Measuring the Complexity of Simplicity.

By T. H. Ray*

"*Everything should be made as simple as possible, but no simpler.*" ~ A. Einstein

Abstract

Yaneer Bar-Yam[1] based his theory of multi-scale variety on the law of requisite variety [2] that underlies information science. Once we have accepted requisite variety as a theorem for complex systems science, we are compelled to ask:

How *deep* is the structure of complexity? In complex systems science, it *has* to be deep enough to include gravity, because there can be no coherent system, simple or complex, without negative feedback—and gravity, even though we don't understand it as a unitary theory, is demonstrably such a universal control (negative feedback) mechanism.

We will introduce a mathematical framework by which irreducibly complex spacetime—as the only independent cosmological variable—has the potential to create and interact with matter, and mediate feedback.

What is the " … Many which allows itself to be thought of by us as a One", in the words of Georg Cantor, [3] such that gravity is contained in the Many that manifests in our rational judgments as a One? The question begs a set-theoretic construction, and thus we present our result in a set-theoretic context.

I. Introduction

All physics is local. [4]

1.1 Given that information has the requisite variety to be complete and not perfect, † locality generates information variety. "Complete" is contrasted to "complex" in that complexity is not a necessary element of completeness, while completeness is a necessary element of complexity. How does local complexity generate variety?

Because the last critical element is the first element of the new structure, ‡ the evolution of information is indifferent to past and future. *Events* have measured

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† Perfect information is finite and knowable; e.g. the number of slots on a roulette wheel. Complete information is finite and unknowable; e.g. a fractal pattern.

‡Cf. Dedekind; a cut in the number line is made between "the least of the most and the most of the least", both of which are coherent independent parts.

correspondence, assuming causality—events have a beginning and end, correspondent to cause and effect. Past events, as relativity shows, are observer-dependent. We observers assign a temporal meaning.

1.2 Theories based on probability—e.g., quantum mechanics—assume a domain of perfect information. Being products of dimensionality, measurement domains cannot be assumed in a physical sense; i.e., we measure events in three dimensions of space and what we call one dimension of time. Often overlooked is that if time has no preference for past and future, it is not observer-dependent. If *spacetime* is physically real—if space and time are fully integrated—the 4-dimension sphere is a 1-to-1 projection of the 2-dimension flat world.

Using topological terms, given a projection between S^1 and S^3, $\frac{\sqrt{4}}{2} = \sqrt{\frac{4}{2}}$.

Identity inversion in the most natural terms, then, $2^4 = 4^2$, informs us that *transitivity implies identity*. [5] So we take for granted that our 2-dimension measures adjoin 4 dimensions, where the "boundary of a boundary is zero." [§] Yet that's not what we experience. We experience time (transitivity) as a continuous phenomenon—to objectify the concept, we assign an event a coordinate number in relation to the spatial coordinates, and call it continuous with space when we index it in a cumulative record. [6] It is always a *completed* event, however. Transience has a limit, equal to the locality—the scale—at which an event is measured. What are the boundary conditions of locality? Because locality is 1-dimensional, [7] there are infinite varieties of locality embedded in every measured event, just differentiating right and left. Variety comes with every bit of information we receive.

1.3 Complex systems science enters when we appeal to the law of large numbers—interpreted as the beginning and end of the real number line meeting in a single non-zero point. If beginning and end are of opposite polarities, the limit ± 0.5 is the origin of an oscillating unit. So one dimension quite directly implies two dimensions.[**] And two dimensions quite clearly implies four dimensions. Our measure space, however, is three dimensions—literally, we put the cart before the horse. The Riemann hypothesis validates this: taking the starting condition $s = 1$ for the Riemann zeta function $\zeta_s = \sum_{n=1}^{n} \frac{1}{n^s}$ within the bound [0,1] in the complex plane, a two-dimension space. [8]

[§] Attributed to John Wheeler.

[**] "My desire and wish is that the things I start with should be so obvious that you wonder why I spend my time stating them. This is what I aim at because the point of philosophy is to start with something so simple as not to seem worth stating, and to end with something so paradoxical that no one will believe it." ~ Bertrand Russell, *The Philosophy of Logical Atomism (1918)* as quoted in Wikipedia.

1.4 Einstein suggests, however, that field influence is primary even in the 1-dimension spacetime, and this result confirms it. While there does exist a net-negative field, symmetric to the positive, as required by $s = 1$, the interaction is null. Continuity is inevitably restored by pair anti-correlation, **-+-+-+,** an equilibrium state. Zero potential.

Suppose three coordinates define a minimum 3-dimension field in 1 dimension. Coordinates > 3 are required, because there must exist two such fields, $\langle \psi | \psi' \rangle$, that vary in time as a function of the coordinates, yet only self-interact. (Cf. quarks, M. Gell-Mann)

Figure 1.3. Independent self interacting fields.

Neither self-interacting case separately is physical. Yet what do minimally interacting fields tell us?—interactive events carry twice the active potential of self-interacting *systems* alone. Einstein introduced spacetime (*The Meaning of Relativity*) by carefully defining it: "… independent in its physical properties; having a physical effect, but not itself influenced by physical conditions," suggesting that the origin of the spacetime field is the origin of mass (as quantum field theory, and its string theory extension, emphasize).

Kevin Brown wrote in his marvelous collection of essays *Reflections on Relativity:*

"*This image of a photon as a single unified event with a coordinated emission and absorption seems unsatisfactory to many people, partly because it doesn't allow for the concept of a 'free photon', i.e., a photon that was never emitted and is never absorbed. However, it's worth remembering that we have no direct experience of 'free photons', nor of any 'free particles', because ultimately all our experience is comprised of completed interactions. (Whether this extends to gravitational interactions is an open question.)*" [9]

1.5 In a real sense, time ends whenever we record an event (information). One dimension is the simplest admitting bilateral symmetry—a distinction between left and right for which we do not need boundary conditions, because each direction is a vector bounded by zero, on the half-open interval $[0,1)$.

A random walk is sufficiently models a path in one dimension with one degree of freedom, yet we do not record an event in one dimension, and we do not distinguish past from future. We assign cause and effect, relying on mathematical methods to tell us that observed causes all lie in the past. Except that if we don't allow unobserved causes, we have no future effects that we can predict. The lesson of chaos theory is that the unexpected is unpredictable, suggesting that there is no single path; rather, a network of *n* intersecting paths.

1.6 Einstein said: [10]

"The interference phenomena would really not be so difficult to arrange as one imagines and indeed on the following ground[s]: one must not assume that the radiation consists of quanta that are not in interaction. That would be impossible for the clarification of the interference phenomena. I think of a quantum as a singularity surrounded by a large vector field. A large number of quanta compose a vector field that differs little from what we currently accept as radiation. I can imagine that a compartmentalization of the quanta takes place at the impingement of the radiation on the boundary surface through [its] effect on the boundary surface, [wall], perhaps every [one] according to the phase of the resultant field with which the quanta reach the separating surface."

More or less obvious is that the singularity, which Einstein views as a quantum, is a *physical* singularity, a physical observer—possessing dimension—and not a mathematical singularity (space collapsed to a point). Transitivity implies identity. Also, these singularities carry a charge, controlling the direction of the vector field at every scale. Important to note is that averaging does not change the dimension—it is still a fundamental 1-dimension line $(-\infty,+\infty)$ that in the complex plane is equivalent to a point $(0,0)$. The extended complex plane $(0,0,0,0)$ as we mentioned, does not complicate the smooth mapping of the point to 4 dimensions. We live, however, in 3 dimensions of 6 coordinate points.

1.7 Because we see the point—the mathematical singularity—as a constant with the value of unity, 1, we assume uniformity of time throughout the universe. If *all physics is local*, uniformity of the laws of physics does not imply uniformity of time. If time is a law, this is its principle:

Time is dependent on initial condition, and nothing else. As an element of an independent spacetime, it begins with the initial condition spacetime prescribes, and everywhere ends with an event.

1.8 What is the cardinality of the least element supporting the structure of complex systems? What contains the requisite variety of elements to initiate an action? What is the least variety in the least element of the least dimension?

A set-theoretic model provides sufficient basis independent of the physics, and admitting no boundary between classical and quantum domains. [11]

If the underlying structure of the world is an irreducibly complex system, if spacetime is physically real, and if all physics is local—Bar-Yam's theory of multi-scale variety suggests an *a priori* self-organized, thus self-limiting system.

2.0 Method

2.1 We take a set theory model of self-organized spacetime, [12] as the continued sum of dimension cardinal points—essential coordinate points native to the dimension. That one comprehends a potentially connected set of coordinates of any dimensionality, is fundamental to task coordination (using Bar-Yam's term)—comprehension in this case being the task of compressing information [13] —defined by the variety of cardinal points natively available to the set of a given dimensionality.

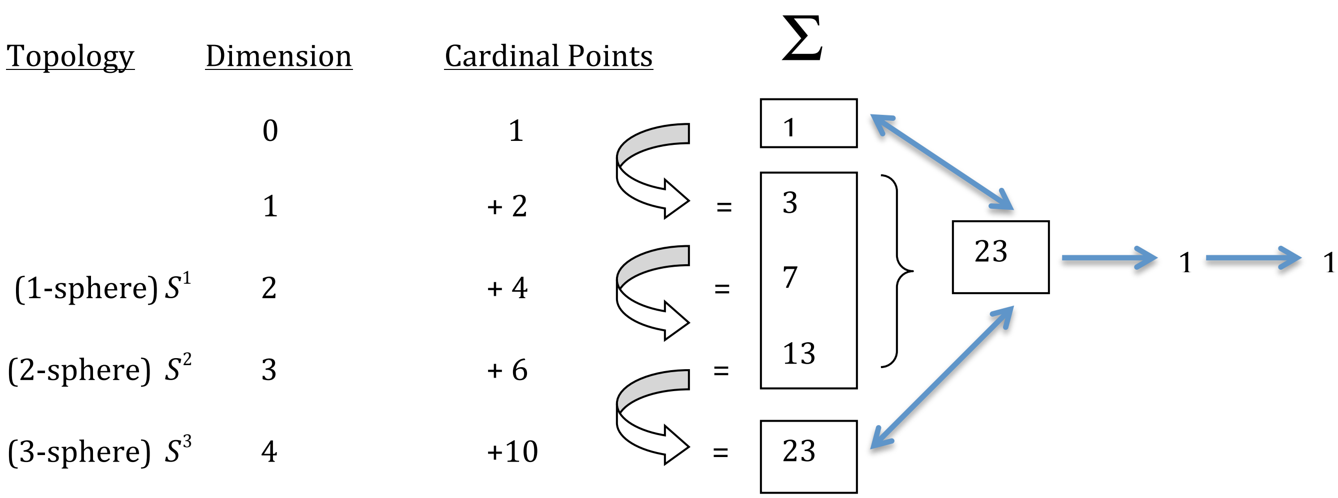


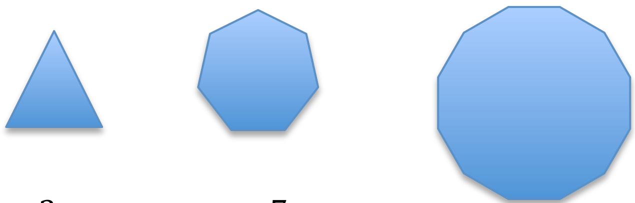
Fig. 2.1 Self-Organized Spacetime.

The greatest variety is found in dimensions 1, 2, and 3. In 0 to 4 dimensions, there is equivalence of cumulative sums in cardinal point sets:

$$\sum_{d=0}^{4} 23 = \sum_{N=0}^{10} 23$$

Yet $d = 3$ cumulative is also $\sum_{\{3,7,13\}} 23$.

We find that these sets are internally consistent, and indecomposable into primes. We can look at them as discrete sets:



3 7 13[††]

Consider the edges and vertices. Prime-valued-gons in 2 dimensions never correspond vertex-to-vertex, only vertex to edge. Call vertices, points, and edges, intervals. We propose that the interval is not the conventional closed interval $[0,1]$ of probability theory in which there is believed to be a definite value—prime-valued-gons correspond by complex analysis on the half closed point-interval $[0,1)$. This restates Brouwer's three-valued logic; [14] all judgments are *true, false*, or *unproven*—and independent of dimension. Which elevates de Castro's proof (see references) to high importance: as $|S| \leq 2$ and $|S| > 2$ simultaneously possess the cardinality of the set $\{0,1\}$ and the cardinality of the continuum $(0,1)$ a "*move of time*" (Brouwer) that transforms 1 into $|1|+|1|$, allows an infinity of zeros. This begs a positivity condition, and exposes the role of time, because $|x|$ is not necessarily identical to $|x'|$. That is, the terms exchange position continually, perhaps randomly, leading to time intervals that are non-differentiable, past and future. The information we process linearly arrives non-linearly, converted to a 1-dimension line with no differentiation of past and future events, because the entropy generated by processing is indifferent to order. Which accounts for global uncertainty—and local meta-stability. [15]

2.2 The 1-dimension 3-gon has 2 degrees of freedom, enough to fill the 1-sphere S^1 with its 7 coordinate points, 3 of which are redundant; S^2 -- the topological version of our ordinary 3 dimensions -- has 13 points, 10 of which are redundant. Do you see where this is going? The Minkowski space-time matrix (fig. 1.2) has 16 points, 6 of them redundant. The 10 non-redundant points of the Minkowski space are the same 10 redundant points that we find by our method. The difference is that our method recognizes the value of redundancy—the 7 excess points of S^2 are enough

[††] Forgive an old man for being too lazy to draw a 13-gon, too clueless to know how to copy and paste an image to the document, and too proud to ask for help.

to make another 3-dimension space, and the excess 13 points two more 3-dimension spaces; 10 more points make S^3 (23), and we have 23 + 1, more than enough for three 3-dimension spaces, and one point short of four 3-dimension spaces.

$$\begin{pmatrix} g_{00}g_{01}g_{02}g_{03} \\ g_{10}g_{11}g_{12}g_{13} \\ g_{20}g_{21}g_{22}g_{23} \\ g_{30}g_{31}g_{32}g_{33} \end{pmatrix}$$

Figure 2.2. Minkowski space matrix.

Consider that the sum of interior angles in a 23-gon is 3780. $3780/360 = 10.5$
The interior angles of a 24-gon sum to 3960, or 11. Not coincidentally, these two results—11 and 10.5—are the upper and lower dimension bounds of M theory (Witten), which may yield to an arithmetic solution, and explain Riemann's zeta function.

2.3 Now we see how important it is that all the sets are prime-valued.

Were the extra point included in 4-dimension space, and not recursively available, as it is, the space would be completely stable. Think of the 24-dimension sphere packing number [16] 196,560—it's not the number that's important, so much as the fact that there *is* an integer solution, an end point. There's no fancy trick: the Leech lattice, as it is known, is a Euclidean lattice packing, and factors of 24—2, 3, 4, 8—are lattice packings as well.

And it's important to the way we define 'dimension'. We have 0 + 1 dimensions to describe real 1-dimensional spacetime. (Identical to string theory.)

2.4 Bekenstein and Mayo [17] made the 1-dimension case for equivalent past and future entropy, for if black holes are 1-dimensional, past and future information enters and leaves randomly, on one channel only. Cf. the Chaitin Ω [18] random result. The constant's one dimension string is independent of all other programs that run the Chaitin algorithm, with different random results.

As random as these sequences appear, in the context of a unified spacetime, a sufficient random number of intersecting delimited random number sequences have the requisite variety to self-organize an ordered sequence. [19] Self-organization contains in principle self-limitation, equivalent to an algorithmically delimited sequence—which leaves open the question:

What is the least Kolmogorov-Chaitin algorithmic information [20] required to induce complexity? What is the ground state of 1 dimension?

2.5 The extra point—there are 7 points in S^1, fig. 1.1—assures by positive feedback that the system never reaches equilibrium globally, while downward-acting negative feedback guarantees meta-stability locally. Global meta-stability demands that, in Chaitin's words, " … comprehension is compression." Negative feedback in the aggregate exceeds the positive by a constant amount, + 1 (Which gives numeric value to Einstein's statement, "The most incomprehensible thing about the world is that it is comprehensible").

2.6 We conjecture data compression is equivalent to length contraction and time dilation—as an extension of the Einstein equivalence principle, i.e., the equivalence of inertial and gravitational mass. Least variety is found in the 6 + 1 points of S^1, dimension 2. To us, physically, it appears as the coordinate system of 3-dimension space—with an observer at the axis. Because general relativity is a coordinate-free geometry, there is no privileged observer frame—making general relativity an observer-free theory also.

2.7 Mathematically and physically, positive mass implies positive time and negative space. That is, we calculate elapsed time—the observed changes in a system, both positive and negative—only between *positive mass* points. By the **positive mass theorem** [21] [22] an observer made of mass and spacetime is integrated into spacetime; however, an inequality between positive mass and negative space implies compression in Chaitin's terms, on the emission-absorption boundary.

3.0 Research

3.1 Let us prove this **theorem** from first principles: *a point {s} can simultaneously approach any set of points {S} of any cardinality and separation, provided it is far enough away.* ■

Taking {s} as origin and assigning {S} cardinality 1, {s} → {S} is a coordinate system, making a mathematical "move of time" suggested by the term 'simultaneously'. A half time cycle would be represented $\{S\} \leftarrow \{s\} \rightarrow \{S\}$

Corollary: *If all points are attracted to the center of their local coordinate system, there are at minimum 3 separable points on a one-dimensional line segment.*

Following from the corollary, $\{S\} \rightarrow \{s\} \leftarrow \{S\}$.

So we generalize the meaning of what we call a 'spacetime cycle' to *n* dimensions

$$\{S\}^{n} \rightarrow \{s\} \rightarrow \{S\}^{m}$$
$$\{S\}^{m'} \leftarrow \{s\} \leftarrow \{S\}^{n'}$$

And it should be obvious that {s} contains infinitely many copies. A theorem due to Brouwer calls for *invariance of dimension*. $X^{n} = Y^{m} IFF m = n$ ■

A point on a 1-dimension line is never "*far enough away*" unless the solution is 1. So the cardinality of S is zero. Yet the cardinality of S cannot be zero because infinitely many copies of s impose themselves in the center of every coordinate field.

3.2 Following from invariance of dimension, the pair of equations above implies least action in a single dimension, *independent of any observer*.

3.3 Notice from fig. 1.1 that once organized into an irreducible set—i.e., once filling all the space available—the +1 point remaining, the extension of set 23, *must* renew the cycle in an infinite string of iterations. And as every point of 4-dimension spacetime contains 6 points of 3-dimension space, every micro scale experiment is in the context of field reaction and the displacement field: *" ... the infinitesimal displacement field ... replaces the inertial system inasmuch as it makes it possible to compare vectors at infinitesimally close points."*

3.4 Consider 3-dimension compactification a function of $x!$, $x = 3$. When $x = 4, x! = 24$. Four compacted dimensions of 24 coordinate points, (more precisely, 23 + 1) are prior to the 16-point Minkowski space matrix.

4.0 Discussion

If there is no boundary between classical and quantum domains, the least dimension short of a dimensionless point already has the potential for complexity. No complex system is isolated; rather, every system is embedded in the ordinary space of measurement where requisite variety is bountiful, and the geometry is free of observer choice of coordinates. By our explicit construction, the primeness of 23 generates closure in 3 and 4 dimensions.

Because all physical measurements are taken relative to a point at infinity, positivity is guaranteed fundamental by the continuum hypothesis—one can choose it or not. [23] One cannot, however, choose the hypothesis and the negation of the hypothesis at the same time. [24]

The potential for complexity is built into the system from the beginning, because the source point is connected to every potential event.

What are the necessary elements of control (what Ashby calls constraint)—i.e., negative feedback? What is positive feedback but a discrete recurrence function? What is negative feedback but a continuous topological function? Thus, gravity is universal negative feedback, assured by recurrence of the discrete source state. This source is everywhere local and nowhere extended—a consequence of the choice function—which as shown in (Ray, 2006) is independent of observer choice. Local meta-stability is dependent on global instability in the form of random oscillations.

For Anu

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References

[1] Bar-Yam "Multiscale Variety in Complex Systems" *Complexity* vol. 9, issue 4 (March 2004) pp. 37-45.

[2] Ashby, W. R. http://www.ecotopia.com/webpress/nurcap/cybernetics198803.htm accessed 27 June 2018

[3] Vilenkin, Nahum Ya., *In Search of Infinity* (translated by Abe Shenitzer) Birkhauser 1995

[4] Einstein, "God does not play dice." Probability space is always nonlocal. John Wheeler, "No phenomenon is physical until it is an observed (measured) phenomenon."

[5] Ray, T. "Self Organization in Real & Complex Analysis" NECSI ICCS 2006

[6] Hess, K. and Philipp, W. http://www.pnas.org/content/101/7/1799?ijkey=8f0ca3c4d4c18947afd59e105d1c598d9266ac4d&keytype2=tf_ipsecs accessed 27 June 2018; also, Hess, K. *Einstein was Right!* Pan-Stanford, 2015

[7] Bekenstein, J. & Mayo, A. "Black holes are one-dimensional" *Gen.Rel.Grav. 33* (2001) 2095-2099 https://arxiv.org/abs/gr-qc/0105055

[8] "Witten and others have pointed out that although superstrings are formulated in a 10-dimension spacetime, in another sense a string field is also a theory about two-dimensional surfaces. The fields are a quantum expression of all the dynamical movements of a world sheet. But these world sheets are two-dimensional; that is, they are created out of a one-dimensional line or loop stretched into the second dimension of time. In this sense a string field theory is therefore about the quantum properties of two-dimension surfaces." Peat, F. David, *Superstrings*, Contemporary Books, 1988. p. 307

[9] "Locality and Temporal Asymmetry" in *Reflections on Relativity*, 9.9 p. 671, Kevin Brown, 2011. Also available online, http://mathpages.com/rr/s9-09/9-09.htm

[10] Einstein, A. *Physikalische Zeitschrift* Vol. 10. No. 22, p. 817 (discussion section)

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[11] We are using the nonstandard term 'cardinal' points because these points exist prior to the level of spatial coordinates, as abstract sets. Alexandre de Castro of Brazil, has a nice set-theoretic proof that eliminates the quantum-classical boundary by equivalence of the Boolean classical domain {0,1} which has cardinality $|S| \leq 2$ and the quantum interval (0,1) with cardinality $|S| > 2$.
https://www.researchgate.net/profile/Alexandre_Castro3

[12] Ray, NECSI ICCS 2006

[13] https://arxiv.org/pdf/math/0411091.pdf accessed 28 June 2018

[14] https://plato.stanford.edu/entries/brouwer/ accessed 9 March 2018

[15] Andrei Khrennikov: ""In principle, chaotic fluctuations in the microworld may generate statistical stability on the macrolevel!" Khrennikov, A. http://arxiv.org/pdf/quant-ph/0301051.pdf accessed 28 June 2018

[16] http://www.math.ups.edu/~bryans/Current/Journal_Spring_2006/ARoberts_LeechLattice.pdf accessed 28 June 2018

[17] Bekenstein-Mayo. *op.cit.*

[18] Chaitin, G. *Meta-Math! The quest for Omega*, Vintage, 2006.

[19] Ray, T. *op cit.*

[20] Chaitin, *op.cit.*

[21] Witten, E. https://pdfs.semanticscholar.org/8e92/e36299c87a185184735c7046a64f49bfd13a.pdf accessed 28 June 2018

[22] Yau, S.T. http://www.doctoryau.com/papers/PositiveMassTheorem.pdf accessed 28 June 2018

[23] Cohen, P. proof https://www.ias.edu/ideas/2011/kennedy-continuum-hypothesis accessed 28 June 2018

[24] R. Gill, *et al*, http://www.univie.ac.at/qfp/publications3/pdffiles/2002-10.pdf (accessed 28 June 2018) purport to refute the Hess-Philipp result (6) with the conclusion: "Time is not an issue in the proof of Bell's theorem. What is crucial is the freedom of the experimenter to choose either of two settings at the same time. Hess and Philipp's hidden variables model is nonlocal." Far from being crucial, the experimenter's choice is part of the experimental apparatus. In fact, no matter which setting an experimenter chooses, the result will be nonlocal; the argument merely finds what it assumed.