The transferable complex belief model

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Abstract—We describe the transferable complex belief model, a model for representing quantified beliefs based on a newly defined complex belief function. The relation between the complex belief function and the probability function is derived when decisions must be made.

Index Terms—Complex belief function, Quantified complex beliefs, Complex number.

I. THE COMPLEX BELIEF FUNCTION

Let $\Omega$ be a set of mutually exclusive and collective non-empty events, defined by

$$\Omega = \{E_1, E_2, \ldots, E_i, \ldots, E_N\},$$

where $\Omega$ represents a frame of discernment.

The power set of $\Omega$ is denoted by $2^\Omega$, in which

$$2^\Omega = \{\emptyset, \{E_1\}, \{E_2\}, \ldots, \{E_N\}, \{E_1, E_2\}, \ldots, \{E_1, E_2, \ldots, E_i\}, \ldots, \Omega\},$$

and $\emptyset$ is an empty set.

Definition 1: (Complex mass function)

A complex mass function $M$ in the frame of discernment $\Omega$ is modeled as a complex number, which is represented as a mapping from $2^\Omega$ to $\mathbb{C}$, defined by

$$M: \ 2^\Omega \rightarrow \mathbb{C},$$

(3)

satisfying the following conditions,

$$M(\emptyset) = 0,$$

$$M(A) = m(A)e^{i\theta(A)}, \ A \in 2^\Omega$$

(4)

$$\sum_{A \in 2^\Omega} M(A) = 1,$$

where $i = \sqrt{-1}$; $m(A) \in [0, 1]$ representing the magnitude of the complex mass function $M(A)$; $\theta(A) \in [-\pi, \pi]$ denoting a phase term.

In Eq. (4), $M(A)$ can also expressed in the “rectangular” form or “Cartesian” form, denoted by

$$M(A) = x + yi, \ A \in 2^\Omega$$

(5)

with

$$\sqrt{x^2 + y^2} \in [0, 1].$$

(6)

By using the Euler’s relation, the magnitude and phase of the complex mass function $M(A)$ can be expressed as

$$m(A) = \sqrt{x^2 + y^2}, \ \text{and} \ \theta(A) = \arctan\left(\frac{y}{x}\right),$$

(7)

where $x = m(A) \cos(\theta(A))$ and $y = m(A) \sin(\theta(A)).$

The square of the absolute value for $M(A)$ is defined by

$$|M(A)|^2 = M(A)\overline{M}(A) = x^2 + y^2,$$

(8)

where $\overline{M}(A)$ is the complex conjugate of $M(A)$, such that $\overline{M}(A) = x - yi$.

These relationships can be then obtained as

$$m(A) = |M(A)|, \ \text{and} \ \theta(A) = \angle M(A),$$

(9)

where if $M(A)$ is a real number (i.e., $y = 0$), then $m(A) = |x|$.

The complex mass function $M$ modeled as a complex number in the generalized Dempster–Shafer (GDS) evidence theory can also be called a complex basic belief assignment (CBBA).

If $|M(A)|$ is greater than zero, where $A \in 2^\Omega$, $A$ is called a focal element of the complex mass function. The value of $|M(A)|$ represents how strongly the evidence supports the proposition $A$.

Definition 2: (Complex belief function)

A complex belief function can be defined by a mapping, $M: 2^\Omega \rightarrow \mathbb{C}$, called a complex basic belief assignment, satisfying the following axioms:

- $M(\emptyset) = 0$,
- $\sum_{A \in 2^\Omega} M(A) = 1$.

With regards to the complex basic belief assignment, the complex belief function in a proposition $A \in 2^\Omega$ can be defined by

$$Bel(A) = \sum_{B \subseteq A} M(B),$$

(10)

Definition 3: (Complex plausibility function)

The complex plausibility function of proposition $A$, denoted as $Pl(A)$ is defined by a mapping from $2^\Omega$ to $\mathbb{C}$

$$Pl(A) = 1 - Bel(\bar{A}) = \sum_{B \cap A \neq \emptyset} M(B),$$

(11)

where $\bar{A}$ is the complement of $A$, such that $\bar{A} = \Omega - A$.

II. THE PIGNISTIC PROBABILITY DERIVED FROM A COMPLEX BELIEF FUNCTION

Definition 4: (Complex pignistic probability transformation)

Let $M$ be a complex basic belief assignment on the frame of discernment $\Omega$ and $A$ be a proposition where $A \subseteq \Omega$, the complex pignistic probability transformation function is defined by

$$Bet(B) = \sum_{A \in 2^\Omega} \frac{|M(A)|}{|B \cap A|} \frac{|B \cap A|}{|A|},$$

(12)
where $|A|$ represents the cardinality of $A$, $|B \cap A|$ represents the cardinality of intersection of $|B \cap A|$, and $|M(A)|$ represents the absolute value of $M(A)$. 