

What is the conclusion of Newton's equations of motion by using differential equations?

Author: MOHAMED AHMED ABOUZEID

STEM LUXOR-EGYPT

<https://orcid.org/0000-0002-5836-7871>

mohamed.abouzeid@stemluxor.moe.edu.eg

0201014419135

ABSTRACT

Conclusion of Newton's equations of motion by using differential equations

Keywords

Newton's equations, differential equations

Contribution

$$y = y_0 + c_1 \frac{dy}{dx} + c_2 \frac{d^2y}{dx^2}$$

$$x = x_0 + c_1 v + c_2 a$$

$$\text{Let } c_1 = k_1 t \text{ and } c_2 = k_2 t^2$$

$$x = x_0 + k_1 t v + k_2 t^2 a \quad a = \text{constant}$$

$$v = 0 + k_1 t a + v k_1 + a k_2 (2t) + k_2 t^2 (0)$$

$$v = k_1 a t + k_1 v + 2 k_2 t a$$

Compare the coefficient of v

$$k_1 = 1$$

Compare the coefficient of at

$$k_1 + 2 k_2 = 0$$

$$2 k_2 = - k_1$$

$$k_2 = -\frac{1}{2}$$

$$x = x_o + vt - \frac{1}{2} at^2 \text{----- (1)}$$

$$y = y_o + c_1 \frac{dy}{dx}$$

$$v = v_o + c_3 a \quad c_3 = k_3 t$$

$$v = v_o + k_3 t \quad a \quad v_o = \text{constant}$$

$$a = 0 + k_3 t \quad (0) + a \quad k_3$$

Compare the coefficient of a

$$k_3 = 1$$

$$v = v_o + at \text{----- (2)}$$

From (1) and (2)

$$x = x_o + vt - \frac{1}{2} at^2 \text{----- (1)}$$

$$v = v_o + at \text{----- (2)}$$

$$x = x_o + (v_o + at) t - \frac{1}{2} at^2$$

$$x = x_o + v_o t + at^2 - \frac{1}{2} at^2$$

$$x = x_o + v_o t + \frac{1}{2} at^2 \text{----- (3)}$$

If $x_o = 0$

$$x = vt - \frac{1}{2} at^2 \text{----- (4)}$$

$$v = v_o + at \text{----- (5)}$$

$$x = v_o t + \frac{1}{2} at^2 \text{----- (6)}$$

Conclusion

$$v = v_o + at \text{----- (5)}$$

$$x = v_o t + \frac{1}{2} at^2 \text{----- (6)}$$