Refutation of collection theory as the set of universal closure of sentences

Abstract: We evaluate collection theory as the set of universal closure of sentences in a schema equation. It is *no*t tautologous. This refutes Collection as the conjectured schema. Therefore collection theory is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Quinsey, J.E. (1980). Applications of Kripke's notion of fulfilment. Dissertation. St. Catherine's College, Oxon. arxiv.org/pdf/1904.10540.pdf jquinsey@i2msystems.com

When we say, for example, that Collection is the schema

$$\forall x \in a\ \exists y\theta \supset \exists b\ \forall x \in a\ \exists y \in b\theta\ , \tag{1.1}$$

we mean that Collection is the set of universal closure of sentences of this form, where θ ranges over all formulae of the language under consideration, and where suitable precautions are taken to avoid collision of variables. A *theory* is a set of sentences.

LET p, q, x, y, t: *a*, *b*, *x*, *y*, θ.

(#x<(p&(%y&t)))>((%q&#x)<((p&%y)<(q&t))) ;

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TTTT TTTT TTTT TTTT(16), CCTT CCTT CCTT CCTT(16),
TTTT TTTT TTTT TTTT(16),
CCTC CCTC CCTC CCTC, CTTT CTTT CTTT CTTT}x8=(16)      (1.2)
```

Eq. 1.2 as rendered is not tautologous. This refutes Collection as the conjectured schema.