I try to write the differential equation for a continuous electric circuit (an electric circuit with the meshses that tend to zero dimension), so that I can write the continuous Kirchhoff's laws like a differential equation for the currents and voltages: a wave equation must be exist for the continuous circuit (similarly to the electromagnetic wave for Maxwell’s equations).

An optimal circuit can be built (for example an optimal band-pass filter), changing the complex impedances in the space: the continuous RLC circuit. It should be possible to study lattices with inhomogeneous ions, or metal coating (for example superconductors) using the mechanical-electrical analogy (rlc circuit like an ion interaction analogy)

The continuous Kirchhoff’s laws are:

\[
\begin{align*}
\nabla \cdot \mathbf{I} &= 0 \quad \text{Kirchhoff's current law} \\
\nabla \times \mathbf{V} &= 0 \quad \text{Kirchhoff's voltage law}
\end{align*}
\]

if there is a continuous variation of the impedances:

\[
\mathbf{V}(\mathbf{x}, \omega) = Z(\mathbf{x}, \omega) \mathbf{I}(\mathbf{x}, \omega)
\]

\[
\begin{align*}
\nabla \cdot \mathbf{I} &= 0 \\
\nabla \times (Z \mathbf{I}) &= 0
\end{align*}
\]

if the voltages are irrotational, then

\[
\mathbf{V} = Z \mathbf{I} = \nabla \phi
\]

\[
\mathbf{I} = Y \nabla \phi
\]

\[
\nabla \cdot (Y \nabla \phi) = 0
\]

\[
\nabla Y \cdot \nabla \phi + Y \Delta \phi = 0
\]
\[ \nabla \phi \cdot \nabla \ln Y + \Delta \phi = 0 \]

this is the Kirchhoff’s law for a continuous circuit.

There is a solution in a neighbourhood of \( x \):

\[ \phi(x + \epsilon) = e^{ik \cdot \epsilon} \]

\[ k \cdot \nabla \ln Y(x, \omega) + k \cdot k \simeq 0 \]

\[ k \cdot [\nabla \ln Y(x, \omega) + k] = 0 \]

there are two solutions for \( k \):

\[ k = 0 \]

\[ k = -\nabla \ln Y(x, \omega) \]

then the local wave solution is:

\[ \phi(x + \epsilon) \simeq e^{-ik \cdot \nabla \ln Y(x_0, \omega) + i\omega t} \]

the velocity of the local wave equation is:

\[ c(x, \omega) = \frac{\omega}{|\nabla \ln Y(x, \omega)|} \]

and the wave equation for the local current is:

\[ \Delta \phi - \frac{|\nabla \ln Y|^2}{\omega^2} \partial_{tt} \phi = 0 \]

the current value, for local wave equation is:

\[ I = Y \ \nabla \phi = \phi \ \nabla \ln Y \]

\[ I = \nabla \phi = \phi \nabla \ln Y \]

There is a solution for constant current, and constant voltages:

\[ \phi = k \cdot x \]

\[ V = \nabla \phi = k \]

\[ I = Y \ \nabla V = Y \ k \]

for stationary currents, the current divergence is null: this is a solution for coaxial cable, or telegraphic cable.

If there is a periodic variability of the impedance

\[ Y(x) = e^{i\omega_p x} \]

then there is a periodicity of the currents for the Kirchhoff’s continuous law (with not zero gradient, and progressive and regressive waves): the phonons.