

# Theory of Gravitation

**Kenneth Dalton**

email: [kxdalton@yahoo.com](mailto:kxdalton@yahoo.com)

## **Abstract**

Despite its successes, general relativity has failed to define a stress-energy-momentum tensor for the gravitational field. In what follows, the scalar, three-vector theory solves the problem of gravitational dynamics. It yields covariant expressions for the energy, momentum, stress, force and power. The weak-field approximation predicts both longitudinal and transverse gravitational waves. It is time to launch a search for longitudinal waves, in the data at LIGO and Virgo.

## 1. Introduction

The precis of general relativity is that it provides covariant expressions (tensors) for all physical quantities and for the laws that relate those quantities. However, no such expressions exist for gravitational energy, momentum, stress, force and power. This, in itself, shows that general relativity cannot be correct. The four-dimensional tensor formalism is incapable of describing gravitational dynamics [1].

The theory presented here derives from the fact that special relativity was invented without the use of four-vectors. Only scalars and three-vectors were used by the founders. This is evident in the basic physical elements of the theory: (time, space), (energy, momentum), (charge, current), etc. It is by expressing these elements in terms of coordinates that a new theory of gravitation emerges. It yields a fully covariant treatment of gravitational dynamics.

## 2. Field equations

The theory of special relativity concerns the motion and orientation of orthonormal frames of reference. A displacement  $d\mathbf{r}$  is projected onto an orthonormal 3-frame:  $\mathbf{i} \cdot d\mathbf{r}$ ,  $\mathbf{j} \cdot d\mathbf{r}$ ,  $\mathbf{k} \cdot d\mathbf{r}$ . These projections, together with the time interval  $dt$ , undergo a Lorentz transformation, which leaves the fundamental interval invariant

$$ds^2 = c^2 dt^2 - d\mathbf{r}^2 \quad (1)$$

The physical displacements may also be expressed in terms of a coordinate system  $\{x^\mu\}$

$$c dt = e_0(x) dx^0 \quad d\mathbf{r} = \mathbf{e}_i(x) dx^i \quad (2)$$

where  $e_\mu = (e_0, \mathbf{e}_i)$  is a scalar, 3-vector basis. The interval (1) then takes the form

$$\begin{aligned} ds^2 &= (e_0 dx^0)^2 - \mathbf{e}_i \cdot \mathbf{e}_j dx^i dx^j \\ &= g_{\mu\nu} dx^\mu dx^\nu \end{aligned} \quad (3)$$

where

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & 0 & 0 & 0 \\ 0 & & & \\ 0 & & g_{ij} & \\ 0 & & & \end{pmatrix} \quad (4)$$

The theory of gravitation concerns the structure of this metrical coordinate system. A Lorentz transformation may take place at any point. It will not involve the coordinates  $\{x^\mu\}$ .

An observer is free to choose a new coordinate system  $\{x^{\mu'}\}$ . In order to retain the distinction between scalars and 3-vectors, the coordinate transformations are restricted to the form

$$x^{0'} = x^{0'}(x^0) \quad x^{i'} = x^{i'}(x^j) \quad (5)$$

Displacements (2) will then be invariant, while the metric transforms as a tensor

$$g_{0'0'} = \frac{\partial x^0}{\partial x^{0'}} \frac{\partial x^0}{\partial x^{0'}} g_{00} \quad g_{i'j'} = \frac{\partial x^m}{\partial x^{i'}} \frac{\partial x^n}{\partial x^{j'}} g_{mn} \quad (6)$$

The Christofel symbols

$$\Gamma_{\nu\lambda}^\mu = \frac{1}{2} g^{\mu\rho} (\partial_\lambda g_{\nu\rho} + \partial_\nu g_{\rho\lambda} - \partial_\rho g_{\nu\lambda}) \quad (7)$$

yield the Ricci tensor

$$R_{\mu\nu} = \partial_\nu \Gamma_{\mu\lambda}^\lambda - \partial_\lambda \Gamma_{\mu\nu}^\lambda + \Gamma_{\rho\nu}^\lambda \Gamma_{\mu\lambda}^\rho - \Gamma_{\lambda\rho}^\lambda \Gamma_{\mu\nu}^\rho \quad (8)$$

The gravitational field equations

$$\frac{c^4}{8\pi G} \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) + T_{\mu\nu}^{(m)} = 0 \quad (9)$$

follow from the Einstein-Hilbert action

$$\delta \int \frac{c^4}{16\pi G} g^{\mu\nu} R_{\mu\nu} \sqrt{-g} d^4x + \delta \int L^{(m)} \sqrt{-g} d^4x = 0 \quad (10)$$

There are seven field equations, corresponding to the seven variations  $\delta g^{\mu\nu} = (\delta g^{00}, \delta g^{ij})$ . Components  $R_{0i}$  and  $T_{0i}^{(m)}$  do not appear in (9).

### 3. Gravitational energy, momentum and stress

The rate of change of the basis system is defined in terms of connection coefficients  $Q_{\mu\nu}^\lambda$

$$\nabla_\nu e_\mu = e_\lambda Q_{\mu\nu}^\lambda \quad (11)$$

This formula separates into scalar and 3-vector parts

$$\nabla_\nu e_0 = e_0 Q_{0\nu}^0 \quad (12)$$

$$\nabla_\nu \mathbf{e}_i = \mathbf{e}_j Q_{i\nu}^j \quad (13)$$

where  $Q_{0\nu}^j = Q_{i\nu}^0 \equiv 0$ . In terms of the metrical functions (4),

$$\partial_\lambda g_{00} = 2g_{00}Q_{0\lambda}^0 \quad (14)$$

$$\partial_0 g_{ij} = g_{in}Q_{j0}^n + g_{jn}Q_{i0}^n \quad (15)$$

$$\partial_k g_{ij} = g_{in}Q_{jk}^n + g_{jn}Q_{ik}^n \quad (16)$$

If  $Q_{jk}^i = Q_{kj}^i$  and if the two terms in (15) are assumed to be equal, then

$$Q_{0\lambda}^0 = \Gamma_{0\lambda}^0 = \frac{1}{2}g^{00}\partial_\lambda g_{00} \quad (17)$$

$$Q_{j0}^i = \Gamma_{j0}^i = \frac{1}{2}g^{in}\partial_0 g_{nj} \quad (18)$$

$$Q_{jk}^i = \Gamma_{jk}^i = \frac{1}{2}g^{in}(\partial_k g_{jn} + \partial_j g_{nk} - \partial_n g_{jk}) \quad (19)$$

Together, they comprise the formula

$$Q_{\nu\lambda}^\mu = \Gamma_{\nu\lambda}^\mu + g^{\mu\rho}g_{\lambda\eta}Q_{[\nu\rho]}^\eta \quad (20)$$

where

$$Q_{[\nu\lambda]}^\mu \equiv Q_{\nu\lambda}^\mu - Q_{\lambda\nu}^\mu \quad (21)$$

The non-zero components of  $Q_{[\nu\lambda]}^\mu$  are

$$Q_{[0i]}^0 = Q_{0i}^0 = \frac{1}{2}g^{00}\partial_i g_{00} \quad Q_{[j0]}^i = Q_{j0}^i = \frac{1}{2}g^{in}\partial_0 g_{nj} \quad (22)$$

They transform as tensor components

$$Q_{[0'i']}^{0'} = \frac{\partial x^n}{\partial x^{i'}} Q_{[0n]}^0 \quad Q_{[j'0']}^{i'} = \frac{\partial x^{i'}}{\partial x^m} \frac{\partial x^n}{\partial x^{j'}} \frac{\partial x^0}{\partial x^{0'}} Q_{[n0]}^m \quad (23)$$

This field strength tensor serves to define the gravitational energy tensor

$$T_{\mu\nu}^{(g)} = \frac{c^4}{8\pi G} \left\{ Q_{[\lambda\mu]}^\rho Q_{[\rho\nu]}^\lambda + Q_\mu Q_\nu - \frac{1}{2} g_{\mu\nu} g^{\eta\tau} (Q_{[\lambda\eta]}^\rho Q_{[\rho\tau]}^\lambda + Q_\eta Q_\tau) \right\} \quad (24)$$

where  $Q_\mu = Q_{[\rho\mu]}^\rho$ . For a static Newtonian potential  $\psi$

$$g_{00} = 1 + \frac{2}{c^2} \psi \quad (25)$$

so that  $Q_{[\nu\lambda]}^\mu$  is given by

$$Q_{[0i]}^0 = \frac{1}{c^2} \partial_i \psi \quad Q_{[j0]}^i = 0 \quad (26)$$

It follows that

$$T_{00}^{(g)} = \frac{1}{8\pi G} (\nabla\psi)^2 \quad (27)$$

$$T_{0i}^{(g)} = 0 \quad (28)$$

$$T_{ij}^{(g)} = \frac{1}{4\pi G} \left\{ \partial_i \psi \partial_j \psi - \frac{1}{2} \delta_{ij} (\nabla\psi)^2 \right\} \quad (29)$$

which is the Newtonian stress-energy tensor.

The conservation law for energy and momentum is found by summing the expression  $e_\mu T^{\mu\nu} dV_\nu$  over a closed, infinitesimal region  $\delta R$

$$\begin{aligned} \sum_{\delta R} e_\mu T^{\mu\nu} dV_\nu &= \left\{ e_\mu \partial_\nu (\sqrt{-g} T^{\mu\nu}) + (\nabla_\nu e_\mu) \sqrt{-g} T^{\mu\nu} \right\} d^4 x \\ &= e_\mu \left\{ \frac{1}{\sqrt{-g}} \partial_\nu (\sqrt{-g} T^{\mu\nu}) + Q_{\lambda\nu}^\mu T^{\lambda\nu} \right\} \sqrt{-g} d^4 x \end{aligned} \quad (30)$$

where  $T^{\mu\nu} = T_{(g)}^{\mu\nu} + T_{(m)}^{\mu\nu}$  is the total energy tensor and

$$dV_\nu = \sqrt{-g} (dx^1 dx^2 dx^3, dx^0 dx^2 dx^3, \dots) \quad (31)$$

Energy and momentum are conserved, if

$$\operatorname{div} T^{\mu\nu} = \frac{1}{\sqrt{-g}} \partial_\nu (\sqrt{-g} T^{\mu\nu}) + Q_{\lambda\nu}^\mu T^{\lambda\nu} = 0 \quad (32)$$

Make use of (20) to find

$$\operatorname{div} T^{\mu\nu} = T^{\mu\nu}_{;\nu} + g^{\mu\nu} Q_{[\alpha\nu]}^\beta T_\beta^\alpha \quad (33)$$

where  $T^{\mu\nu}_{;\nu}$  is the (contracted) covariant derivative. The divergence of the mixed tensor is

$$\operatorname{div} T_\mu^\nu = T_{\mu;\nu}^\nu + Q_{[\alpha\mu]}^\beta T_\beta^\alpha \quad (34)$$

The conservation law has been used to calculate the gravitational energy of the Robertson-Walker metric [2]. It was found that gravity accounts for two-thirds of the energy in the Universe—it is the “dark energy.” The energy tensor (24) was used in [3] to show that gravitational coupling can account for the fermion mass in a gauge invariant manner.

#### 4. Gravitational force and power

The motion of a particle in a gravitational field is described by the Lagrangian

$$L = mc \sqrt{g_{\mu\nu}(x) u^\mu u^\nu} \quad (35)$$

and the resulting equation of motion

$$mc \left\{ \frac{du^\mu}{ds} + \Gamma_{\nu\lambda}^\mu u^\nu u^\lambda \right\} = 0 \quad (36)$$

where  $u^\mu = dx^\mu/ds$ . The energy and momentum of the particle are given by

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \quad \mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - v^2/c^2}} = \frac{E}{c^2} \mathbf{v} \quad (37)$$

They yield the power formula

$$\frac{dE}{ds} = \mathbf{v} \cdot \frac{d\mathbf{p}}{ds} \quad (38)$$

where  $\mathbf{v} = d\mathbf{r}/dt$  is the physical velocity. The gravitational power and force are found by expressing the energy and momentum in terms of coordinates

$$E = mc^2 e_0 u^0 \quad \mathbf{p} = mc \mathbf{e}_i u^i \quad (39)$$

The rate of change of  $e_\mu u^\mu$  is

$$\begin{aligned} \frac{d(e_\mu u^\mu)}{ds} &= e_\mu \frac{du^\mu}{ds} + \frac{de_\mu}{ds} u^\mu = e_\mu \left\{ \frac{du^\mu}{ds} + Q_{\nu\lambda}^\mu u^\nu u^\lambda \right\} \\ &= e_\mu \left\{ \frac{du^\mu}{ds} + \Gamma_{\nu\lambda}^\mu u^\nu u^\lambda \right\} + e^\mu Q_{[\nu\mu]}^\lambda u^\nu u_\lambda \end{aligned} \quad (40)$$

where (20) has been used. Substitute the equation of motion (36) to find

$$mc \frac{d(e_\mu u^\mu)}{ds} = e^\mu mc Q_{[\nu\mu]}^\lambda u^\nu u_\lambda \quad (41)$$

Separate this formula into scalar and 3-vector parts, then substitute the components (22) to find that the energy and momentum change as follows:

$$\frac{dE}{ds} = e^0 \frac{mc^2}{2} \left\{ -\partial_n g_{00} u^n u^0 + \partial_0 g_{mn} u^m u^n \right\} \quad (42)$$

$$\frac{d\mathbf{p}}{ds} = \mathbf{e}^i \frac{mc}{2} \left\{ \partial_i g_{00} u^0 u^0 - \partial_0 g_{in} u^0 u^n \right\} \quad (43)$$

These equations express the power and force which are exerted by the gravitational field. In the Newtonian limit (25),  $u^0 = 1$  and  $u^n = v^n/c$  so that

$$\frac{dE}{dt} = -m \nabla \psi \cdot \mathbf{v} \quad \frac{d\mathbf{p}}{dt} = -m \nabla \psi \quad (44)$$

If other forces are present, they will appear in the equation of motion. For example, a charged particle in combined gravitational and electromagnetic fields is described by

$$L = mc \sqrt{g_{\mu\nu}(x) u^\mu u^\nu} + \frac{q}{c} A_\mu(x) u^\mu \quad (45)$$

and

$$mc \left\{ \frac{du^\mu}{ds} + \Gamma_{\nu\lambda}^\mu u^\nu u^\lambda \right\} = \frac{q}{c} F_{\nu}^\mu u^\nu \quad (46)$$

It follows that (compare (41))

$$mc \frac{d(e_\mu u^\mu)}{ds} = e^\mu \left\{ mc Q_{[\nu\mu]}^\lambda u^\nu u_\lambda + \frac{q}{c} F_{\mu\nu} u^\nu \right\} \quad (47)$$

The scalar and 3-vector parts of the Lorentz force are then added to the right-hand side of (42) and (43).

## 5. The weak-field approximation

(In this section,  $T_{(m)}^{\mu\nu} = T^{\mu\nu} = \rho c^2 u^\mu u^\nu$ .)

If the coordinate system is nearly rectangular, then the metric tensor may be expanded

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad |h_{\mu\nu}| \ll 1 \quad (48)$$

Substitution into (8) yields

$$R_{\mu\nu} = \frac{1}{2} \left\{ \eta^{\lambda\rho} \partial_\lambda \partial_\rho h_{\mu\nu} + \partial_\mu \partial_\nu h_\lambda^\lambda - \partial_\mu \partial_\lambda h_\nu^\lambda - \partial_\nu \partial_\lambda h_\mu^\lambda \right\} \quad (49)$$

The four conditions

$$\partial_\nu h_\lambda^\lambda = 2 \partial_\lambda h_\nu^\lambda \quad (50)$$

leave three independent components  $h_{\mu\nu}$ , and they greatly simplify the Ricci tensor

$$R_{\mu\nu} = \frac{1}{2} \partial^\lambda \partial_\lambda h_{\mu\nu} \quad (51)$$

Rewrite the field equations in the form

$$R_\nu^\mu = -\frac{8\pi G}{c^4} \left( T_\nu^\mu - \frac{1}{2} \delta_\nu^\mu T \right) \quad (52)$$

in order to obtain

$$\partial^\lambda \partial_\lambda h_\nu^\mu = -\frac{16\pi G}{c^4} \left( T_\nu^\mu - \frac{1}{2} \delta_\nu^\mu T \right) \quad (53)$$

The retarded solution is

$$h_\nu^\mu(\mathbf{x}, t) = -\frac{4G}{c^4} \int \frac{(T_\nu^\mu - \frac{1}{2} \delta_\nu^\mu T)}{|\mathbf{x} - \mathbf{x}'|} \Big|_{\text{ret}} d^3 x' \quad (54)$$



If the source is at rest, then  $T_0^0 = T = \rho c^2$  and  $T_j^i = 0$  so that (54) yields

$$ds^2 = \left(1 + \frac{2}{c^2} \psi\right) (dx^0)^2 - \left(1 - \frac{2}{c^2} \psi\right) (dx^2 + dy^2 + dz^2) \quad (55)$$

where  $\psi$  is the Newtonian potential.

### 5.1 Wave generation

When motion of the source is significant, radiation is produced which propagates into distant regions. In those regions, the solution (54) simplifies to

$$h_\nu^\mu(\mathbf{x}, t) = -\frac{4G}{c^4 r} \int (T_\nu^\mu - \frac{1}{2} \delta_\nu^\mu T) d^3x \quad (56)$$

Here, the long wavelength limit is assumed, so that ret simply means time  $(t - r/c)$ . Formula (50) for  $\nu = 0$  imposes a constraint on this solution, viz.,  $\partial_0 h_0^0 = \partial_0 h_n^n$ . Explicitly,

$$h_0^0(x) = -\frac{2G}{c^4 r} \int (T_0^0 - T_n^n) d^3x \quad (57)$$

$$h_n^n(x) = -\frac{2G}{c^4 r} \int (-3T_0^0 - T_n^n) d^3x \quad (58)$$

so that the constraint becomes

$$\frac{d}{dt} \int T_0^0 d^3x = 0 \quad (59)$$

The material energy changes very little in the weak-field approximation, i.e., little gravitational energy is produced. Thus, the  $T_0^0$  term will not contribute to the radiation, which leaves

$$h_0^0(x) = \frac{2G}{c^4 r} \int T_n^n d^3x \quad (60)$$

$$h_j^i(x) = -\frac{4G}{c^4 r} \int (T_j^i - \frac{1}{2} \delta_j^i T_n^n) d^3x \quad (61)$$

These integrals may be transformed by means of the identity [4-6]

$$\int T^{ij} d^3x = \frac{1}{2} \int x^i x^j \partial_k \partial_l T^{kl} d^3x \quad (62)$$

The conservation law  $\partial_\mu T^{\mu\nu} = 0$  (again, ignoring the gravitational part) gives  $\partial_k \partial_l T^{kl} = \partial_0 \partial_0 T^{00}$ , and it follows that

$$\int T^{ij} d^3x = \frac{1}{2} \frac{d^2}{dt^2} \int \rho x^i x^j d^3x = \frac{1}{2} \frac{d^2 I^{ij}}{dt^2} \quad (63)$$

Finally,

$$h^0_0(x) = \frac{G}{c^4 r} \frac{d^2 I^n_n}{dt^2} = -\frac{G}{c^4 r} \frac{d^2 I}{dt^2} \quad (64)$$

$$h^i_j(x) = -\frac{2G}{c^4 r} \left( \frac{d^2 I^i_j}{dt^2} + \frac{1}{2} \delta^i_j \frac{d^2 I}{dt^2} \right) \quad (65)$$

The above expressions give the radiation field in terms of the motion of the source. The  $h_{\mu\nu}$  also satisfy the field equations (53) for matter-free space,  $\partial^\lambda \partial_\lambda h_{\mu\nu} = 0$ , which admit plane wave solutions such as

$$h^\mu_\nu = A^\mu_\nu \cos(-k_\lambda x^\lambda) \quad (k^0 = k) \quad (66)$$

Conditions (50) now take the form

$$A^0_0 = A^n_n \quad k_i A^n_n = k_n A^n_i \quad (67)$$

If a particular direction is chosen (say  $k^3$ ) then the following components remain

$$h^0_0 = h^3_3 \quad h^1_1 = -h^2_2 \quad h^1_2 = h^2_1 \quad (68)$$

while  $h^2_3 = h^3_1 = 0$ . The transverse part of  $A_{ij}$  is traceless

$$h_{\mu\nu} = \begin{pmatrix} h_{00} & 0 & 0 & 0 \\ 0 & h_{11} & h_{12} & 0 \\ 0 & h_{12} & -h_{11} & 0 \\ 0 & 0 & 0 & -h_{00} \end{pmatrix} \quad (69)$$

This can be proved for the general case by using the projection operator [7]  $P_{ij} = \delta_{ij} - n_i n_j$  to define  $A^T_{ij} = P_{in} P_{jm} A_{nm}$ . Then (67) gives  $A^T_n^n = 0$ .

## 5.2 Radiative energy flow

The flow of gravitational energy is determined by (24)

$$\begin{aligned}
T_{0i}^{(g)} &= \frac{c^4}{8\pi G} \{ Q_{[0n]}^0 Q_{[i0]}^n + Q_{[n0]}^n Q_{[0i]}^0 \} \\
&= \frac{c^4}{32\pi G} \{ \partial_n h_0^0 \partial_0 h_i^n + \partial_i h_0^0 \partial_0 h_n^n \}
\end{aligned} \tag{70}$$

A plane wave satisfies  $k \partial_i h_\nu^\mu = k_i \partial_0 h_\nu^\mu$ , so that

$$T_{0i}^{(g)} = \frac{c^4}{32\pi G} \left\{ \frac{k_n}{k} \partial_0 h_0^0 \partial_0 h_i^n + \frac{k_i}{k} \partial_0 h_0^0 \partial_0 h_n^n \right\} \tag{71}$$

This shows that the transverse-traceless components do not contribute to the energy flow. Conditions (67) now give

$$T_{0i}^{(g)} = \frac{c^4}{16\pi G} \frac{k_i}{k} (\partial_0 h_0^0)^2 = \frac{c^4}{16\pi G} n_i (\partial_0 h_0^0)^2 \tag{72}$$

The scalar product

$$c T_0^{(g)i} n^i = \frac{c^5}{16\pi G} (\partial_0 h_0^0)^2 \tag{73}$$

gives the flow in the radial direction. There is no angular dependence and so the total power is found by substituting (64)

$$\frac{dE}{dt} = \int c T_0^{(g)i} n^i r^2 d\Omega = \frac{G}{4c^5} \left( \frac{d^3 I}{dt^3} \right)^2 \tag{74}$$

In a binary system, [6]

$$\frac{d^3 I}{dt^3} = -\frac{2m_1 m_2}{a(1-e^2)} e \sin \theta \dot{\theta} \tag{75}$$

Substitution into (74) gives the power

$$\frac{dE}{dt} = \frac{G m_1^2 m_2^2}{c^5 a^2 (1-e^2)^2} e^2 \sin^2 \theta \dot{\theta}^2 \tag{76}$$

while the average over one period is

$$\left\langle \frac{dE}{dt} \right\rangle = \frac{G m_1^2 m_2^2 (m_1 + m_2)}{2c^5 a^5} e^2 \left(1 + \frac{e^2}{4}\right) (1-e^2)^{-7/2} \tag{77}$$

This formula exhibits a strong dependence upon eccentricity. In particular, circular orbits will not radiate energy ( $e = 0$ ), suggesting that they are more

stable than eccentric orbits. Nevertheless, they do emit transverse waves in accordance with (65)

$$h^i_j(x) = -\frac{2G}{c^4 r} \frac{d^2 I^i_j}{dt^2} \quad (78)$$

### 5.3 Wave detection

The force (43) exerted on a detector initially at rest is clearly due to  $\partial_i g_{00}$  alone

$$\frac{d\mathbf{p}}{dt} = \frac{mc^2}{2} \mathbf{e}^i \partial_i g_{00} = \frac{mc^2}{2} \partial_0 h_{00} \mathbf{e}_i n^i \quad (79)$$

which is along the direction of propagation. Substitute (64) to find

$$\frac{d\mathbf{p}}{dt} = -\frac{Gm}{2c^3 r} \frac{d^3 I}{dt^3} \mathbf{e}_i n^i \quad (80)$$

Apart from the acceleration of the mirrors (referring to LIGO), the gravitational field changes the wavelength of light in the storage chambers, much as in the static redshift. In this regard, it is important that each mode possesses energy:

$$\begin{aligned} T_{00}^{(g)} &= \frac{c^4}{16\pi G} \left\{ Q_{n0}^m Q_{m0}^n + Q_{m0}^m Q_{n0}^n - 2\eta_{00}\eta^{mn} Q_{0m}^0 Q_{0n}^0 \right\} \\ &= \frac{c^4}{64\pi G} \left\{ \partial_0 h_n^m \partial_0 h_m^n + \partial_0 h_m^m \partial_0 h_n^n + 2(\partial_0 h_0^0)^2 \right\} \\ &= \frac{c^4}{32\pi G} \left\{ (\partial_0 h_1^1)^2 + (\partial_0 h_2^2)^2 + 2(\partial_0 h_0^0)^2 \right\} \end{aligned} \quad (81)$$

where the final line pertains to motion along  $k^3$  (see (69)). The effect on light can then be viewed as an exchange of energy.

The gravitational stress is given by

$$T_{ij}^{(g)} = \frac{c^4}{8\pi G} \left\{ 2Q_{0i}^0 Q_{0j}^0 - \frac{1}{2}\eta_{ij} \left[ \eta^{00} (Q_{n0}^m Q_{m0}^n + Q_{m0}^m Q_{n0}^n) + 2\eta^{mn} Q_{0m}^0 Q_{0n}^0 \right] \right\} \quad (82)$$

which is diagonal for the field (69)

$$T_{11}^{(g)} = T_{22}^{(g)} = \frac{c^4}{32\pi G} \{(\partial_0 h_1^1)^2 + (\partial_0 h_2^1)^2\} \quad (83)$$

$$T_{33}^{(g)} = \frac{c^4}{32\pi G} \{(\partial_0 h_1^1)^2 + (\partial_0 h_2^1)^2 + 2(\partial_3 h_0^0)^2\} \quad (84)$$

These stresses are compressive, with the transverse modes exerting an equal pressure in all directions, and the longitudinal mode exerting pressure along  $k^3$ .

#### 5.4 LIGO and Virgo

The search for longitudinal waves could begin with the experiments at LIGO and Virgo. At any given installation, if a gravitational wave arrives vertically, then its transverse components would alter the wavelength of light in the storage chambers. For other directions of arrival, both transverse and longitudinal components would contribute. A new analysis of the data could reveal the existence of longitudinal gravitational waves.

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