Refutation Arrow's impossibility theorem

Abstract: We evaluate two versions of Arrow's impossibility theorem with disjunctive or conjunctive results. Both as rendered are *not* tautologous. This means Arrow's framework is refuted, hence coloring the conjecture of Arrow's theorem before pivotal voters or dictators can be derived. Therefore Arrow's impossibility theorem forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⇻ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ⇷, ≲ ;
= Equivalent, ≡, :=, ⟺, ↔, ≙, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Bisht, H.; Kuber, A. (2019). Aggregating relational structures. arxiv.org/pdf/1904.12482.pdf
en.wikipedia.org/wiki/Arrow's_impossibility_theorem

Say there are three choices for society, call them **A**, **B**, and **C**. (1.1)

LET p, q, r, s: choice A, choice B, choice C, society;
unanimity, everyone, everything; #~p everything not p

s>((p&q)&r) ; `TTTT TTTT` **`FFFF FFF`**`T` (1.2)

Suppose first that everyone prefers option **B** the least: (2.1)

#s>~q ; `TTTT TTTT TTCC TTCC` (2.2)

everyone prefers **A** to **B**, and everyone prefers **C** to **B**. (3.1)

(#s>(p>q))&(#s>(r>q)) ; `TTTT TTTT TCTT CCTT` (3.2)

By unanimity, society must also prefer both **A** and **C** to **B**. (4.1)

#(s>((p&r)>q)) = (p=p) ; `NNNN NNNN NNNN N`**`F`**`NN` (4.2)

… On the other hand, if everyone preferred **B** to everything else, then society would have to prefer **B**

to everything else by unanimity. (5.1)

(#s>(q>#~q))>#(s>(q>#~q)) ; NNNN NNNN NNNN NNNN (5.2)

Remark 1.1-5.1: The argument then becomes, If Eqs. 1.1, Then ((2.1 And (3.1 and 4.1)) Or 5.1). (6.1)

(s>((p&q)&r)) >
(((#s>~q)&(((#s>(p>q))&(#s>(r>q)))&#(s>((p&r)>q)))) + ((#s>(q>#~q))>#(s>(q>#~q)))) ;
NNNN NNNN TTTT TTTN (6.2)

Remark 6.1: If the disjunctive phrase in Eq. 6.1 is changed to conjunctive (Or connective is changed to And), the argument is weakened as, If Eqs. 1.1, Then ((2.1 And (3.1 and 4.1)) And 5.1). (7.1)

(s>((p&q)&r)) >
(((#s>~q)&(((#s>(p>q))&(#s>(r>q)))&#(s>((p&r)>q)))) & ((#s>(q>#~q))>#(s>(q>#~q)))) ;
NNNN NNNN TTTT TTT**F** (7.2)

Eqs. 6.2 and 7.2 as rendered are *not* tautologous. This means Arrow's impossibility framework as stated is refuted, hence coloring the conjecture of Arrow's impossibility theorem before pivotal voters or dictators are derived.