About the congruent number

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Abstract

The three sides of the right triangle are rational numbers, and those with natural numbers are congruent numbers.

Theorem 1 Pythagorean theorem

\[(m^2 + n^2)^2 = (2mn)^2 + (m^2 - n^2)^2\]

Definition 2 \( S \) is a congruent number.

\[S' = k^2 S \quad (k > 0)\]
\[S' = mn (m^2 - n^2) \quad m \neq n\]

Proposition 3

If \( mn \) is a rational number, it is a natural number.

Proof 4

\[m = \frac{b}{a} \quad n = \frac{d}{c}\]

\[S' = \frac{b}{a} \cdot \frac{d}{c} \left( \frac{b^2}{a^2} - \frac{d^2}{c^2} \right)\]

\[S' = \frac{bd}{ac} \left( \frac{b^2c^2 - a^2d^2}{a^2c^2} \right)\]

\[S' = \frac{bd \left((bc)^2 - (ad)^2\right)}{(ac)^3}\]
\[ S' = \frac{y}{z} \]
\[ 1 = \frac{y}{S'z} \Rightarrow 1 = \frac{S'z}{y} \]
\[ 2 = \frac{y}{S'z} + \frac{S'z}{y} \]
\[ 2 = \frac{y^2 + S'^2z^2}{S'zy} \]
\[ 2S' = \frac{y^2 + S'^2z^2}{zy} \]

Proposition 5
The multiplication of the hypotenuse and one side of a right triangle is a congruent number.

Proof 6

\[ S' = mn(m^2 - n^2) \quad m \neq n \]
\[ m = M^2 + N^2 \quad n = 2MN \]
\[ S' = 2MN(M^2 + N^2)(M^2 - N^2)^2 \quad M \neq N \]
\[ S'' = 2MN(M^2 + N^2) \]

Example 7

\[ 3^2 + 4^2 = 5^2 \]
\[ S = 3 \cdot 5 = 15 \]
\[ S'' = 4 \cdot 5 = 2^2 \cdot 5 \]