About the congruent number

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May 2, 2019

Abstract

The three sides of the right triangle are rational numbers, and those with natural numbers are congruent numbers.

Theorem 1 Pythagorean theorem

\[(m^2 + n^2)^2 = (2mn)^2 + (m^2 - n^2)^2\]

Definition 2

\[\text{ace}(m_0^2 + n_0^2) = \text{ace} \cdot \frac{f}{e} = aef\]
\[\text{ace}(2m_0n_0) = \text{ace} \cdot \frac{b}{a} = bce\]
\[\text{ace}(m_0^2 - n_0^2) = \text{ace} \cdot \frac{d}{c} = ade\]

\[S' = \frac{bd}{2ac} \quad (bd = \text{even})\]

Definition 3 \(S\) is a congruent number. \((m, n = \mathbb{N})\)

\[(\text{ace})^2S' = \frac{\text{ace}^2bd}{2} = mn (m^2 - n^2) = k^2S \quad (k \geq 1 \quad , \quad m \neq n)\]

about \((k = 1)\)

\[m_1n_1 (m_1^2 - n_1^2) = A \quad (A \neq k'^2\mathbb{N})\]
\[k'm_1k'n_1 ((k'm_1)^2 - (k'n_1)^2) = k'^4A\]
\[mn (m^2 - n^2) = k^2S\]

\[A = S\]
**Proposition 4** The multiplication of the hypotenuse and one side of a right triangle is a congruent number.

**Proof 5**

\[
m = M^2 + N^2 \quad n = 2MN
\]

\[
k^2S = 2MN(M^2 + N^2)(M^2 - N^2)^2
\]

\[
S'' = 2MN(M^2 + N^2)
\]

\[
m = M' \quad n = N'
\]

\[
k^2S = M'^2N'^2(M'^4 - N'^4) \Rightarrow M'^4 - N'^4 = (M'^2 - N'^2)(M'^2 + N'^2)
\]

\[\square\]

**Corollary 6**

\[
S'' = 2 \cdot 2m'^2n'^2(2m'^4 + n'^4) \Rightarrow 2^2m'^4 + n'^4
\]

\[
S'' = 2m'^2n'^2(m'^4 + n'^4) \Rightarrow 2m'^4 + 2n'^4
\]

**Corollary 7** If 1 is not a congruence number, Fermat’s last theorem case 4 is equivalent.

\[
1 \cdot k^2 \neq z^4 - y^4
\]