

# A New Hypothetical Solution to Gravitational Singularities

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## Abstract

An expected behaviour of space-time and matter within a black hole is postulated regarding a model based on the mathematical indications of other well-known and empirically confirmed theories, without invalidating such theories when applied where they correspond, and solving the mathematical problems such as the gravitational singularity and the information paradox.

## 1. Introduction

What happens inside the event horizon of a black hole is unknown. General theory of relativity<sup>1</sup> describes with enormous precision almost all space-time regions we know, but when applied to the inside of a black hole, a gravitational singularity is suggested to appear, that is, a location where the gravitational field is predicted to become infinite, as well as the density of matter.

This kind of incoherences are usually considered wrong predictions, just like the forces of gravity between two particles that should increase asymptotically to infinity when approaching at very small distances by the Newton's equation

$$\vec{F}_{12} = -G \frac{m_1 m_2}{r_{21}^2} \hat{r}_{21}$$

This equation, which does not give us wrong predictions when applied to the correct cases, certainly presents many indications of how does matter behaves in the general relativity. We will study the indications of Newtons equation for a special case: two equal but negative masses.

Following the equation, both negative signs in the masses are cancelled and we obtain the traditional forces of the same value, equal direction and different sense (attractive) for every particle. The Newtons acceleration equation

$$\vec{a} = \frac{\vec{F}}{m}$$

results in equal accelerations, in the same direction, but different (repulsive) sense, taking into account the negative of the inertial mass for each particle. This indicates that both particles would experience a repulsive effect, equivalent to antigravity, in a way that the weak principle of equivalence holds (a negative mass would have the same gravitational and inertial mass).<sup>2</sup>

Problem arises when applying this development to two particles of different mass signs. The results are two accelerations in the same direction, which implies that both particles would accelerate, one chasing each other, infinitely or at least up to the speed of light, together with paradoxes like perpetual motion machines or infinite energy sources.

## 2. Indications

The existence of negative mass universes is not prohibited by general relativity<sup>3</sup>, and it is consistent with the theory and its proven application since it was discovered. It is suggested that antimatter's negative energy solution from Dirac's equation

$$\psi = u(E, \vec{p})e^{i(\vec{p}\cdot\vec{r}-Et)}$$

taking into consideration the time dependence with  $p=0$

$$\psi = u(E, 0)e^{-iEt}$$

with solution for negative energy

$$E = -mc^2; \text{ with } c=1; E = -m$$

(notice the negative inertial mass or rest mass), gives us two spinor states

$$\psi_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{+imt}; \text{ and } \psi_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} e^{+imt}$$

that can be interpreted as a negative energy solution with negative time (particles moving backwards in time), following the Feynman-Stueckelberg interpretation<sup>4</sup> with  $p \neq 0$

$$\begin{aligned} v_1(E, p)e^{-i(Et-\mathbf{x}\cdot\mathbf{p})} &= u_4(-E, -p)e^{i(Et-\mathbf{x}\cdot\mathbf{p})} \\ v_2(E, p)e^{-i(Et-\mathbf{x}\cdot\mathbf{p})} &= u_3(-E, -p)e^{i(Et-\mathbf{x}\cdot\mathbf{p})} \end{aligned}$$

with the phase of the wave function as  $(Et - \mathbf{x}\cdot\mathbf{p})$ ;

for changing to positive energy  $-E \rightarrow E$ ;

we have to change  $-t \rightarrow t$ , and swap  $p \rightarrow -p$  for consistency,

(antimatter would have positive energy and positive mass in a positive time universe and negative energy and negative mass in a negative time universe, due to the non-symmetrical parity-time transformation, consistent with positive energy results of antiparticle-particle annihilation in our positive time universe), which could be a real physical solution<sup>5</sup> (Figure 1.).

The change in the sign of time (which is not what we usually think for reversing time as playing a film backwards, a common misconception about what a time transformation really means) is the key point for relating this case to black holes, and the change in the sign of momentum will also be discussed for this particular case.

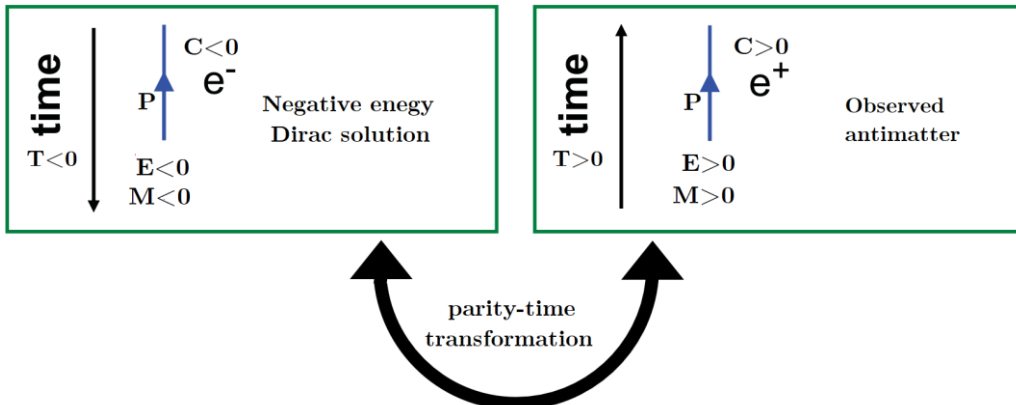


Fig. 1. Relation between the particle Dirac solution and an antiparticle

It is said that moving in space becomes partially moving in time inside a black hole. A better explanation is given by the common solution to the Einstein's field equations by the Schwarzschild metric<sup>6</sup>

$$ds^2 = -c^2 d\tau^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad r_s = \frac{2GM}{c^2}$$

in a way in which substituting and simplifying we obtain

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) (cdt)^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

so that, for  $r < r_s$  (Schwarzschild radius), the sign of the term  $\left(1 - \frac{2GM}{c^2 r}\right)$  turns to be negative, and by that change, the metric term  $dt^2$  changes its sign to positive.

Notice that the external Schwarzschild metric is not applied at  $r < 2M$  but just used as an indicator.

The sign of the term  $\left(1 - \frac{2GM}{c^2 r}\right)^{-1}$  independent from  $d\theta^2$  and  $d\phi^2$  is changed as well, so that the metric term  $dr^2$  changes its sign to negative. For the exterior Schwarzschild solution with  $M < 0$

$$ds^2 = -\left(1 + \frac{2GM}{c^2 r}\right) (cdt)^2 + \left(1 + \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

this change is equivalent to  $r < 0$  for the inside of the black hole.<sup>7</sup> Considering the spherical coordinates and the conversion to Cartesian coordinates

$$ds^2 = -c^2 dt^2 + dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$x = r \sin(\theta) \cos(\phi)$$

$$y = r \sin(\theta) \sin(\phi)$$

$$z = r \cos(\theta)$$

with

$$r^2 = x^2 + y^2 + z^2$$

$$r = \pm \sqrt{x^2 + y^2 + z^2}$$

a change in sign in  $dr^2$  could be understood as the negative solution for  $r^2$  (-r transformation takes the sphere centered on the origin into itself). By letting  $r$  be negative, we have two sets of  $(r, \theta, \phi)$  and a single set of  $(x, y, z)$ , so that we are defining a new independent space-time not covered by the Cartesian coordinates of  $r > 0$ . In this case, we can assume that the switching of signs affects the metric term  $dt^2$  and a possible parity-like transformation in spherical coordinates. Although the metric term that changes in sign is not the time coordinate, this change leads us to think that a reversal in sign could take place.

The  $dr$  coordinate has been a source of controversy. Schwarzschild solution defines  $r$  as a polar coordinate, strictly positive, and an auxiliary quantity for the external solution  $R$  as

$$R = \sqrt[3]{(r^3 + \alpha^3)}$$

so, if  $r=0$ , then

$$R = \sqrt[3]{(\alpha^3)} = \alpha$$

with  $\alpha$  been the Schwarzschild radius. Thus,  $R$  and  $r$  would not be the same although for  $R \ll r$  they are practically identical. The real Schwarzschild solution proposed in 1916 was

$$ds^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \frac{dr^2}{1 - \frac{\alpha}{R}} - R^2 d\vartheta^2 - R^2 \sin^2 \vartheta d\varphi^2$$

been regular for any value of  $r < 0$  and with its singularity at  $r = \alpha$  that can be avoided by a change in coordinates. But what happens at  $r < \alpha$ ? We are led to think that that interior doesn't exist.

Different metric solutions and coordinates describing a black hole will be studied in relation to these transformations of parity, time, and energy (mass).

- For the Gullstrand–Painlevé coordinates, the new dT coordinate

$$dT = dt - \frac{v}{1 - v^2} dr$$

under a time-reverse  $dt' = -dt$  and  $dr' = dr$  results in

$$dT = dt + \frac{v}{1 - v^2} dr$$

where dT' is still the same sign as dt', changing only the dr term (the radial coordinate is unchanged in going from the Schwarzschild chart to the Gullstrand-Painlevé chart). Thus, the new dT coordinate in the Painleve metric changes sign under time reversal. Time reverse of ingoing Gullstrand-Painlevé coordinates are outgoing Gullstrand-Painlevé coordinates. Eddington-Finkelstein coordinates ( $v, r, \theta, \phi$ ) are also time independent.

- Kerr-Schild coordinates

$$ds^2 = -\frac{\Delta}{\rho^2} (dt - a \sin^2 \theta d\phi)^2 + \frac{\sin^2 \theta}{\rho^2} \left( (r^2 + a^2) d\phi - a dt \right)^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2$$

$$\Delta = r^2 - 2Mr + a^2 + Q^2 \quad a = \frac{J}{M}$$

are invariant under a  $(-M, -dr, -dt)$  transformation (for a time reversal,  $-M$  and  $-r$  are required).

- Further research has been done in relation to these cases, for example, the inversion of the Kruskal-Szekeres coordinates ( $U, V$ ) been equivalent to reversing the sing of mass, parity and time, equivalent also to the full reversion of the Kruskal-Szekeres time ( $T$ ) and coordinate ( $R$ ), for the watching point of the negative masses, corresponding to the white hole region IV.<sup>8</sup>

- But the most significant research has been done by defining a new metric by a determine change of variable, connecting two Minkowski spaces with a pt symmetry (inversion of the arrow of time and a double inversion of space, accompanied by a mass inversion) and avoiding the central singularity.<sup>9</sup> Notice that inadequate coordinate choice can lead to wrong image of geometrical objects.

The Schwarzschild line element limited to  $(r, \phi)$  coordinates

$$d\Sigma^2 = \frac{dr^2}{1 - \frac{R_s}{r}} + r^2 d\varphi^2$$

for  $r < R_s$ ,  $(+, +)$  changes to  $(-, +)$ . With the change of variable

$$r = R_s(1 + \text{Log ch } \rho)$$

which gives

$$d\Sigma^2 = R_s^2 \left[ \frac{(1 + \text{Log ch } \rho)}{\text{Log ch } \rho} \text{th}^2 \rho d\rho^2 + (1 + \text{Log ch } \rho)^2 d\varphi^2 \right]$$

and results in all singularities disappearing at  $r=R_s$ , corresponding to  $\rho=0$ . The determinant along the sphere of the metric is no longer zero

$$\det g = R_s^4 \frac{(1 + \text{Log ch } \rho)^2}{\text{Log ch } \rho} \text{th}^2 \rho$$

and the metric is well defined for all values of  $\rho$ . From Lagrange equations we can calculate the geodesics in  $(\rho, \phi)$ . If embedded, the surface owns a throat circle whose perimeter is  $2\pi R_s$ . We can shape the surface as a twofold  $F^{(+)}$  and  $F^{(-)}$  cover of a  $M_2$  manifold with a 1D common circular border, and create induced mapping between adjacent points  $M^{(+)}$  and  $M^{(-)}$ . If we introduce the 3D metric

$$d\Sigma^2 = \frac{dr^2}{1 - \frac{R_s}{r}} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

Euclidean at infinity, when  $r < R_s$  the signature  $(+, +, +)$  is changed into  $(-, +, +)$  so we can obtain with our change of variable the metric

$$ds^2 = R_s^2 \left[ \frac{(1 + \text{Log ch } \rho)}{\text{Log ch } \rho} \text{th}^2 \rho d\rho^2 + (1 + \text{Log ch } \rho)^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right]$$

Euclidean at infinity and defined for all values of  $\rho$ . Adjacent points in 3D  $(\rho, \phi, \theta)$  are defined by

$$M: (\rho, \phi, \theta) \rightarrow M': (-\rho, \phi, \theta)$$

In conclusion, the change of variable

$$R = \alpha \left[ 1 + \text{Log} \frac{e^\rho + e^{-\rho}}{2} \right]$$

results in the metric solution

$$ds^2 = \frac{\text{Log ch } \rho}{1 + \text{Log ch } \rho} c^2 dt^2 - \alpha^2 \left[ \frac{(1 + \text{Log ch } \rho)}{\text{Log ch } \rho} \text{th}^2 \rho d\rho^2 + (1 + \text{Log ch } \rho)^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right]$$

equivalent to a pt symmetry. Notice that the metric is proposed within the Janus cosmological model and the author suggests that only particles of different sign in mass would interact repulsively.

• The parity transformation (which might as well help the inflation or bounce of space within the event horizon, considering an initial high kinetical energy star or full momentum space of the matter within), could also be explained due to the formulas for time dilation and length contraction, which are connected by the Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \beta^2}} = \frac{dt}{d\tau}$$

so that a clock with negative time would imply a negative Lorentz factor  $\gamma = -1$  (which would also imply negative energy, consistent with the model), and taking it into account in the length contraction formula  $L_1=L_0/\gamma$  would imply a negative scaling along one space dimension, equivalent to the mirroring of the parity transformation.

This change in time (not the classical time evolution we are used to) <sup>10</sup> will indicate that particles inside the black hole would behave as if they were proper antiparticles, and matter as proper antimatter, with negative mass and antigravitational interactions. Also, the common need of changes in coordinates for the event horizon could indicate the parity transformation, required for the inside of the black hole to be rotating in the same direction as the outside when reversing time.<sup>11</sup>

• Additionally, the parameter  $m$  for the Schwarzschild solution is nothing but an integration constant, which can be chosen positive or negative<sup>12</sup>. For the internal Schwarzschild solution

$$ds^2 = \left[ \frac{3}{2} \sqrt{1 - \frac{r_0^2}{A}} - \frac{1}{2} \sqrt{1 - \frac{r^2}{A}} \right] c^2 dt^2 - \frac{dr^2}{1 - \frac{r^2}{A}} - r^2 (d\theta^2 + \sin^2(\theta) d\phi^2)$$

with  $r \leq r_0$  and  $A = \frac{3c^2}{8\pi G \rho}$  ;  $\rho$  can be chosen negative, with the coupling condition:

$$m = -\frac{4\pi G r_0^3}{c^2} \rho$$

Supposing matter inside a black hole behaves as antigravitational antimatter does not imply any other problems, because both sides of this divided space time would never be in contact (no annihilation would occur), and changing all the particles charge would result in no special change for a neutrally charged black hole (the common assumption for real black holes). It would also be impossible for two masses of different sign to coexist in the same causal space-time region (no runaway motion would occur, and conservation of momentum would hold). Within an antigravitational space-time region, no infinite density volumes could form because of the repulsive forces growing asymptotically when trying to create very big negative energy densities of negative mass, so we would be looking at a solution for gravitational singularities, and also a solution without the disappearance of physical information (information paradox).<sup>13</sup>

The theoretical questions of why does the stellar mass of the original star that formed the black hole becomes the mass of the black hole, or where does the mass of the black hole come from, with the problem that arises when considering that mass the one compacted in the singularity, are also easily answered with this model (its mass would be the mass inside the space-time within the event horizon, understood as positive mass from outside it).

The existence of a white hole at the other side of the black hole is an assumption of the model, plausible according to the mathematics of general relativity regarding white holes (i.e. their existence is not prohibited in the universe, and their no-observational evidence would be explained if they would only exist at the other side of black holes, so they would be treated as real physical solutions), and consistent with the accepted idea of a white hole been a “time-reversed” black hole, with entropy related problems (second law of thermodynamics) of this entity solved when considering the reversed time in which exists.

Consequently, we should be looking for the metric solution and coordinates describing a black hole in which for a time transformation (or a time-parity transformation), the resulting metric is a mass sign reversal (or and a parity reversal, or and a charge reversal), turning the interior and exterior solutions into a white hole, invariant under a cmpt transformation.

These ideas fit the description of the approximated scheme (Figure 2.) for the non-changing light cones evolution when approaching the event horizon and with the pt transformation they experience when crossing such horizon.

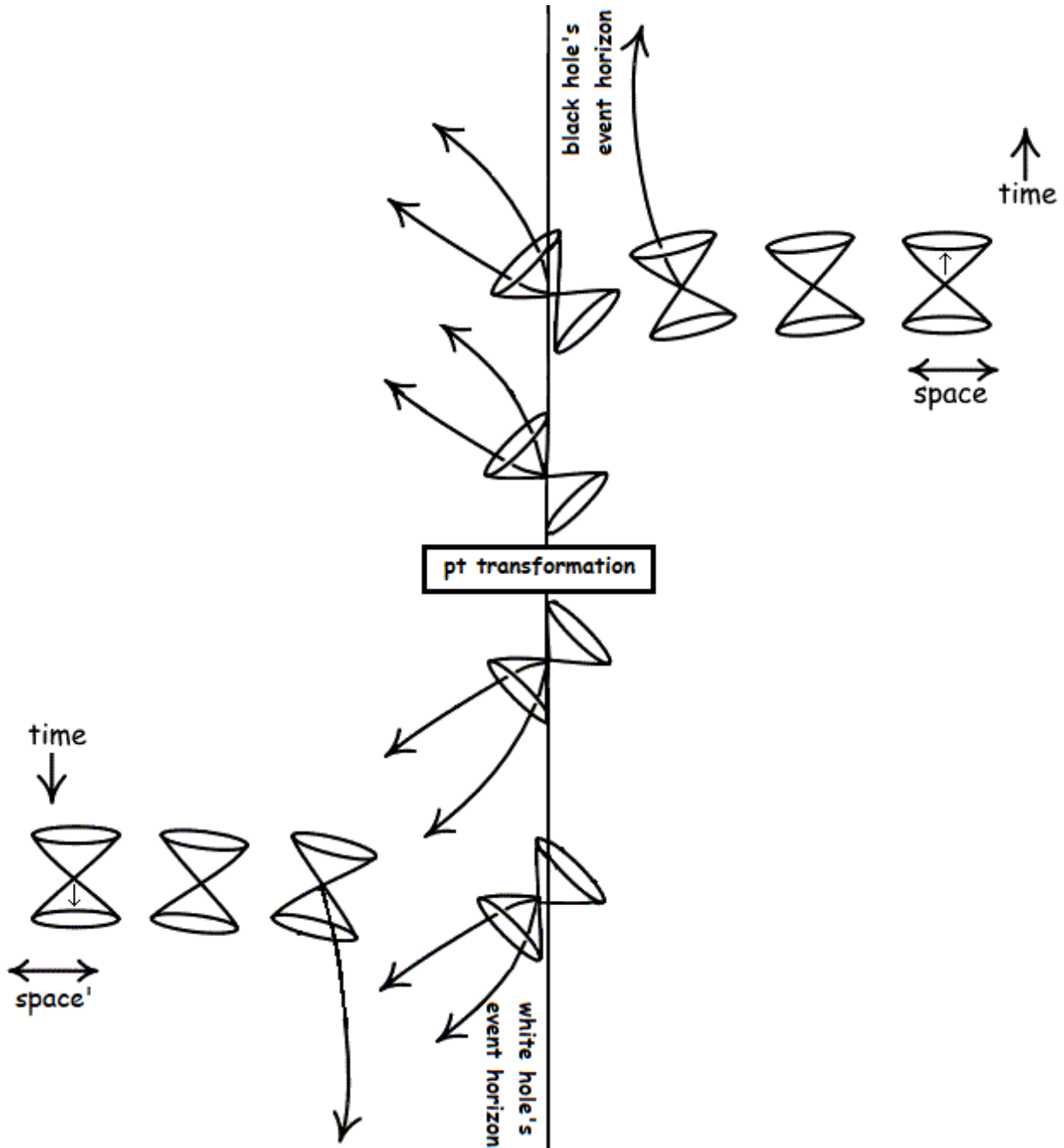


Fig. 2. Light cones evolution (notice that the present time line is divided into different present times for a better description when crossing the horizon, and light cones shape do not change), with their past cone in the white hole space-time coinciding with their actual past state in the black holes space-time

Even though common sense is the least indication for a model of this nature to be grounded, the author makes an invitation to think that if the “flow of time” is approaching zero when getting close to the event horizon for an external observer, and “the flow of time” stopping at the right edge, then in the other side we should not think of the “flow of time” as a continuation of been stopped, but to begin reversing in the opposite sense (time transformation) (Figure 3.). This thought experiment helps to understand the real phenomena of the hypothesis.

Thus, the event horizon would not be more than the place in space in which time begins to move backwards, and the asymmetry of that transformation within the laws of physics should be considered when describing the interior of a black hole.

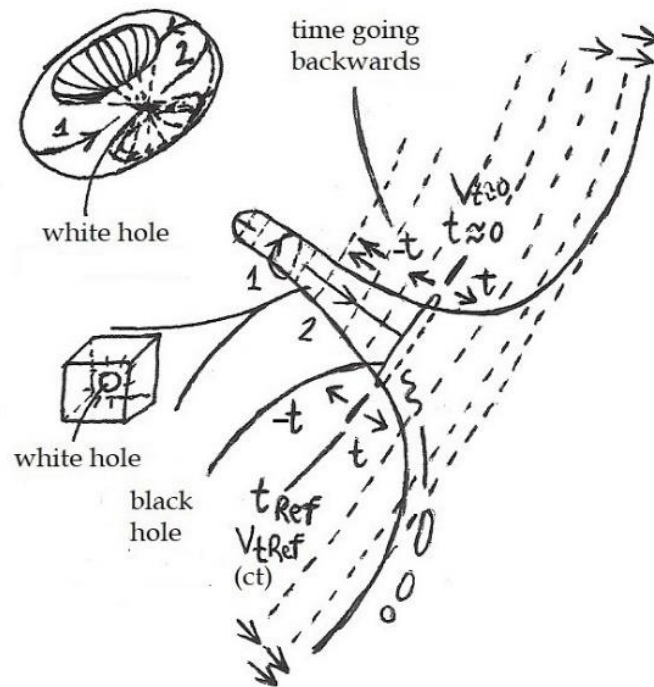


Fig. 3. Visualization of the thought experiment by the approximated model of extrinsic curvature representation with space dimensions reduced to 2D surfaces and a space-time shape is proposed following what a region like that, with a white hole and antigravitational matter inside, would be shaping the space-time according to what it's been explained together with the rest of physical phenomena, such as expansion or contraction, developing like in the well-known outer space-time.



### 3. Interior description

Once an event horizon is formed under an initial massive neutron star, the mass of the star inside it would be disconnected from the rest of the star, experiencing the parity-time transformation discussed before. A similar scenario has already been considered in other research<sup>8</sup> in which a negative mass universe is postulated. Particles will then describe the orbits corresponding to the Newtonian case of repulsion, so that all bodies will be repelled. These masses would create repulsive gravitational fields, and no stars or singularities could form only because of gravity. It is suggested that the stability that the star (made up of degenerate matter) had would be broken, and the gravitational forces would be added to the forces of degeneracy pressure, creating an inflation phase similar to a big bang. Notice that this phenomenon cannot be assigned a time for an external observer in the positive time universe.

Following the principle of equivalence of inertial and gravitational (passive and active) masses, having always the same sign, when an inversion in sign occurs as discussed before, only negative masses should exist in this isolated region of space-time (inertial mass will also be considered dependent of this inversion, and thus, negative) and so, all energies of the region must be negative for keeping the laws of physics as we know them (i.e. conservation of momentum), and for causality to be preserved. No information about the negative masses and their repulsive gravitational fields could influence the outside of the black hole.

For conservation of energy to be preserved, we know that total energy ( $E_T$ ) measured in both universes (considering absolute values of energy), should be the same. In the first universe, we can measure the total energy ( $E_{u1}$ ) as the total energy minus the energy within the black hole, plus the energy of the black hole. In the second universe, we can measure the total energy ( $E_{u2}$ ) as the total energy minus the energy within the white hole, plus the energy of the white hole. For both total energies to be equal in both universes, we obtain that the energy of the white hole should be the total energy minus the energy of the black hole, or in other words, the sum of the energy of the black hole plus the energy of the white hole should be the same as the total energy in any of both universes.

$$\begin{aligned}
 E_{u1} &= E_{u2} = E_T \\
 E_{u1} &= (E_T - E_{bh}) + E_{bh} \\
 E_{u2} &= (E_T - E_{wh}) + E_{wh} \\
 E_{wh} &= E_T - E_{bh} \\
 E_{u2} &= (E_T - E_T + E_{bh}) + E_T - E_{bh} = E_T
 \end{aligned}$$

If we apply the pt transformation to an expanding black hole, we also get an expanding white hole in the negative time universe. This could be seen as a region of apparently infinite negative energy, distributed within its event horizon following Gauss law for gravity, because we could not talk about a singularity when in our universe no energy singularities can form due to antigravity. Light in the initial white hole event horizon could reach every point of this new region of space-time, and light emitted after the white hole formation could only reach part of the space-time region.

Thus, the black hole could be seen just like a wormhole, consistent with the fact that in order to maintain an open wormhole, very big negative masses are required.<sup>14</sup> Note that reversing entropy would create a much different universe than the one we are used to.

## 4. Conclusions

This paper shows multiple mathematical hints which suggest that a parity-time transformation (or at least a time transformation) takes place when crossing the event horizon of a black hole, equivalent to a charge transformation, and that matter inside might behave as proper antimatter with negative energy corresponding to negative masses, which would interact antigravitationally, solving all questions, paradoxes and mathematical singularities that arise from the study of these astrophysical entities, and giving real physical sense to other predictions of fundamental theories, without the need of a theory of quantum gravity.

A new cmpt symmetry is proposed for this solution (no to be confused with magnetic pole conjugation), and demonstrating that cpt symmetry alone, is violated, could be the biggest proof of the theory, and a full quantum theory including gravity should be cmpt symmetric. It is suggested that traditional cpt symmetry will hold as long as m symmetry is bounded into c symmetry, and t transformation includes the reversal of the sign in mass.

## References:

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- <sup>1</sup> A. Einstein, *Zur allgemeinen relativitätstheorie*, Sitzungsber. preuss. Akad. Wiss. (1915)
- <sup>2</sup> J. S. Farnes, *A Unifying Theory of Dark Energy and Dark Matter: Negative Masses and Matter Creation within a Modified  $\Lambda$ CDM Framework*, (2018)
- <sup>3</sup> Luttinger, J. M, *On "Negative" mass in the theory of gravitation*, (1951)
- <sup>4</sup> Lancaster, Tom; Blundell, Stephen J.; Blundell, Stephen, *Quantum Field Theory for the Gifted Amateur*, (2014)
- <sup>5</sup> N. Debergh; J.-P. Petit; G. D'Agostini, *On evidence for negative energies and masses in the Dirac equation through a unitary time-reversal operator*, (2018)
- <sup>6</sup> K. Schwarzschild, *Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie*, Sitzungsber. Preuss. Akad. Wiss. Berlin, pp.189-196, (1916)
- <sup>7</sup> W. B. Bonnor, *Negative Mass in General Relativity*, (1989)
- <sup>8</sup> S. Bondarenko, *Negative mass scenario and Schwarzschild spacetime in general relativity*, (2019)
- <sup>9</sup> J-P. Petit; G. D'Agostini, *Cancellation of the central singularity of the Schwarzschild solution with natural mass inversion process*, (2015)
- <sup>10</sup> J. Belletete; M. B. Paranjape, *On negative mass*, International Journal of Modern Physics D 1341017, (2013)
- <sup>11</sup> S. W. Hawking, *The Large Scale Structure of Space-Time*, (1973)
- <sup>12</sup> H. Bondi, *Negative Mass in General Relativity*, (1957)
- <sup>13</sup> L. Smolin, S. Hossenfelder, *Conservative solutions to the black hole information problem*, (2009)
- <sup>14</sup> M. S. Morris; K. S. Thorne, *Wormholes, time machines, and the weak energy condition*, (1988)