About the congruent number

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Abstract

The three sides of the right triangle are rational numbers, and those with natural numbers are congruent numbers.

Theorem 1 Pythagorean theorem

\[(m^2 + n^2)^2 = (2mn)^2 + (m^2 - n^2)^2\]

Definition 2

\[\begin{align*}
ace(m_0^2 + n_0^2) &= ace \cdot \frac{f}{c} = acf \\
ace(2m_0n_0) &= ace \cdot \frac{b}{a} = bce \\
ace(m_0^2 - n_0^2) &= ace \cdot \frac{d}{c} = ade
\end{align*}\]

\[S' = \frac{bd}{2ac} \quad (bd = \text{even})\]

\[\begin{align*}
ac(2m_0n_0) &= ac \cdot \frac{b}{a} = bc \\
ac(m_0^2 - n_0^2) &= ac \cdot \frac{d}{c} = ad
\end{align*}\]

Definition 3 S is a congruent number. \((m, n \in \mathbb{N})\)

\[(ac)^2S' = \frac{bcad}{2} = mn \left( m^2 - n^2 \right) = k^2S \quad (k \geq 1, \ m \neq n)\]

about \((k = 1)\)

\[\begin{align*}
m_1n_1 (m_1^2 - n_1^2) &= A \quad (A \neq k'^2\mathbb{N}) \\
k'm_1k'n_1 \left( (k'm_1)^2 - (k'n_1)^2 \right) &= k'^4A \\
nm (m^2 - n^2) &= k^2S \\
A &= S
\end{align*}\]
Proposition 4  The multiplication of the hypotenuse and one side of a right triangle is a congruent number.

Proof 5

\[
m = M^2 + N^2 \quad n = 2MN
\]
\[
S' = 2MN(M^2 + N^2)(M^2 - N^2)^2
\]
\[
S'' = 2MN(M^2 + N^2)
\]
\[
m = M' \quad n = N'
\]
\[
S' = M'^{2}N'^{2}(M'^{4} - N'^{4}) \Rightarrow M'^{4} - N'^{4} = (M'^{2} - N'^{2})(M'^{2} + N'^{2})
\]

\[\Box\]

Corollary 6

\[
S'' = 2 \cdot 2m'^{2}n'^{2}(2m'^{4} + n'^{4}) \Rightarrow 2^{2}m'^{4} + n'^{4}
\]

Corollary 7  If 1 is not a congruence number, Fermat’s last theorem case 4 is equivalent.

\[1 \cdot k^2 \neq z^4 - y^4 \]