# Spherical Quantum Spacetime 

Tomasz Kobierzycki<br>E-mail: shizykfizyk@gmail.com

Understanding gravity at Planck scale is biggest goal of quantum gravity theory. In this paper $i$ will present idea of quantum spacetime that can be thought as gravity in quantum scale, this spacetime is fixed it means this idea is not background independent it lives on specific modified spherical spacetime. That spherical spactime does not break at inside of black hole and works in low energy very close to general relativity- it changes mostly after passing event horizon. Key idea is to use Planck energy units of energy and momentum as measure of curvature of spacetime. Model predicts that if energy goes to Planck energy time stops-all light cones are frozen and it happens from point of view of observer falling into black hole. From field equation there is calculated wave function vector that represents state of quantum system and thus leads to it's gravity effects.

## Contents

1 Field Equations 3
1.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3
1.2 Field Equation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4
1.3 Simplest solutions . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 5
1.4 Many system equation and measurement . . . . . . . . . . . . . . . . . . . . . . 6

| 2 | Consequences of field equation | 7 |
| :--- | :--- | :--- |

2.1 Cosmological Model . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 7
2.2 Black holes model . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 8
2.3 Massless particle spacetime . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 10
2.4 General field equation and symmetries. . . . . . . . . . . . . . . . . . . . . . . . 12
2.5 Summary and meaning of spherical spacetime . . . . . . . . . . . . . . . . . . . 14

## Section 1: Field Equations

### 1.1 Introduction

In this paper i will assume background dependent spacetime, that is extension of 3-sphere metric with rotation in four dimension- that four dimension is 4 -sphere dimension and rotation goes only from zero to $\pi$ or from zero to $-\pi$. So it means there is only rotation by $\pi$ angle in four dimension from this there is metric build. For each spacetime direction $x, y, z, c t$ there one of this metric, so there is 3 -sphere and rotation in fourth direction of 4 -sphere for each of that direction. There is a term that says how much rotation in fourth direction (4-sphere one) each coordinate has, that term is equal to:

$$
\begin{equation*}
d x^{\mu} d x^{v}\left(1-\left|T_{\mu v}\right|\right) \tag{1.1.1}
\end{equation*}
$$

Where $T_{\mu \nu}$ is energy tensor, that term is in square form like in metric, for $\pi$ rotation that term gives zero- so energy tensor is equal to one. For half rotation energy tensor is equal to one half and so on. I can write the rotation angle as: $\varphi= \pm \pi\left|T_{\mu \nu}\right|$. Rotation to full $\pi$ means that it goes from point A to point B where point B goes to point A and so on creating loop, that loop does not appear in lower energy scale. From it i can calculate relative change in length by using sec function of angle between two systems. I can write it as (where $L^{\prime}$ is distance change):

$$
\begin{equation*}
L^{\prime}=\frac{\sec (\varphi)}{\sec \left(\varphi^{\prime}\right)} \tag{1.1.2}
\end{equation*}
$$

Where $\sec (\varphi)$ comes from coordinate pointing in rotated angle to base angle of coordinate. If angle is half of $\pi$ this means direction is perpendicular to normal angle and from point of any other direction it does not point in direction of that coordinate. This angle is rotation in four dimension that comes from 4-sphere, each coordinate moves forward in direction of 4-sphere and it moves in direction perpendicular to rotated 3-sphere angle.

### 1.2 Field Equation

Energy tensor is extension of Einstein energy momentum relation (1) to sixteen parts where only ten of them are independent. Tensor itself has four indexes but i use contraction of one index to match metric tensor. I can write energy tensor components as, where indexes $i, j, l$ go from one to three and $k$ goes from zero to three :

$$
\begin{gather*}
T_{k 00}^{k}=\left(E_{0}^{0}\right)^{2}-\left(E_{1}^{1}\right)^{2}-\left(E_{2}^{2}\right)^{2}-\left(E_{3}^{3}\right)^{2}  \tag{1.2.1}\\
T_{0 i j}^{0}=-p_{i} p_{j}  \tag{1.2.2}\\
T_{k i j}^{k}=-p_{0 i}^{0} p_{0 j}^{0}+p_{l i}^{l} p_{l j}^{l}  \tag{1.2.3}\\
T_{k 0 j}^{k}=T_{k j 0}^{k}=-E_{0}^{0} p_{0 j}^{0}+E_{l}^{l} p_{l j}^{l} \tag{1.2.4}
\end{gather*}
$$

Energy and momentum are dived by Planck units, it means energy is equal to energy divided by Planck energy and same with momentum- it is so maximum value of energy and momentum can be one. Field equation with wave function vector $\psi_{\mu}$ energy tensor and metric tensor is equal to :

$$
\left\{\begin{array}{c}
\Delta \psi_{\mu}-T_{\gamma \mu \nu}^{\gamma} \psi^{v}=0  \tag{1.2.5}\\
T_{\gamma \mu \nu}^{\gamma} \psi^{v}-g_{\mu \nu} \psi^{v}=0
\end{array}\right.
$$

Where $\Delta$ (2) is Laplace operator, metric tensor then is equal to (3):

$$
\begin{equation*}
g_{\mu \nu}=s(\mu, v)\left[\left(1-\left|T_{0 \mu v}^{0}\right|\right)-\left[\left(d \phi_{3}^{2}+\sin ^{2}\left(\phi_{3}\right)\left(d \phi_{2}^{2}+\sin ^{2}\left(\phi_{2}\right) d \phi_{1}^{2}\right)\right]_{\mu v}\right]\right. \tag{1.2.6}
\end{equation*}
$$

Function $s(\mu, v)$ is a sign function (4), for zero part is has plus sign $s(0,0)=1$ for rest part it has minus $\operatorname{sign} s(\mu, v)=-1$. Radius is equal to one. Differential of angle is defined by $d \theta \rightarrow \frac{\theta_{1}-\theta_{0}}{2 \pi}$, that means that for angle $2 \pi$ its equal to one, where $\theta_{0}$ means begin angle and $\theta_{1}$ final angle. When i want to calculate relative change in length in one frame of reference to another it's equal to:

$$
\begin{equation*}
d x^{\mu^{\prime}} d x^{\nu^{\prime}}=d x^{\mu} d x^{v} \sec \left(\left|T_{0 \mu \nu}^{0}\right| \pi\right)\left[\sec \left(\left|T_{0 \mu \nu}^{0^{\prime}}\right| \pi\right)\right]^{-1} \tag{1.2.7}
\end{equation*}
$$

### 1.3 Simplest solutions

Simplest solutions to this equations are plane waves. First i write down energy tensor, where zero component is changing with radius, i can write component $T_{000}^{0}=\frac{M l_{P}}{m_{P} R}, M$ means mass, $R$ is radius and $l_{p}, m_{P}$ is Planck length and mass. Rest of the components follow same rule just with momentum, so i can write energy tensor solutions as:

$$
\begin{gather*}
T_{00}=\left(\frac{M l_{P}}{m_{P} R}\right)^{2}-\left(\frac{p_{01}^{0} l_{P}}{p_{P} R}\right)^{2}-\left(\frac{p_{02}^{0} l_{P}}{p_{P} R}\right)^{2}-\left(\frac{p_{03}^{0} l_{P}}{p_{P} R}\right)^{2}  \tag{1.3.1}\\
T_{11}=-\left(\frac{p_{10}^{1} l_{P}}{p_{P} R}\right)^{2}+\left(\frac{p_{11}^{1} l_{P}}{p_{P} R}\right)^{2}+\left(\frac{p_{12}^{1} l_{P}}{p_{P} R}\right)^{2}+\left(\frac{p_{13}^{1} l_{P}}{p_{P} R}\right)^{2}  \tag{1.3.2}\\
T_{22}=-\left(\frac{p_{20}^{2} l_{P}}{p_{P} R}\right)^{2}+\left(\frac{p_{21}^{2} l_{P}}{p_{P} R}\right)^{2}+\left(\frac{p_{22}^{2} l_{P}}{p_{P} R}\right)^{2}+\left(\frac{p_{23}^{2} l_{P}}{p_{P} R}\right)^{2}  \tag{1.3.3}\\
T_{33}=-\left(\frac{p_{30}^{3} l_{P}}{p_{P} R}\right)^{2}+\left(\frac{p_{31}^{3} l_{P}}{p_{P} R}\right)^{2}+\left(\frac{p_{32}^{3} l_{P}}{p_{P} R}\right)^{2}+\left(\frac{p_{33}^{3} l_{P}}{p_{P} R}\right)^{2} \tag{1.3.4}
\end{gather*}
$$

From it follows plane wave solutions so i get wave function (5) vector as (6):

$$
\psi_{\mu}=\left(\begin{array}{llll}
-\Psi_{0} e^{i x^{a} k_{a 0}-i \omega_{0} t} & \Psi_{1} e^{i x^{a} k_{a 1}-i \omega_{1} t} & \Psi_{2} e^{i x^{a} k_{a 2}-i \omega_{2} t} & \Psi_{3} e^{i x^{a} k_{a 3}-i \omega_{3} t} \tag{1.3.5}
\end{array}\right)
$$

There is relation between energy tensor and wave numbers of wave function that has to be fulfilled, they have to be equal:

$$
\begin{gather*}
T_{00}=\omega_{0}^{2}-k_{10}^{2}-k_{20}^{2}-k_{30}^{2}  \tag{1.3.6}\\
T_{11}=-\omega_{1}^{2}+k_{11}^{2}+k_{21}^{2}+k_{31}^{2}  \tag{1.3.7}\\
T_{22}=-\omega_{2}^{2}+k_{12}^{2}+k_{22}^{2}+k_{32}^{2}  \tag{1.3.8}\\
T_{33}=-\omega_{3}^{2}+k_{13}^{2}+k_{23}^{2}+k_{33}^{2} \tag{1.3.9}
\end{gather*}
$$

Those are simplest solutions to wave equation, from them i can calculate metric tensor. thus geometry of spacetime for given wave function vector.

### 1.4 Many system equation and measurement

If i have one system equation is in really simple form as expressed in first chapter. But it can be extended to many system using tensor product. First i write probability for one system, it's just sum of vector wave function components with it's complex conjugate (7):

$$
\begin{equation*}
P=\int_{x_{1}, t_{1}}^{x_{2}, t_{2}} \sum_{\mu=0}^{3} \psi_{\mu} \psi_{\mu}^{*} d^{4} x \tag{1.4.1}
\end{equation*}
$$

Probability tells what is change of particle being in position of spactime $x_{1}, t_{1}$ to $x_{2}, t_{2}$ where $x$ has three components and $d^{4} x$ means spacial and time components. Whole probability has to be equal to one so that integral for whole spacetime is one:

$$
\begin{equation*}
P=\int_{X} \sum_{\mu=0}^{3} \psi_{\mu} \psi_{\mu}^{*} d^{4} x=1 \tag{1.4.2}
\end{equation*}
$$

For many system i use tensor product and change Laplace operator to be sum for many coordinates, first operator is sum of operators for each particle so that i can write first part of equation as:

From it i can write second part of field equation that is equality between metric tensor and energy tensor by:

$$
\begin{equation*}
T_{\gamma_{1} \ldots \gamma_{n} \mu_{1} \ldots \mu_{n} v_{1} \ldots v_{n}}^{\gamma_{1}}\left(\psi^{v_{1}} \otimes \psi^{v_{2}} \ldots \otimes \psi^{v_{n}}\right)-g_{\mu_{1} v_{1} \ldots g_{\mu_{n}} v_{n}}\left(\psi^{v_{1}} \otimes \psi^{v_{2}} \ldots \otimes \psi^{v_{n}}\right)=0 \tag{1.4.4}
\end{equation*}
$$

Those are many systems field equations, now i can write probability for $n$ system state by just extending vector sum to many indexes:

$$
\begin{equation*}
P=\int_{x_{1}, t_{1}}^{x_{2}, t_{2}} \ldots \int_{x_{1 n}, t_{1 n}}^{x_{2_{n}}, t_{2_{n}}} \sum_{\mu=0}^{3} \psi_{\mu} \psi_{\mu \cdots}^{*} \sum_{\mu_{n}=0}^{3} \psi_{\mu_{n}} \psi_{\mu_{n}}^{*} d^{4} x_{1} \ldots d x_{n}^{4} \tag{1.4.5}
\end{equation*}
$$

## Section 2: Consequences of field equation

### 2.1 Cosmological Model

Field equation solutions can be extended to universe as whole, to do it i just need to calculate balance between expansion and contraction of spacetime. Key idea is that universe can expand or contract or stay static it depends on energy of matter in universe and radius of visible universe. Universe is understood as all casual events that we can get information from so it means our light cone can reach it. Universe is understood only local, from our frame of reference. First i write $\Lambda_{D}$ that is equal to:

$$
\begin{equation*}
\Lambda_{D}=1-\frac{2 M l_{P}}{R m_{P}} \tag{2.1.1}
\end{equation*}
$$

If $\Lambda_{D}$ is equal to zero i get static universe that is equal to black hole event horizon, for positive value i get expanding universe because expansion has more energy than matter, for negative value i get contracting universe (energy of matter is bigger than expansion). Now i can write radius dependent on time as:

$$
\begin{equation*}
R(t)=R_{0}+c t\left(1-\frac{2 M l_{P}}{c t m_{P}}\right) \tag{2.1.2}
\end{equation*}
$$

If mass stay constant i can write whole $\Lambda_{D}$ as function of time:

$$
\begin{equation*}
\Lambda_{D}(t)=1-\frac{2 M l_{P}}{\left[R_{0}+c t\left(1-\frac{2 M l_{p}}{c t m_{P}}\right)\right] m_{P}} \tag{2.1.3}
\end{equation*}
$$

If mass does not stay constant $i$ need to make it a function of time. From those i can calculate energy density of expansion by:

$$
\begin{equation*}
\rho=\frac{M c^{2}\left(1-\frac{M l_{P}}{R m_{P}}\right)}{\frac{4}{3} \pi R^{2} \frac{M l_{P}}{m_{P}}} \tag{2.1.4}
\end{equation*}
$$

From this model i can calculate universe as whole futures, this model focus only on energy tensor first component $T_{00}$ it means it does not have momentum in it and rest of field equation components. But for low momentum universe it works as good approximation, for direct cosmological model all part of equation has to be taken into account like in this chapter.

### 2.2 Black holes model

Black holes are most extreme system for gravity theory, black hole has event horizon that is in this idea equal to twice of distance to singularity it means, inside of a black hole has half of event horizon radius. Black hole geometry is very complex and depends on how observer inside it is moving, relative simple equation that are presented here get more complex when black holes are taken into account. First i write metric for event horizon of black hole that is equal to (where $d S_{\mu \nu}^{2}$ is equal to 3 -sphere part of metric):

$$
\begin{gather*}
d s^{2}=\frac{1}{2} c^{2} d t^{2}-c^{2} d t^{2} d S_{00}^{2}-d x^{2}\left(1-\left|T_{011}^{0}\right|\right)+d x^{2} d S_{11}^{2}-d y^{2}\left(1-\left|T_{022}^{0}\right|\right) \\
+d y^{2} d S_{22}^{2}-d z^{2}\left(1-\left|T_{033}^{0}\right|\right)+d z^{2} d S_{33}^{2} \tag{2.2.1}
\end{gather*}
$$

This is for observer that is falling into a black hole, for observer far away from black hole i need to use transform or relative length, and for addition here there has to be equality of space directions summing up to one half:

$$
\begin{equation*}
d x^{2}\left(1-\left|T_{011}^{0}\right|\right)+d y^{2}\left(1-\left|T_{022}^{0}\right|\right)+d z^{2}\left(1-\left|T_{033}^{0}\right|\right)=\frac{1}{2} c^{2} d t^{2} \tag{2.2.2}
\end{equation*}
$$

It means object at event horizon is moving with at least half of speed of light. When i use relative length change for observer far away i will get infinity from sec functions that angle is half $\pi$. This agrees with General Relativity prediction of event horizon, but here it's both for time and space. Another part of a black hole is it's center i can write metric as:

$$
\begin{equation*}
d s^{2}=-c^{2} d t^{2} d S_{00}^{2}+d x^{2} d S_{11}^{2}+d y^{2} d S_{22}^{2}+d z^{2} d S_{33}^{2}=0 \tag{2.2.3}
\end{equation*}
$$

It comes form a fact that special part needs to be equal to time part so it gives zero and only 3 -sphere part stay in equation:

$$
\begin{equation*}
d x^{2}\left(1-\left|T_{011}^{0}\right|\right)+d y^{2}\left(1-\left|T_{022}^{0}\right|\right)+d z^{2}\left(1-\left|T_{033}^{0}\right|\right)=c^{2} d t^{2}\left(1-\left|T_{000}^{0}\right|\right)=0 \tag{2.2.4}
\end{equation*}
$$

Terms that come from 3-sphere have give summed zero it comes from fact that energy tensor is equal to one and sign flips so i add to it, it means inside of a black hole move in loops. I can write energy tensor as components:

$$
\begin{align*}
& T_{033}^{0}+T_{022}^{0}+T_{011}^{0}=T_{000}^{0}=1 \\
& T_{300}^{3}+T_{200}^{2}+T_{100}^{1}=T_{000}^{0}=1 \\
& T_{311}^{3}+T_{211}^{2}+T_{111}^{1}=T_{011}^{0}=1  \tag{2.2.5}\\
& T_{322}^{3}+T_{222}^{2}+T_{122}^{1}=T_{022}^{0}=1 \\
& T_{333}^{3}+T_{233}^{2}+T_{133}^{1}=T_{033}^{0}=1
\end{align*}
$$

So summed energy tensor components give zero, it means i have equality on both side of field equation. To make 3 -sphere part give zero it can't move around third axis ( $d \phi_{3}$ has to be equal to zero) and sin functions have to give zero unless system is rotating only in one direction $d \phi_{1}$ then only one of sine functions has to be equal to zero $\left(\sin \left(\phi_{2}\right)=0\right)$. If it's rotating wit two axis ( $d \phi_{1}, d \phi_{2}$ ) then both of sine functions has to be equal to zero so they take value of zero or $\pi$, last case is when only rotation is in direction of $d \phi_{2}$ then $\sin \left(\phi_{3}\right)=0$. All those rotations are how spacetime is curved in inside of a black hole- it leads to loops in spacetime, i can write now field equation in full form:

$$
\begin{align*}
& {\left[T_{000}^{0}-\left(T_{300}^{3}+T_{200}^{2}+T_{100}^{1}\right)\right] \psi_{0}+d S_{00}^{2} \psi_{0}=0} \\
& {\left[-T_{011}^{0}+\left(T_{311}^{3}+T_{211}^{2}+T_{111}^{1}\right)\right] \psi_{0}-d S_{11}^{2} \psi_{1}=0} \\
& {\left[-T_{022}^{0}+\left(T_{322}^{3}+T_{222}^{2}+T_{122}^{1}\right)\right] \psi_{0}-d S_{22}^{2} \psi_{2}=0}  \tag{2.2.6}\\
& {\left[-T_{033}^{0}+\left(T_{333}^{3}+T_{233}^{2}+T_{331}^{1}\right)\right] \psi_{0}-d S_{33}^{2} \psi_{3}=0}
\end{align*}
$$

Those are solutions for black holes in this model of spherical spactime, they lead to loops that are 2-sphere parts. Finally i can write first part of equation as (zero means it moves with speed of light):

$$
\begin{equation*}
\Delta \psi_{\mu}=T_{\gamma \mu \nu}^{\gamma}=0 \tag{2.2.7}
\end{equation*}
$$

### 2.3 Massless particle spacetime

Particle like photons have always zero spacetime interval, here loops are help to create how information about event in specetime can be coded with zero distance. First i write metric for massless particle:

$$
\begin{gather*}
d s^{2}=c^{2} d t^{2}\left(1-\left|T_{000}^{0}\right|\right)-c^{2} d t^{2} d S_{00}^{2}-d x^{2}\left(1-\left|T_{011}^{0}\right|\right)+d x^{2} d S_{11}^{2}-d y^{2}\left(1-\left|T_{022}^{0}\right|\right) \\
+d y^{2} d S_{22}^{2}-d z^{2}\left(1-\left|T_{033}^{0}\right|\right)+d z^{2} d S_{33}^{2}=0 \tag{2.3.1}
\end{gather*}
$$

For spacetime interval to be zero there has to be two things that are equal to each other, first one is space and time part another is 3 -sphere part. I can write both equality as:

$$
\begin{gather*}
d x^{2}\left(1-\left|T_{011}^{0}\right|\right)+d y^{2}\left(1-\left|T_{022}^{0}\right|\right)+d z^{2}\left(1-\left|T_{033}^{0}\right|\right)=c^{2} d t^{2}\left(1-\left|T_{000}^{0}\right|\right)  \tag{2.3.2}\\
d x^{2} d S_{11}^{2}+d y^{2} d S_{22}^{2}+d z^{2} d S_{33}^{2}=c^{2} d t^{2} d S_{00}^{2} \tag{2.3.3}
\end{gather*}
$$

First equality says that distance between any two points in spacetime is equal to zero, second one states that distance in 3-sphere part is to equal to zero. Both of those equality have information about how spacetime looks from point of view of massless particle. Lets say i have $N$ spacetime distances, each of them represents a points that are on same point of and a 3 -sphere point, each movement in 3 -sphere with respect to time and space are equal. When i move around 3-sphere it moves in all three space and one time direction- each point lies on same point (each $d x, d y, d z, c d t$ lies on same point) of 3 -sphere. So picture of it is four 3-spheres pointing to a point that fallows some loop that is equal to distance of event $\left(c^{2} d t^{2}=d x^{2}+d y^{2}+d z^{2}\right)$ for each of $N$ events there is another four spheres that point each point is equal to point where all first 3-sphere point. To make that complex picture more simple- lets say i have a vector on 3-sphere for each coordinate, sum of four of those vectors for space and time (movement in space and time are equal) gives for any given instant a point - that point is where all other bigger or smaller
four 3-spheres that come from another $d x, d y, d z, c d t$ and from it there is some loop created out of this movement. I can write it as using $N$ differential of coordinates as (first all those meet in one point, where that point is equal to differential coordinates) first i write a position vector as:

$$
\begin{gather*}
d X^{\mu} d X_{\mu}=c^{2} d t^{2}\left(1-\left|T_{000}^{0}\right|\right)-c^{2} d t^{2} d S_{00}^{2}-d x^{2}\left(1-\left|T_{011}^{0}\right|\right)+d x^{2} d S_{11}^{2}-d y^{2}\left(1-\left|T_{022}^{0}\right|\right) \\
+d y^{2} d S_{22}^{2}-d z^{2}\left(1-\left|T_{033}^{0}\right|\right)+d z^{2} d S_{33}^{2}=0 \tag{2.3.4}
\end{gather*}
$$

Now i take $N$ of those vectors to where all points meet, from that i can get points where they do not meet, i will use another symbol $Y$ for vectors that do not give zero first i write them as:

$$
\begin{gather*}
d Y^{\mu} d Y_{\mu}=c^{2} d t^{2}\left(1-\left|T_{000}^{0}\right|\right)-c^{2} d t^{2} d S_{00}^{2}-d x^{2}\left(1-\left|T_{011}^{0}\right|\right)+d x^{2} d S_{11}^{2}-d y^{2}\left(1-\left|T_{022}^{0}\right|\right) \\
+d y^{2} d S_{22}^{2}-d z^{2}\left(1-\left|T_{033}^{0}\right|\right)+d z^{2} d S_{33}^{2} \neq 0 \tag{2.3.5}
\end{gather*}
$$

From this i need condition to make that those differential are not equal and so they are not in a point where first vector is pointing so where massless particle is, i can write it simply by:

$$
\begin{gather*}
c^{2} d t_{X}^{2} \neq c^{2} d t_{Y}^{2} \\
d x_{X}^{2} \neq d x_{Y}^{2}  \tag{2.3.6}\\
d y_{X}^{2} \neq d y_{Y}^{2} \\
d z_{X}^{2} \neq d z_{Y}^{2}
\end{gather*}
$$

Now i can get information about any given point where massless particle is by using calculation where it is not. I get a distance from a point where it is to point where it is not and from it i can calculate how spacetime information looks for massless particle. I just use $N$ steps to calculate it:

$$
\begin{equation*}
d Y_{N}^{\mu} d Y_{\mu_{N}} \neq d X_{N}^{\mu} d X_{\mu_{N}} \tag{2.3.7}
\end{equation*}
$$

### 2.4 General field equation and symmetries

Field equation explains gravity only- but there is a way to make it explain any force by changing second part of field equation. Gravity has symmetry between energy and curvature of spacetime- they are both equal. But for other forces there can be more energy system has than it goes to curvature of spacetime. First i re-write field equation and add term to second part of it this term:

$$
\left\{\begin{array}{c}
\Delta \psi_{\mu}-T_{\gamma \mu \nu}^{\gamma} \psi^{v}=0  \tag{2.4.1}\\
T_{\gamma \mu \nu}^{\gamma} \psi^{v}-g_{\mu \nu} \psi^{v}=K_{\gamma \mu v}^{\gamma} \psi^{v}
\end{array}\right.
$$

It means that for any other force than gravity there can be more energy than it goes to curvature of spacetime or there can be less energy than curvature of spacetime- it depends of the force. This equation leads to two symmetries that come from extending field equation. First symmetry says that system is massless so in first equation both Laplace operator times wave function vector gives zero and energy tensor gives zero. Second symmetry says that tensor $K$ is equal to zero or more. Both of those symmetries can be fulfilled or not fulfilled. I can write those symmetries as:

$$
\left\{\begin{align*}
& S_{1} \Leftrightarrow \Delta \psi_{\mu}=0 \rightarrow T_{\gamma \mu \nu}^{\gamma} \psi^{v}=0  \tag{2.4.2}\\
& S_{2} \Leftrightarrow T_{\gamma \mu \nu}^{\gamma} \psi^{v}-g_{\mu \nu} \psi^{v} \geq 0 \rightarrow K_{\gamma \mu \nu}^{\gamma} \psi^{v} \geq 0
\end{align*}\right.
$$

From those symmetries i can create a combination of all possible symmetries, that will gives four pair $\left(+S_{1},+S_{2}\right),\left(-S_{1},+S_{2}\right),\left(+S_{1},-S_{2}\right),\left(-S_{1},-S_{2}\right)$ each of those pair has a specific wave function solution, so i need to modify field equation to match all possible symmetry states system does obey. I can think of symmetry pairs as an operator that does change wave function, matrix is a $(1,1)$ tensor so i can write field equation as:

$$
\left\{\begin{array}{c}
\Delta \psi_{v} S_{\mu}^{v}-\partial_{\mu} T_{\gamma \mu \nu}^{\gamma} \psi^{v}=0  \tag{2.4.3}\\
\partial_{\mu} T_{\gamma \mu \nu}^{\gamma} \psi^{v}-\partial_{\mu} g_{\mu \nu} \psi^{v}=\partial_{\mu} K_{\gamma \mu \nu}^{\gamma} \psi^{v}
\end{array}\right.
$$

Where $\partial_{\mu}$ is equal to $\frac{\partial}{\partial x_{\mu}}$, symmetry matrix has four components of summation ( $v$ index ) each of them represents one symmetry pair. So first one is first symmetry and so on, it gives four
possible wave vector solutions that are summed. For equation to work i need to take derivative with respect to vector wave function with index $\mu$, it just states that any change in of wave function vector due symmetry state is equal it's change in energy. It leads to change in metric and $K$ tensor to rest part of equation. Idea behind symmetries is that sum of symmetries for any given system states constant- it does not change. I can write symmetries for any system as sum of vector components, that vector $N$ is equal to:

$$
N_{v}=\left(\begin{array}{llll}
S_{1}+S_{2} & -S_{1}+S_{2} & S_{1}-S_{2} & -S_{1}-S_{2} \tag{2.4.4}
\end{array}\right)
$$

Where $S$ can have value of one, minus one or zero. For each of this value there is assign matrix $S_{\mu}^{v}$ components. So it means that if $v=1$ o get first symmetry from $N_{v}$ matrix and it leads to first $S_{\mu}^{\nu}$ component. Because symmetries stay constant for any system i can write that for any given wave function vector derivative with respect to $v$ index gives zero:

$$
\begin{equation*}
\partial_{v} \psi_{\mu} N_{v}^{v}=0 \tag{2.4.5}
\end{equation*}
$$

Where i use sum of vector $N$ components that stays constant. First symmetry states that system does move with speed of light (it's massless), second symmetry states that energy of system is equal or greater than it's effect on curvature of spacetime. Those two symmetries are natural ways to take basics properties of field equation and they lead to general conservation of those symmetries. Idea behind how system changes is that when it breaks symmetry it leads to energy change, for example electron emitting photon breaks symmetry and creates a photon (virtual one) then it absorbs it and gets extra energy that did come from symmetry breaking, that's general approach to how interaction of other than gravity system works, there is a symmetry breaking that generates force particle carrier and it has to give zero in symmetry change so it has to be absorbed or create another particle that is absorbed to match symmetry change equal to zero. Symmetry states are natural propeties of field equation so extension to general field equation is just grouping field equation solutions.

### 2.5 Summary and meaning of spherical spacetime

This simple idea of background dependent spacetime that is extension of 4-sphere comes from idea of creating spacetime geometry from rotation of arrows. That rotation of arrows comes from wave function and idea is that each direction that arrow from wave function can rotate is equal to spacetime geometry, that's why this idea is based on background dependent spacetime. Because wave function has to be in vector form and each part of wave function is dependent on four scalar components $x, y, z, c t$ each part of metric tensor has to match all four components to make an equality. That's why each component of metric tensor has one component coming from vector part of wave function $d x, d y, d z, c d t$ and three components that come from scalar part of wave function. It leads to energy tensor that for each wave function component has four components that are summed - that come from scalar parts of wave function. In general case there are ten independent wave function components and ten independent metric tensor components. Field equation says that for a given wave function vector there is sum of four metric tensor components and sixteen energy tensor components that lead to one wave function vector, in presented case there are only one metric tensor component summation and thus it lead to four energy tensor component summation. Another idea is that energy is measured in Planck's units, so one is equal to Planck unit of energy, that leads to very weak gravity effects on single particle level but bigger when there is a lot of mass and energy coming from many particles or gravity get stronger at Planck energy level, where quantum effects come to play and spacetime freezes. This model does not have any point where it leads to singularity of spacetime, that stop of light cone movement is key idea of solving black hole and begin of universe mystery. Cosmological model i presented is simplest solution to expansion or contraction of spacetime depending on it's energy and radius, it leads to expansion of universe till energy of expansion will turn universe into black hole (9).

## References and Notes

1. Energy Momentum Relation
https://en.wikipedia.org/wiki/Energy-momentum_relation

## 2. Laplacian

http://mathworld.wolfram.com/Laplacian.html
3. 3-sphere metric tensor
http://mathworld.wolfram.com/Hypersphere.html
4. It's just written this way to make it more compact it comes from fact that used metric signature is $(+,-,-,-)$, so it gives only plus sign to time component rest is understood us multiplication of metric sign so it gives minus.

## 5. Wave equation

http://mathworld.wolfram.com/WaveEquation.html
6. Wave function sign changes in covariant components for time to minus sign. It does not change in contravariant components.
7. Wave function for many systems https://en.wikipedia.org/wiki/Wave_function
8. Freezing means that point on spacetime loops back to itself so it does not change. As showed in (1.1) chapter.
9. Photon travels from one Planck length to another goes back to start and it reaches singularity, it means universe will be some kind of cyclic model. Expansion will lead to black hole global state from which universe did come out.

