The Mechanics' Contradiction:

I proved in my report "Mémoire en physiques" that:
We can't use $\sin d\varphi \simeq d\varphi$ and $\cos d\varphi \simeq 1$.
Now, if we use: $\sin d\varphi \simeq d\varphi$ and $\cos d\varphi \simeq \sqrt{1-d\varphi^2}$ than:
(with e and h as in the page 3 of my report where the trigonometric circle is)

$$e \simeq 1 - \sqrt{1-d\varphi^2} \Rightarrow e^2 \simeq 2 - d\varphi^2 - 2\sqrt{1-d\varphi^2}$$

than: $$e^2 + h^2 \simeq 2 - 2\sqrt{1-d\varphi^2} \simeq d\varphi^2$$

hence: $$2 \simeq d\varphi^2 + 2\sqrt{1-d\varphi^2} \Rightarrow 4 \simeq d\varphi^4 + 4 - 4d\varphi^2 + 4\,d\varphi^2 \cdot \sqrt{1-d\varphi^2}$$

Consequently: $$1 \simeq \frac{d\varphi^2}{4} + \sqrt{1-d\varphi^2} \Rightarrow \left(1-\frac{d\varphi^2}{4}\right)^2 \simeq 1 - d\varphi^2$$

We conclude that: $$1 + \frac{d\varphi^4}{16} - \frac{d\varphi^2}{2} \simeq 1 - d\varphi^2 \Rightarrow d\varphi^2 \simeq -8$$

* We proved that: $\sin d\varphi = d\varphi$ and $\cos d\varphi = \sqrt{1-d\varphi^2} \Rightarrow d\varphi^2 \simeq -8$ which is an impossible result.

* We finally conclude that instead of using $\sin d\varphi \simeq d\varphi$ and $\cos d\varphi \simeq \sqrt{1-d\varphi^2}$, we should use, to avoid contradictions, $\cos d\varphi = 1 - \frac{d\varphi^2}{2} \Rightarrow \sin d\varphi = d\varphi \cdot \sqrt{1-\frac{d\varphi^2}{4}}$ which is explained in my work: "mémoire en physiques" with the correct results.

The real time at the satellite:
(The case of the GPS system satellites)

* When the speed V of the satellite is constant $V = k_1$, and the initial altitude of the satellite is $h = k_2$, than we have by using my derivation in physics:

$$\frac{d\overrightarrow{OM}}{dt} = \frac{d\rho(t)}{dt}\cdot\vec{e}_\rho - \frac{d\varphi}{2}\cdot\dot{\varphi}\cdot\rho(t)\cdot\vec{e}_\rho + \rho(t)\cdot\dot{\varphi}\cdot\sqrt{1-\frac{d\varphi^2}{4}}\cdot\vec{e}_\varphi$$

M stands for the position of the satellite which has a circular orbit, so we must have:

$$\frac{d\rho(t)}{dt} = \dot{\varphi}\cdot\frac{d\varphi}{2}\cdot\rho(t) \Longleftrightarrow \rho(t) = e^{\frac{d\varphi}{2}\cdot\varphi + \ln h} = h\cdot e^{\frac{d\varphi}{2}\cdot\varphi} \quad (1)$$

* since V is constant than: $\rho(t)\cdot\frac{d\varphi}{dt}\sqrt{1-\frac{d\varphi^2}{4}} = V \Longleftrightarrow \frac{d\varphi}{dt} = e^{-\frac{d\varphi}{2}\cdot\varphi + \ln\frac{V}{h\sqrt{1-\frac{d\varphi^2}{4}}}}$

$$\Longleftrightarrow \frac{d\varphi}{dt} = \frac{V}{h\cdot\sqrt{1-\frac{d\varphi^2}{4}}}\cdot e^{-\frac{d\varphi}{2}\cdot\varphi} \quad (2)$$

Remarks:

1) $\rho(t)$ increases because of the speed of the launching of the satellite.
2) $\varphi = 0$ at the position where the satellite started orbiting after being launched.
3) $d\varphi$ is the smallest variation of angle that we can detect, and dt is the time needed for that variation.
4) $\frac{d\varphi}{dt}$ is always positive so we consider φ always positive ($\varphi > 0$).

* Since $d\varphi$ is constant but dt not constant, we conclude that:

$$dt = \frac{h}{V}\cdot\sqrt{1-\frac{d\varphi^2}{4}}\cdot d\varphi\cdot e^{\frac{d\varphi}{2}\cdot\varphi} \quad \text{(from (2))}$$

Consequently, dt really depends on the angle φ of the satellite, and that angle φ is always increasing.

Conclusion:

* The real time needed for a satellite to make one "lap" around the earth is:

$$\Delta T = \sum_{i=1}^{K}\frac{h}{V}\cdot\sqrt{1-\frac{d\varphi^2}{4}}\cdot e^{\frac{d\varphi}{2}\cdot i\cdot d\varphi}\cdot d\varphi + \Delta t = \frac{h}{V}\cdot d\varphi\cdot\sqrt{1-\frac{d\varphi^2}{4}}\cdot\sum_{i=1}^{K}e^{i\frac{d\varphi^2}{2}} + \Delta t$$

with $K = \left[\frac{2\pi}{d\varphi}\right]$ and $\Delta t = \frac{h}{V}\cdot\sqrt{1-\frac{d\varphi^2}{4}}\cdot d\varphi_1\cdot e^{\frac{d\varphi_1}{2}\cdot 2\cdot\pi}$ with: $d\varphi_1 = 2\pi - K\cdot d\varphi$