been omitted for brevity. With a few rearrangements, one obtains:

$$S = -\frac{1}{Z} \sum_{q\in\mathbb{Q}} \exp\left(-f X_0[q] e_0 + ...\right) \left[\ln \exp\left(-f X_0[q] e_0 + ...\right) - \ln Z\right] \tag{76}$$

The logarithm of the exponential of a matrix is equal to the matrix $\ln \exp A = A$. Therefore,

$$S = -\frac{1}{Z} \sum_{q\in\mathbb{Q}} \exp\left(-f X_0[q] e_0 + ...\right) \left[\left(-f X_0[q] e_0 + ...\right) - \ln Z\right] \tag{77}$$

$$= -\frac{1}{Z} \sum_{q\in\mathbb{Q}} \left(-f X_0[q] e_0 + ...\right) \exp\left(-f X_0[q] e_0 + ...\right) + \frac{1}{Z} \sum_{q\in\mathbb{Q}} \exp\left(-f X_0[q] e_0 + ...\right) \ln Z \tag{78}$$

From definition (61), $Z = \sum_{q\in\mathbb{Q}} \exp\left(-f X_0[q] e_0 + ...\right)$. Therefore,

$$S = -\frac{1}{Z} \sum_{q\in\mathbb{Q}} \left(-f X_0[q] e_0 + ...\right) \exp\left(-f X_0[q] e_0 + ...\right) + \ln Z \tag{79}$$

From definition (60), the average of a quantity $\langle X_i e_i\rangle = \sum_{q\in\mathbb{Q}} X_i[q] e_i \rho(q)$. Therefore,

$$S = f\langle X_0 e_0\rangle + \langle ...\rangle + \ln Z \tag{80}$$

□

Definition 6 (Geometric equation of state). *The equation of state of the ensemble of events is:*

$$d\hat{S} = \underbrace{f d\langle X_0 e_0\rangle + ... + f d\langle X_{n-1} e_{n-1}\rangle}_{\text{time terms}} + \underbrace{\tilde{k} d\langle X_n e_n\rangle + ... + \tilde{k} d\langle X_{n+m-1} e_{n+m-1}\rangle}_{\text{space terms}} \tag{81}$$

3.1 Main problem: Going from geometric entropy to a scalar entropy

We note that geometric entropy $\hat{S}$ is not an element of the reals, but a vector of the geometric algebra. This situation is undesirable —one would prefer a definition of entropy that is a real number— and, in fact, it will be by resolving this problem that we will obtain the interesting physics connecting the quantum entropy to the curvature of space-time. In this section, we will explore how to express this entropy more attractively as an element of $\mathbb{R}_{\geq 0}$.

As a first attempt, let us investigate the Von Neumann entropy. One can define a transformation on $\hat{S}$ such that $\hat{S} \to S \in \mathbb{R}_{\geq 0}$. The suggestion by Von Neumann is to define the entropy not by the usual Shannon/Boltzmann definition (as we have done here) but instead in reference to the choice of basis associated with the minimal value for the entropy for S. To do so, one first notices that the entropy $\hat{S}$ depends on the choice of basis. One can then rotate the basis until one finds the lowest value for the entropy. Finally, one redefines the entropy to be this minimal value instead of the previous definition. It is possible to show (proof omitted) that this is equivalent to diagonalizing the density matrix and then tracing over it according to the typical definition of the Von Neumann entropy.

$$S = \mathrm{Tr}\hat{\rho} \ln \hat{\rho} \tag{82}$$

In this case S is a scalar and it has the precious property $S(\rho) = S(U\rho U^\dagger)$.

However, the obvious weakness with this approach is that the resulting entropy is not the full and complete entropy of the initial system! The resulting Von Neumann entropy is instead an entropy defined according to, roughly, the departure away from a pure quantum state. Consequently, one who defines the entropy using the Shannon entropy (as we have done) will obtain a greater value for the entropy of the system than the value obtained by the Von Neumann entropy.

Thus, in practice, one ought to be able to extract more information from a quantum system than the Von Neumann entropy suggests (and indeed one can). So, what information can one extract from a quantum system that is missing from the Von Neumann entropy? The answer is the measurement entropy.

Definition 7 (Measurement entropy)**.** *The Shannon entropy defines the amount of information one gains by knowing which element q has been randomly select from a set $\mathbb{Q}$ under some probability distribution $\rho(q)$. In base e, it is defined as:*

$$S = -\sum_{q \in \mathbb{Q}} \rho(q) \ln \rho(q) \tag{83}$$

Suppose a quantum system defined by a function:

$$|\psi\rangle = \sum_{u \in \mathbb{U}} a(u) |u\rangle \tag{84}$$

where $\mathbb{U}$ is an eigenbasis of $|\psi\rangle$, and where $a(u)$ returns the probability amplitude associated with the eigenstate $|u\rangle$.

Then, recall that a quantum measurement is defined as a projection P

$$P(u) |\psi\rangle \to |u\rangle \text{, with probability } a(u)a(u)^* \tag{85}$$

where $a(u)^*$ is the complex conjugate of $a(u)$, such that after measurement the system $|\psi\rangle$ finds itself in one eigenstate $|u\rangle$ randomly selected from the set $\mathbb{U}$ according to a probability of $a(u)a(u)^*$.

Since the measurement involves the random selection of one element from a set, one can then use the Shannon entropy to associate an entropy to the measurement. In this case,

$$S = -\sum_{u \in \mathbb{U}} a(u)a(u)^* \ln \left[a(u)a(u)^*\right] \tag{86}$$

This entropy corresponds to the amount of information one gains by measuring a quantum system $|\psi\rangle$ within some eigenbasis $\mathbb{U}$.

The measurement entropy represents the "information distance" between the system and the final measurement outcome within some chosen basis.

We note that the Von Neumann entropy, unlike the measurement entropy, does not associate a gain of information to the measurement process. Indeed, consider the measurement of a quantum system in a pure state before and after the measurement, say from $\psi_{\text{before}} = \alpha\,|0\rangle + \beta\,|1\rangle$ to $\psi_{\text{after}} = |0\rangle$. The Von Neumann entropy, quantifying the departure from a pure state, is thus zero both before and after measurement. Both ψ_{before} and ψ_{after} are pure states.

We note that the measurement entropy, unlike the Von Neumann entropy, has the property that it is dependant on the choice of eigenbasis, and therefore the entropy is not invariant with respect to a unitary transformation U on $|\psi\rangle$.

Theorem 3. *Let us show that the measurement entropy is not invariant with respect to the unitary transformation.*

Proof. Suppose a quantum system:

$$|\psi\rangle = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{87}$$

where $\alpha\alpha^* + \beta\beta^* = 1$. The measurement entropy along the basis $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is

$$S = \alpha\alpha^* \ln\left[\alpha\alpha^*\right] + \beta\beta^* \ln\left[\beta\beta^*\right] \tag{88}$$

After a rotation of basis by the action of a unitary operator U, defined as

$$U = \begin{pmatrix} a & -b^* \\ b & a^* \end{pmatrix}, \text{ where } a, b \in \mathbb{C} \text{ and } aa^* + bb^* = 1 \tag{89}$$

Then,

$$U\,|\psi\rangle = \begin{pmatrix} a & -b^* \\ b & a^* \end{pmatrix} \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} a & -b^* \\ b & a^* \end{pmatrix} \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{90}$$

$$= (\alpha a - \beta b^*) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + (\alpha b + \beta a^*) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{91}$$

and the entropy of a measurement within the rotated basis is now:

$$\begin{aligned} S = {} & (\alpha a - \beta b^*)(\alpha a - \beta b^*)^* \ln\left[(\alpha a - \beta b^*)(\alpha a - \beta b^*)^*\right] + \\ & (\alpha b + \beta a^*)(\alpha b + \beta a^*)^* \ln\left[(\alpha b + \beta a^*)(\alpha b + \beta a^*)^*\right] \end{aligned} \tag{92}$$

We note that S in equation (88) is not equal to S in equation (92).

□

To define the measurement entropy, one must choose a basis. But what if no basis is chosen (or at least not yet); can we define an entropy which is basis-invariant (like the Von Neumann entropy) while also aware of the possibility of an eventual basis-dependant information-yielding measurement (like the measurement entropy) — thereby achieving the best of both worlds? The answer, at least for ensemble of events, will be the space-time entropy.

3.2 Main result: Space-time entropy

Theorem 4 (Space-time entropy)**.** *Due to the peculiar non-commutative properties of the algebra of events, the multivector $\hat{S}$ becomes a real number simply by squaring the equation of state:*

$$(dS)^2 = \left(\underbrace{f d\langle X_0 e_0\rangle + ... + f d\langle X_{n-1} e_{n-1}\rangle}_{\text{time terms}} + \underbrace{\tilde{k} d\langle X_n e_n\rangle + ... + \tilde{k} d\langle X_{n+m-1} e_{n+m-1}\rangle}_{\text{space terms}} \right)^2 \tag{93}$$

Unlike redefining the entropy using the Von Neumann definition, squaring $d\hat{S}$ does not change the meaning of the equation nor the entropy, yet it nonetheless erases base-specific information making $(dS)^2$ a scalar as desired.

To help us interpret the squared equation, we now divide each side of the above equation by $\tilde{k}^2$. Finally, we identify the term $\tilde{k}^{-2}(dS)^2$ as the space-time interval $(ds)^2$. Then, the equation of state is easily recognized as the interval of general relativity (the metric) in $m+n$ space.

$$\underbrace{(ds)^2}_{\text{interval}} = \left(\underbrace{cd\langle X_0 e_0\rangle + ... + cd\langle X_{n-1}e_{n-1}\rangle}_{\text{time terms}} + \underbrace{d\langle X_n e_n\rangle + ... + d\langle X_{n+m-1}e_{n+m-1}\rangle}_{\text{space terms}}\right)^2 \tag{94}$$

where $(ds)^2 = \tilde{k}^{-2}(dS)^2$.

Proof. Expanding the power of two of the right-hand side of the equation and rearranging, it is straightforward to recover the generalized metric;

$$\underbrace{\tilde{k}^{-2}(dS)^2}_{\text{entropic distance}} = \underbrace{(ds)^2}_{\text{interval}} = \sum_{\mu\nu} g_{\mu\nu} d\langle X_\mu\rangle d\langle X_\nu\rangle \tag{95}$$

□

We will now provide two related results, then we will discuss the main result.

3.3 Einstein field equations

Deriving the Einstein field equations from curved space-times can be done straightforwardly by appealing to the principle of least action. The action is defined as:

$$S = \int \mathcal{L} d^{(4)}V \tag{96}$$

In curved space-time, the 4-volume element $d^{(4)}V$ is given by:

$$d^{(4)}V = \sqrt{-g}d^4x \tag{97}$$

where g is the determinant of the metric tensor matrix. We then take the Ricci scalar as the simplest curvature invariant which produces a scalar, and we pose $\mathcal{L} := R$. We then obtain:

$$S = \int R\sqrt{-g}d^4x \tag{98}$$

which we recognize as the Hilbert-Einstein action (up to a multiplication constant).

3.4 Generalization to C-space

Our ensemble can be extended to C-space[Castroa and Pavšicb(2007)] (Clifford-space) fairly easily. In this case, the formalism can account for the description of extended elements, such as surfaces and volumes, and their distribution in space-time. The quantum properties discussed thus far will, of course, be inherited within this extension, and therefore the theory becomes a quantum theory of extended objects in space-time. The object of study of this space is no longer the space-time event, but its generalization in geometric algebra; the geometric event.

Definition 8 (Geometric event)**.** *To define a geometric event, one simply needs to make use of the full descriptive freedom afforded by Clifford algebra by including bivectors ($e_0e_1, e_0e_2, e_1e_2, ..$), trivectors ($e_0e_1e_2$), etc. into the multivector describing an event. The algebraic definition of an event then becomes:*

$$\begin{aligned}
\vec{q} :=& a && \text{(scalar)}\\
&+ X_0e_0 + X_1e_1 + ... && \text{(vectors)}\\
&+ A_0e_0e_1 + A_1e_0e_2 + ... && \text{(areas)}\\
&+ V_0e_0e_1e_2 + ... && \text{(volumes)}\\
&+ H_0e_0e_1e_2e_3 + ... && \text{(hyper-volumes)}\\
&+ etc... && (99)
\end{aligned}$$

One can then use the extended event $\vec{q}$ to define the partition function of an ensemble of extended events, as:

$$Z = \sum_{q\in\mathbb{Q}} \exp(-c\vec{q}) \tag{100}$$

With the equation of state:

$$d\hat{S} = c\langle\vec{q}d\rangle \tag{101}$$

By squaring the entropy, the non-commutative properties of the basis ensure that basis information is erased and the entropy is a scalar (this is true even with the elements of the higher dimensional basis, but only if the scalar $a = 0$). In this case, one obtains the interval of extended special relativity in C-space, which is equated to the interval entropy:

$$(ds)^2 = \tilde{k}^{-2}(dS)^2 = (c\langle\vec{q}\rangle)^2 \tag{102}$$

4 Discussion

The main result implies an entropic model of time. First, let us provide two examples, then we will list the conjectures.

1. Euclidean space: The entropy is:
$$\tilde{k}^{-2}(dS)^2 = (dx)^2 + (dy)^2 + (dz)^2 \tag{103}$$
We conclude that all events (points) have a positive entropic distance from the origin.
2. Minkowski space-time: The entropy is
$$\tilde{k}^{-2}(dS)^2 = (dx)^2 + (dy)^2 + (dz)^2 - (dt)^2 \tag{104}$$
We conclude the existence of a nullsurface on which all events have zero entropic distance from the origin and with each other.

Conjecture 1 (Present)**.** *The relation $(ds)^2 = \tilde{k}^{-2}(dS)^2$ equates the space-time interval to an entropy. We note that the entropy is constant on the hypersurface traced by light: $0 = (ds)^2 = \tilde{k}^{-2}(dS)^2$ (nullsurface). Since the entropy is constant on the nullsurface, then the entropic distance between all events on the surface is zero. This nullsurface is thus the present of the observer, and it represents the set of all events knowable to the observer on the nullsurface.*

Conjecture 2 (Past/Future)**.** *We note that*

$$(ds)^2 = \tilde{k}^{-2}(dS)^2 \implies ds = \pm\tilde{k}^{-1}dS \tag{105}$$

Splitting the $\pm$, we can then define the past as

$$ds = -\tilde{k}^{-1}dS \tag{106}$$

and the future as

$$ds = \tilde{k}^{-1}dS \tag{107}$$

Conjecture 3 (Growing block universe)**.** *The relation $(ds)^2 = \tilde{k}^{-2}(dS)^2$ quantifies the informational departure of an event from the present. The role of this entropy is to hide from an observer knowledge of events outside its present. This entropy is only eliminated if and when the gap between the observer and the entropically-distant event is reduced to 0 (that is, when the observer moves forward in time until the event occurs). Thus, the future is hidden from the present by entropy.*

Conjecture 4 (Random measurements events)**.** *The future departs informationally from the present due to a positive entropic distance ($\tilde{k}^{-1}dS > 0$). Since the entropy represents the number of microscopic states compatible with the current macroscopic state of the system, an observer pondering about its future will conclude that multi possible futures appear possible but would expect only one to be actual when the interval entropy is reduced to zero.*

Conjecture 5 (Measurement agreement)**.** *Since the entropy is constant along the nullsurface traced by light in space-time, and such entropy represents the unified entropy including measurement entropy, all observers along this surface necessarily agree on the result of all measurement outcomes. A disagreement between observers regarding measurement outcomes is impossible on the nullsurface of the present, as all observers know (or can know) what all others have measured.*

Conjecture 6 (Entropic speed of light)**.** *The speed of light, usually taken as an axiom in special relativity, is here emergent as the ratio of the Lagrange multipliers of the partition function $c := f/\tilde{k}$. The speed of light here fulfills a role similar to the role fulfilled by the temperature in standard thermodynamics. Essentially, the speed of light is the "temperature" of space-time. The speed of light is a property emergent from the random selection of events from a larger set under the principle of maximum entropy. The speed of light is constant in an ensemble of space-time events at equilibrium for the same reason that the temperature is constant in a system at thermodynamic equilibrium. Indeed, when f is the inverse of the Planck time, and $\tilde{k}$ the inverse of the Planck length, we get c, the speed of light:*

$$\left(\sqrt{\frac{c^3}{\hbar G}}\sqrt{\frac{\hbar G}{c^5}}\right)^{-1} = c \tag{108}$$

Conjecture 7 (Arrow of time)**.** *The gradient of space-time entropy always points in the direction an observer's future. Therefore, a space-time system seeking to statistically increase its entropy is powered to evolve forward in time by entropy. Indeed at thermodynamic equilibrium, the entropic frequency becomes an entropy power: $k_B T f = P$.*

5 Conclusion

By defining events using the geometric algebra, we have constructed a quantum statistical ensemble whose entropy is equal to the curvature of space-time (i.e. the interval). In the discussion, we have then argued that the main result implies a model of entropic time in which the future is hidden from the present by entropy and where the present corresponds to the nullsurface of constant entropy delimiting the events knowable to the observer. The pitfalls to avoid / the main-steps were as follows:

- Adopt a premise where, instead of the more general law of conservation of energy, the relation between entropy, information and geometric is the conserved relation. With this premise, the framework of statistical physics can be used to describe events in space-time.
- Recognize the limits, the pros and the cons of the Von Neumann entropy and understand that in the case of a geometric multivector, the entropy can be reduced to a scalar under the application of the geometric product on both sides of the equation. The resulting entropy shares the pros, but not the cons, of the Von Neumann entropy.
- Identifying the space-time entropy to the space-time interval and interpreting the resulting physical implications as a model of entropic time.

References

[Bao et al.(2017a)Bao, Cao, Carroll, and Chatwin-Davies] Ning Bao, ChunJun Cao, Sean M Carroll, and Aidan Chatwin-Davies. de sitter space as a tensor network: Cosmic no-hair, complementarity, and complexity. Physical Review D, 96(12):123536, 2017a.

[Bao et al.(2017b)Bao, Cao, Carroll, and McAllister] Ning Bao, ChunJun Cao, Sean M Carroll, and Liam McAllister. Quantum circuit cosmology: The expansion of the universe since the first qubit. arXiv preprint arXiv:1702.06959, 2017b.

[Bekenstein(1973)] Jacob D Bekenstein. Black holes and entropy. Physical Review D, 7(8):2333, 1973.

[Bekenstein(1974)] Jacob D Bekenstein. Generalized second law of thermodynamics in black-hole physics. Physical Review D, 9(12):3292, 1974.

[Bekenstein(1980)] Jacob D Bekenstein. Black-hole thermodynamics. Physics Today, 33(1):24–31, 1980.

[Cai et al.(2010)Cai, Cao, and Ohta] Rong-Gen Cai, Li-Ming Cao, and Nobuyoshi Ohta. Friedmann equations from entropic force. Physical Review D, 81(6):061501, 2010.

[Cao et al.(2017)Cao, Carroll, and Michalakis] ChunJun Cao, Sean M Carroll, and Spyridon Michalakis. Space from hilbert space: recovering geometry from bulk entanglement. Physical Review D, 95(2):024031, 2017.

[Carroll and Remmen(2016)] Sean M Carroll and Grant N Remmen. What is the entropy in entropic gravity? Physical Review D, 93(12):124052, 2016.

[Castroa and Pavšicb(2007)] C Castroa and M Pavšicb. The extended relativity theory in clifford spaces. Quantization in Astrophysics, Brownian Motion, and Supersymmetry, page 97, 2007.

[Czech et al.(2016)Czech, Lamprou, McCandlish, and Sully] Bartlomiej Czech, Lampros Lamprou, Samuel McCandlish, and James Sully. Tensor networks from kinematic space. Journal of High Energy Physics, 2016(7):100, 2016.

[Easson et al.(2011)Easson, Frampton, and Smoot] Damien A Easson, Paul H Frampton, and George F Smoot. Entropic accelerating universe. Physics Letters B, 696(3):273–277, 2011.

[Evenbly and Vidal(2011)] Glen Evenbly and Guifré Vidal. Tensor network states and geometry. Journal of Statistical Physics, 145(4):891–918, 2011.

[Faulkner et al.(2014)Faulkner, Guica, Hartman, Myers, and Van Raamsdonk] Thomas Faulkner, Monica Guica, Thomas Hartman, Robert C Myers, and Mark Van Raamsdonk. Gravitation from entanglement in holographic cfts. Journal of High Energy Physics, 2014(3):51, 2014.

[Faulkner et al.(2017)Faulkner, Haehl, Hijano, Parrikar, Rabideau, and Van Raamsdonk] Thomas Faulkner, Felix M Haehl, Eliot Hijano, Onkar Parrikar, Charles Rabideau, and Mark Van Raamsdonk. Nonlinear gravity from entanglement in conformal field theories. Journal of High Energy Physics, 2017(8):57, 2017.

[Freund(2010)] Peter GO Freund. Emergent gauge fields. arXiv preprint arXiv:1008.4147, 2010.

[Gao(2011)] Shan Gao. Is gravity an entropic force? Entropy, 13(5):936–948, 2011.

[Hawking(1974)] Stephen W Hawking. Black hole explosions? Nature, 248(5443):30, 1974.

[Hossenfelder(2010)] Sabine Hossenfelder. Comments on and comments on comments on verlinde's paper" on the origin of gravity and the laws of newton". arXiv preprint arXiv:1003.1015, 2010.

[Hu(2011)] BL Hu. Gravity and nonequilibrium thermodynamics of classical matter. International Journal of Modern Physics D, 20(05):697–716, 2011.

[Jacobson(1995)] Ted Jacobson. Thermodynamics of spacetime: The einstein equation of state. Phys. Rev. Lett., 75: 1260–1263, Aug 1995. doi: 10.1103/PhysRevLett.75.1260. URL `https://link.aps.org/doi/10.1103/PhysRevLett.75.1260`.

[Jacobson(2016)] Ted Jacobson. Entanglement equilibrium and the einstein equation. Physical review letters, 116(20): 201101, 2016.

[Kobakhidze(2011a)] Archil Kobakhidze. Gravity is not an entropic force. Physical Review D, 83(2):021502, 2011a.

[Kobakhidze(2011b)] Archil Kobakhidze. Once more: gravity is not an entropic force. arXiv preprint arXiv:1108.4161, 2011b.

[Li and Wang(2010)] Miao Li and Yi Wang. Quantum uv/ir relations and holographic dark energy from entropic force. Physics Letters B, 687(2-3):243–247, 2010.

[Maldacena(1999)] Juan Maldacena. The large-n limit of superconformal field theories and supergravity. International journal of theoretical physics, 38(4):1113–1133, 1999.

[Maldacena and Susskind(2013)] Juan Maldacena and Leonard Susskind. Cool horizons for entangled black holes. Fortschritte der Physik, 61(9):781–811, 2013.

[Padmanabhan(2010)] Thanu Padmanabhan. Thermodynamical aspects of gravity: new insights. Reports on Progress in Physics, 73(4):046901, 2010.

[Sanches and Weinberg(2016)] Fabio Sanches and Sean J Weinberg. Holographic entanglement entropy conjecture for general spacetimes. Physical Review D, 94(8):084034, 2016.

[Smolin(2010)] Lee Smolin. Newtonian gravity in loop quantum gravity. arXiv preprint arXiv:1001.3668, 2010.

[Susskind(2016)] Leonard Susskind. Entanglement is not enough. Fortschritte der Physik, 64(1):49–71, 2016.

[Susskind(2017)] Leonard Susskind. Dear qubitzers, gr= qm. arXiv preprint arXiv:1708.03040, 2017.

[Swingle(2012)] Brian Swingle. Entanglement renormalization and holography. Physical Review D, 86(6):065007, 2012.

[Swingle and Van Raamsdonk(2014)] Brian Swingle and Mark Van Raamsdonk. Universality of gravity from entanglement. arXiv preprint arXiv:1405.2933, 2014.

[Van Raamsdonk(2010)] Mark Van Raamsdonk. Building up spacetime with quantum entanglement. General Relativity and Gravitation, 42(10):2323–2329, 2010.

[Verlinde(2011)] Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. doi: 10.1007/JHEP04(2011)029. URL `https://doi.org/10.1007/JHEP04(2011)029`.

[Visser(2011)] Matt Visser. Conservative entropic forces. Journal of High Energy Physics, 2011(10):140, 2011.

[Wang(2010)] Tower Wang. Coulomb force as an entropic force. Physical Review D, 81(10):104045, 2010.