Theory of Gravitation

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Abstract

It has now been proved that, in general relativity, gravitational energy, momentum, stress, force and power do not exist. A new theory of gravitation is required. In the new theory, gravitational dynamics is expressed in terms of a field strength tensor. This tensor arises from the scalar, three-vector structure of time and space. The theory predicts that longitudinal waves are responsible for the flux of gravitational energy. It is time to launch a dedicated search for longitudinal gravitational waves.

1. Introduction

In relativistic particle dynamics, the energy and momentum

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \qquad \mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - v^2/c^2}} = \frac{E}{c^2}\,\mathbf{v} \tag{1}$$

yield the power formula

$$\frac{dE}{ds} = \mathbf{v} \cdot \frac{d\mathbf{p}}{ds} \tag{2}$$

where $\mathbf{v} = d\mathbf{r}/dt$ is the physical velocity. This shows that a displacement $d\mathbf{r}$ is required, if energy is to be exchanged with the particle. For example, in cosmology, the galaxies are at rest in the expanding Robertson-Walker coordinate system. They do not exchange energy with the gravitational field. Something similar occurs with the purely transverse gravitational waves of general relativity. They do not produce a displacement of the detector [1]. Therefore, they do not transfer energy to the detector.

In what follows, the problem of radiation is treated in terms of the scalar, three-vector theory of gravitation. Here, the radiation field includes both transverse and longitudinal components. The latter exert a force, produce a displacement and transfer energy to the detector.

2. Field equations

The theory of special relativity concerns the motion and orientation of orthonormal frames of reference. A displacement $d\mathbf{r}$ is projected onto an orthonormal 3-frame: $\mathbf{i} \cdot d\mathbf{r}$, $\mathbf{j} \cdot d\mathbf{r}$, $\mathbf{k} \cdot d\mathbf{r}$. These projections, together with the time interval dt, undergo a Lorentz transformation, which leaves the fundamental interval invariant

$$ds^2 = c^2 dt^2 - d\mathbf{r}^2 \tag{3}$$

The physical displacements may also be expressed in terms of a coordinate system $\{x^\mu\}$

$$c\,dt = e_0(x)dx^0 \qquad d\mathbf{r} = \mathbf{e}_i(x)dx^i \tag{4}$$

where $e_\mu = (e_0, \mathbf{e}_i)$ is a scalar, 3-vector basis. The interval (3) then takes the form

$$
\begin{aligned}
ds^2 &= (e_0 dx^0)^2 - \mathbf{e}_i \cdot \mathbf{e}_j \, dx^i dx^j \\
&= g_{\mu\nu} dx^\mu dx^\nu
\end{aligned}
\tag{5}
$$

where

$$
g_{\mu\nu} = \begin{pmatrix} g_{00} & 0 & 0 & 0 \\ 0 & & & \\ 0 & & g_{ij} & \\ 0 & & & \end{pmatrix}
\tag{6}
$$

The theory of gravitation concerns the structure of this metrical coordinate system. A Lorentz transformation may take place at any point. It will not involve the coordinates $\{x^\mu\}$.

An observer is free to choose a new coordinate system $\{x^{\mu'}\}$. In order to retain the distinction between scalars and 3-vectors, the coordinate transformations are restricted to the form

$$
x^{0'} = x^{0'}(x^0) \qquad\qquad x^{i'} = x^{i'}(x^j)
\tag{7}
$$

Displacements (4) will then be invariant, while the metric transforms as a tensor

$$
g_{0'0'} = \frac{\partial x^0}{\partial x^{0'}} \frac{\partial x^0}{\partial x^{0'}} \, g_{00} \qquad\qquad g_{i'j'} = \frac{\partial x^m}{\partial x^{i'}} \frac{\partial x^n}{\partial x^{j'}} \, g_{mn}
\tag{8}
$$

The Christofel symbols

$$
\Gamma^\mu_{\nu\lambda} = \frac{1}{2} g^{\mu\rho} \left(\partial_\lambda g_{\nu\rho} + \partial_\nu g_{\rho\lambda} - \partial_\rho g_{\nu\lambda} \right)
\tag{9}
$$

yield the Ricci tensor

$$
R_{\mu\nu} = \partial_\nu \Gamma^\lambda_{\mu\lambda} - \partial_\lambda \Gamma^\lambda_{\mu\nu} + \Gamma^\lambda_{\rho\nu} \Gamma^\rho_{\mu\lambda} - \Gamma^\lambda_{\lambda\rho} \Gamma^\rho_{\mu\nu}
\tag{10}
$$

The gravitational field equations

$$
\frac{c^4}{8\pi G} \Big(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \Big) + T^{(m)}_{\mu\nu} = 0
\tag{11}
$$

follow from the Einstein-Hilbert action

$$\delta \int \frac{c^4}{16\pi G}\, g^{\mu\nu} R_{\mu\nu} \sqrt{-g}\, d^4x + \delta \int L^{(m)} \sqrt{-g}\, d^4x = 0 \tag{12}$$

There are seven field equations, corresponding to the seven variations $\delta g^{\mu\nu} = (\delta g^{00}, \delta g^{ij})$. Components R_{0i} and $T^{(m)}_{0i}$ do not appear in (11).

3. Gravitational energy, momentum and stress

The rate of change of the basis system is defined in terms of connection coefficients $Q^{\lambda}_{\mu\nu}$

$$\nabla_\nu e_\mu = e_\lambda Q^{\lambda}_{\mu\nu} \tag{13}$$

This formula separates into scalar and 3-vector parts

$$\begin{aligned} \nabla_\nu e_0 &= e_0 Q^0_{0\nu} && (14)\\ \nabla_\nu \mathbf{e}_i &= \mathbf{e}_j Q^j_{i\nu} && (15) \end{aligned}$$

where $Q^j_{0\nu} = Q^0_{i\nu} \equiv 0$. In terms of the metrical functions (6),

$$\begin{aligned} \partial_\lambda g_{00} &= 2 g_{00} Q^0_{0\lambda} && (16)\\ \partial_0 g_{ij} &= g_{in} Q^n_{j0} + g_{jn} Q^n_{i0} && (17)\\ \partial_k g_{ij} &= g_{in} Q^n_{jk} + g_{jn} Q^n_{ik} && (18) \end{aligned}$$

If $Q^i_{jk} = Q^i_{kj}$ and if the two terms in (17) are assumed to be equal, then

$$\begin{aligned} Q^0_{0\lambda} &= \Gamma^0_{0\lambda} = \frac{1}{2} g^{00} \partial_\lambda g_{00} && (19)\\ Q^i_{j0} &= \Gamma^i_{j0} = \frac{1}{2} g^{in} \partial_0 g_{nj} && (20)\\ Q^i_{jk} &= \Gamma^i_{jk} = \frac{1}{2} g^{in} \left(\partial_k g_{jn} + \partial_j g_{nk} - \partial_n g_{jk}\right) && (21) \end{aligned}$$

Together, they comprise the formula

$$Q^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\nu\lambda} + g^{\mu\rho} g_{\lambda\eta} Q^{\eta}_{[\nu\rho]} \tag{22}$$

where

$$Q^{\mu}_{[\nu\lambda]} \equiv Q^{\mu}_{\nu\lambda} - Q^{\mu}_{\lambda\nu} \tag{23}$$

The non-zero components of $Q^{\mu}_{[\nu\lambda]}$ are

$$Q^{0}_{[0i]} = Q^{0}_{0i} = \frac{1}{2}g^{00}\partial_i g_{00} \qquad\qquad Q^{i}_{[j0]} = Q^{i}_{j0} = \frac{1}{2}g^{in}\partial_0 g_{nj} \tag{24}$$

They transform as tensor components

$$Q^{0'}_{[0'i']} = \frac{\partial x^n}{\partial x^{i'}}\, Q^{0}_{[0n]} \qquad\qquad Q^{i'}_{[j'0']} = \frac{\partial x^{i'}}{\partial x^m}\frac{\partial x^n}{\partial x^{j'}}\frac{\partial x^0}{\partial x^{0'}}\, Q^{m}_{[n0]} \tag{25}$$

This field strength tensor serves to define the gravitational energy tensor

$$T^{(g)}_{\mu\nu} = \frac{c^4}{8\pi G}\left\{Q^{\rho}_{[\lambda\mu]}Q^{\lambda}_{[\rho\nu]} + Q_\mu Q_\nu - \frac{1}{2}g_{\mu\nu}g^{\eta\tau}(Q^{\rho}_{[\lambda\eta]}Q^{\lambda}_{[\rho\tau]} + Q_\eta Q_\tau)\right\} \tag{26}$$

where $Q_\mu = Q^{\rho}_{[\rho\mu]}$. For a static Newtonian potential ψ

$$g_{00} = 1 + \frac{2}{c^2}\psi \tag{27}$$

so that $Q^{\mu}_{[\nu\lambda]}$ is given by

$$Q^{0}_{[0i]} = \frac{1}{c^2}\,\partial_i\psi \qquad\qquad Q^{i}_{[j0]} = 0 \tag{28}$$

It follows that

$$T^{(g)}_{00} = \frac{1}{8\pi G}(\nabla\psi)^2 \tag{29}$$

$$T^{(g)}_{0i} = 0 \tag{30}$$

$$T^{(g)}_{ij} = \frac{1}{4\pi G}\left\{\partial_i\psi\,\partial_j\psi - \frac{1}{2}\delta_{ij}(\nabla\psi)^2\right\} \tag{31}$$

which is the Newtonian stress-energy tensor.

The conservation law for energy and momentum is found by summing the expression $e_\mu T^{\mu\nu}\, dV_\nu$ over an infinitesimal region δR

$$\sum_{\delta R} e_\mu T^{\mu\nu}\, dV_\nu \;=\; \Big\{e_\mu\, \partial_\nu(\sqrt{-g}\, T^{\mu\nu}) + (\nabla_\nu e_\mu)\sqrt{-g}\, T^{\mu\nu}\Big\} d^4x$$

$$= e_\mu \Big\{\frac{1}{\sqrt{-g}}\partial_\nu(\sqrt{-g}\, T^{\mu\nu}) + Q^\mu_{\lambda\nu} T^{\lambda\nu}\Big\}\sqrt{-g}\, d^4x \qquad (32)$$

where $T^{\mu\nu} = T^{\mu\nu}_{(g)} + T^{\mu\nu}_{(m)}$ and

$$dV_\nu = \sqrt{-g}\, (dx^1dx^2dx^3,\, dx^0dx^2dx^3,\, \ldots) \qquad (33)$$

Energy and momentum are conserved, if

$$\operatorname{div} T^{\mu\nu} = \frac{1}{\sqrt{-g}}\partial_\nu(\sqrt{-g}\, T^{\mu\nu}) + Q^\mu_{\lambda\nu} T^{\lambda\nu} = 0 \qquad (34)$$

Make use of (22) to find

$$\operatorname{div} T^{\mu\nu} = T^{\mu\nu}_{;\nu} + g^{\mu\nu} Q^\beta_{[\alpha\nu]} T_\beta{}^\alpha \qquad (35)$$

where $T^{\mu\nu}_{;\nu}$ is the (contracted) covariant derivative. The divergence of the mixed tensor is

$$\operatorname{div} T_\mu{}^\nu = T_\mu{}^\nu{}_{;\nu} + Q^\beta_{[\alpha\mu]} T_\beta{}^\alpha \qquad (36)$$

The conservation law has been used to calculate the gravitational energy of the Robertson-Walker metric [2]. It was found that gravity accounts for two-thirds of the energy in the Universe—it is the "dark energy." The energy tensor (26) was used in [3] to show that gravitational coupling can account for the fermion mass in a gauge invariant manner.

4. Gravitational force and power

The motion of a particle in a gravitational field is described by the Lagrangian

$$L = mc\sqrt{g_{\mu\nu}(x) u^\mu u^\nu} \qquad (37)$$

and the resulting equation of motion

$$mc\Big\{\frac{du^\mu}{ds} + \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda\Big\} = 0 \qquad (38)$$

where $u^\mu = dx^\mu/ds$. The gravitational power and force are found by expressing the energy and momentum (1) in terms of coordinates

$$E = mc^2 e_0 u^0 \qquad\qquad \mathbf{p} = mc\, \mathbf{e}_i u^i \tag{39}$$

The rate of change of $e_\mu u^\mu$ is

$$\begin{aligned} \frac{d(e_\mu u^\mu)}{ds} &= e_\mu \frac{du^\mu}{ds} + \frac{de_\mu}{ds} u^\mu = e_\mu \Big\{ \frac{du^\mu}{ds} + Q^\mu_{\nu\lambda} u^\nu u^\lambda \Big\} \\ &= e_\mu \Big\{ \frac{du^\mu}{ds} + \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda \Big\} + e^\mu\, Q^\lambda_{[\nu\mu]} u^\nu u_\lambda \end{aligned} \tag{40}$$

where (22) has been used. Substitute the equation of motion (38) to find

$$mc\, \frac{d(e_\mu u^\mu)}{ds} = e^\mu\, mc\, Q^\lambda_{[\nu\mu]} u^\nu u_\lambda \tag{41}$$

Separate this formula into scalar and 3-vector parts, then substitute the components (24) to find that the energy and momentum change as follows:

$$\frac{dE}{ds} = e^0\, \frac{mc^2}{2} \Big\{ -\partial_n g_{00}\, u^n u^0 + \partial_0 g_{mn}\, u^m u^n \Big\} \tag{42}$$

$$\frac{d\mathbf{p}}{ds} = \mathbf{e}^i\, \frac{mc}{2} \Big\{ \partial_i g_{00}\, u^0 u^0 - \partial_0 g_{in}\, u^0 u^n \Big\} \tag{43}$$

These equations express the power and force which are exerted by the gravitational field. In the Newtonian limit (27), $u^0 = 1$ and $u^n = v^n/c$ so that

$$\frac{dE}{dt} = -m\, \nabla\psi \cdot \mathbf{v} \qquad\qquad \frac{d\mathbf{p}}{dt} = -m\, \nabla\psi \tag{44}$$

If other forces are present, they will appear in the equation of motion. For example, a charged particle in combined gravitational and electromagnetic fields is described by

$$L = mc\, \sqrt{g_{\mu\nu}(x) u^\mu u^\nu} + \frac{q}{c}\, A_\mu(x) u^\mu \tag{45}$$

and

$$mc \Big\{ \frac{du^\mu}{ds} + \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda \Big\} = \frac{q}{c}\, F^\mu_{\ \nu} u^\nu \tag{46}$$

It follows that (compare (41))

$$mc\,\frac{d(e_\mu u^\mu)}{ds} = e^\mu \Big\{ mc\, Q^\lambda_{[\nu\mu]} u^\nu u_\lambda + \frac{q}{c}\, F_{\mu\nu} u^\nu \Big\} \tag{47}$$

The scalar and 3-vector parts of the Lorentz force are then added to the right-hand side of (42) and (43).

5. The weak-field approximation

(In this section, $T^{\mu\nu}_{(m)} = T^{\mu\nu} = \rho c^2 u^\mu u^\nu$.)
If the coordinate system is nearly rectangular, then the metric tensor may be expanded

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad\qquad |h_{\mu\nu}| \ll 1 \tag{48}$$

Substitution into (10) yields

$$R_{\mu\nu} = \frac{1}{2}\Big\{ \eta^{\lambda\rho} \partial_\lambda \partial_\rho h_{\mu\nu} + \partial_\mu \partial_\nu h^\lambda{}_\lambda - \partial_\mu \partial_\lambda h^\lambda{}_\nu - \partial_\nu \partial_\lambda h^\lambda{}_\mu \Big\} \tag{49}$$

The four conditions

$$\partial_\nu h^\lambda{}_\lambda = 2\,\partial_\lambda h^\lambda{}_\nu \tag{50}$$

leave three independent components $h_{\mu\nu}$, and they greatly simplify the Ricci tensor

$$R_{\mu\nu} = \frac{1}{2}\,\partial^\lambda \partial_\lambda h_{\mu\nu} \tag{51}$$

Rewrite the field equations in the form

$$R^\mu{}_\nu = -\frac{8\pi G}{c^4}\Big(T^\mu{}_\nu - \frac{1}{2}\,\delta^\mu{}_\nu\, T \Big) \tag{52}$$

in order to obtain

$$\partial^\lambda \partial_\lambda h^\mu{}_\nu = -\frac{16\pi G}{c^4}\Big(T^\mu{}_\nu - \frac{1}{2}\,\delta^\mu{}_\nu\, T \Big) \tag{53}$$

The retarded solution is

$$h^\mu{}_\nu(\mathbf{x}, t) = -\frac{4G}{c^4} \int \frac{(T^\mu{}_\nu - \frac{1}{2}\delta^\mu{}_\nu\, T)}{|\mathbf{x} - \mathbf{x}'|}\,\Big|_{\rm ret}\, d^3x' \tag{54}$$

If the source is at rest, then $T^0_{\ 0} = T = \rho c^2$ and $T^i_{\ j} = 0$ so that (54) yields

$$ds^2 = (1 + \frac{2}{c^2}\,\psi)(dx^0)^2 - (1 - \frac{2}{c^2}\,\psi)(dx^2 + dy^2 + dz^2) \tag{55}$$

where ψ is the Newtonian potential.

5.1 Wave generation

When motion of the source is significant, radiation is produced which propagates into distant regions. In those regions, the solution (54) simplifies to

$$h^\mu_{\ \nu}(\mathbf{x}, t) = -\frac{4G}{c^4 r}\int (T^\mu_{\ \nu} - \frac{1}{2}\,\delta^\mu_{\ \nu}\,T)\,d^3x \tag{56}$$

Here, the long wavelength limit is assumed, so that ret simply means time $(t - r/c)$. Formula (50) for $\nu = 0$ imposes a constraint on this solution, viz., $\partial_0 h^0_{\ 0} = \partial_0 h^n_{\ n}$. Explicitly,

$$h^0_{\ 0}(x) \;=\; -\frac{2G}{c^4 r}\int (T^0_{\ 0} - T^n_{\ n})\,d^3x \tag{57}$$

$$h^n_{\ n}(x) \;=\; -\frac{2G}{c^4 r}\int (-3T^0_{\ 0} - T^n_{\ n})\,d^3x \tag{58}$$

so that the constraint becomes

$$\frac{d}{dt}\int T^0_{\ 0}\,d^3x = 0 \tag{59}$$

The material energy changes very little in the weak-field approximation, i.e., little gravitational energy is produced. Thus, the $T^0_{\ 0}$ term will not contribute to the radiation, which leaves

$$h^0_{\ 0}(x) \;=\; \frac{2G}{c^4 r}\int T^n_{\ n}\,d^3x \tag{60}$$

$$h^i_{\ j}(x) \;=\; -\frac{4G}{c^4 r}\int (T^i_{\ j} - \frac{1}{2}\,\delta^i_{\ j}\,T^n_{\ n})\,d^3x \tag{61}$$

These integrals may be transformed by means of the identity [4-6]

$$\int T^{ij}\,d^3x = \frac{1}{2}\int x^i x^j\,\partial_k\partial_l T^{kl}\,d^3x \tag{62}$$

The conservation law $\partial_\mu T^{\mu\nu} = 0$ (again, ignoring the gravitational part) gives $\partial_k \partial_l T^{kl} = \partial_0 \partial_0 T^{00}$, and it follows that

$$\int T^{ij}\, d^3x = \frac{1}{2}\frac{d^2}{dt^2}\int \rho\, x^i x^j\, d^3x = \frac{1}{2}\frac{d^2 I^{ij}}{dt^2} \tag{63}$$

Finally,

$$h^0{}_0(x) = \frac{G}{c^4 r}\frac{d^2 I^n{}_n}{dt^2} = -\frac{G}{c^4 r}\frac{d^2 I}{dt^2} \tag{64}$$

$$h^i{}_j(x) = -\frac{2G}{c^4 r}\left(\frac{d^2 I^i{}_j}{dt^2} + \frac{1}{2}\delta^i{}_j\frac{d^2 I}{dt^2}\right) \tag{65}$$

The above expressions give the radiation field in terms of the motion of the source. The $h_{\mu\nu}$ also satisfy the field equations (53) for matter-free space, $\partial^\lambda \partial_\lambda h_{\mu\nu} = 0$, which admit plane wave solutions such as

$$h^\mu_{\ \nu} = A^\mu_{\ \nu}\cos(-k_\lambda x^\lambda) \qquad (k^0 = k) \tag{66}$$

Conditions (50) now take the form

$$A^0{}_0 = A^n{}_n \qquad k_i A^n{}_n = k_n A^n{}_i \tag{67}$$

If a particular direction is chosen (say k^3) then the following components remain

$$h^0{}_0 = h^3{}_3 \quad h^1{}_1 = -h^2{}_2 \quad h^1{}_2 = h^2{}_1 \tag{68}$$

while $h^2{}_3 = h^3{}_1 = 0$. The transverse part of A_{ij} is traceless

$$h_{\mu\nu} = \begin{pmatrix} h_{00} & 0 & 0 & 0 \\ 0 & h_{11} & h_{12} & 0 \\ 0 & h_{12} & -h_{11} & 0 \\ 0 & 0 & 0 & -h_{00} \end{pmatrix} \tag{69}$$

This can be proved for the general case by using the projection operator [7] $P_{ij} = \delta_{ij} - n_i n_j$ to define $A^T_{ij} = P_{in} P_{jm} A_{nm}$. Then (67) gives $A^{T\,n}{}_n = 0$.

5.2 Radiative energy flow

The flow of gravitational energy is determined by (26)

$$T^{(g)}_{0i} = \frac{c^4}{8\pi G}\Big\{Q^0_{[0n]}Q^n_{[i0]} + Q^n_{[n0]}Q^0_{[0i]}\Big\}$$
$$= \frac{c^4}{32\pi G}\left\{\partial_n h^0_{\ 0}\,\partial_0 h^n_{\ i} + \partial_i h^0_{\ 0}\,\partial_0 h^n_{\ n}\right\} \tag{70}$$

A plane wave satisfies $k\,\partial_i h^\mu_{\ \nu} = k_i\,\partial_0 h^\mu_{\ \nu}$ so that

$$T^{(g)}_{0i} = \frac{c^4}{32\pi G}\left\{\frac{k_n}{k}\,\partial_0 h^0_{\ 0}\,\partial_0 h^n_{\ i} + \frac{k_i}{k}\,\partial_0 h^0_{\ 0}\,\partial_0 h^n_{\ n}\right\} \tag{71}$$

This shows that the transverse-traceless components do not contribute to the energy flow. Conditions (67) now give

$$T^{(g)}_{0i} = \frac{c^4}{16\pi G}\,\frac{k_i}{k}\,(\partial_0 h^0_{\ 0})^2 = \frac{c^4}{16\pi G}\,n_i\,(\partial_0 h^0_{\ 0})^2 \tag{72}$$

The scalar product

$$c\,T^{(g)i}_{0}n^i = \frac{c^5}{16\pi G}\,(\partial_0 h^0_{\ 0})^2 \tag{73}$$

gives the flow in the radial direction. There is no angular dependence and so the total power is found by substituting (64)

$$\frac{dE}{dt} = \int c\,T^{(g)i}_{0}n^i r^2 d\Omega = \frac{G}{4c^5}\left(\frac{d^3I}{dt^3}\right)^2 \tag{74}$$

In a binary system, [6]

$$\frac{d^3I}{dt^3} = -\frac{2m_1m_2}{a(1-e^2)}\,e\,\sin\theta\,\dot\theta \tag{75}$$

Substitution into (74) gives the power

$$\frac{dE}{dt} = \frac{Gm_1^2m_2^2}{c^5a^2(1-e^2)^2}\,e^2\sin^2\theta\,\dot\theta^2 \tag{76}$$

while the average over one period is

$$\left\langle\frac{dE}{dt}\right\rangle = \frac{Gm_1^2m_2^2(m_1+m_2)}{2c^5a^5}\,e^2(1+\frac{e^2}{4})(1-e^2)^{-7/2} \tag{77}$$

This formula exhibits a strong dependence upon eccentricity. In particular, circular orbits will not radiate energy ($e=0$), suggesting that they are more stable than eccentric orbits. Nevertheless, they do emit transverse waves in accordance with (65)

$$h^i{}_j(x) = -\frac{2G}{c^4 r}\frac{d^2 I^i{}_j}{dt^2} \tag{78}$$

5.3 Wave detection

The force (43) exerted on a detector initially at rest is clearly due to $\partial_i g_{00}$ alone

$$\frac{d\mathbf{p}}{dt} = \frac{mc^2}{2}\,\mathbf{e}^i\,\partial_i g_{00} = \frac{mc^2}{2}\,\partial_0 h_{00}\,\mathbf{e}_i n^i \tag{79}$$

which is along the direction of propagation. Substitute (64) to find

$$\frac{d\mathbf{p}}{dt} = -\frac{Gm}{2c^3 r}\frac{d^3 I}{dt^3}\,\mathbf{e}_i n^i \tag{80}$$

Apart from the acceleration of the mirrors (referring to LIGO), the gravitational field changes the wavelength of light in the storage chambers, much as in the static redshift. In this regard, it is important that each mode possesses energy:

$$\begin{aligned} T^{(g)}_{00} &= \frac{c^4}{16\pi G}\left\{Q^m_{n0}Q^n_{m0} + Q^m_{m0}Q^n_{n0} - 2\eta_{00}\eta^{mn}Q^0_{0m}Q^0_{0n}\right\} \\ &= \frac{c^4}{64\pi G}\left\{\partial_0 h^m{}_n \partial_0 h^n{}_m + \partial_0 h^m{}_m \partial_0 h^n{}_n + 2(\partial_0 h^0{}_0)^2\right\} \\ &= \frac{c^4}{32\pi G}\left\{(\partial_0 h^1{}_1)^2 + (\partial_0 h^1{}_2)^2 + 2(\partial_0 h^0{}_0)^2\right\} \end{aligned} \tag{81}$$

where the final line pertains to motion along k^3 (see (69)). The effect on light can then be viewed as an exchange of energy.

The gravitational stress is given by

$$T^{(g)}_{ij} = \frac{c^4}{8\pi G}\left\{2Q^0_{0i}Q^0_{0j} - \frac{1}{2}\eta_{ij}\left[\eta^{00}(Q^m_{n0}Q^n_{m0} + Q^m_{m0}Q^n_{n0}) + 2\eta^{mn}Q^0_{0m}Q^0_{0n}\right]\right\} \tag{82}$$

which is diagonal for the field (69)

$$T_{11}^{(g)} = T_{22}^{(g)} = \frac{c^4}{32\pi G}\left\{(\partial_0 h^1{}_1)^2 + (\partial_0 h^1{}_2)^2\right\} \tag{83}$$

$$T_{33}^{(g)} = \frac{c^4}{32\pi G}\left\{(\partial_0 h^1{}_1)^2 + (\partial_0 h^1{}_2)^2 + 2(\partial_3 h^0{}_0)^2\right\} \tag{84}$$

These stresses are compressive, with the transverse modes exerting an equal pressure in all directions, and the longitudinal mode exerting pressure along k^3.

6. Conclusion

The precis of general relativity is that it provides covariant expressions (tensors) for all physical quantities and for the laws that relate those quantities. However, no such expressions exist for gravitational energy, momentum, stress, force and power. This, in itself, shows that general relativity cannot be correct. The four-dimensional vector formalism is incapable of describing gravitational dynamics [8].

The theory presented here derives from the fact that special relativity was invented without the use of four-vectors. Only scalars and three-vectors were used by the founders. This is evident in the basic elements of the theory: (time, space), (energy, momentum), (charge, current), etc. It is by expressing these elements in terms of coordinates that a new theory of gravitation emerges. It yields a fully covariant treatment of gravitational dynamics.

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