Spherical Quantum Spacetime

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Understanding gravity at Planck scale is biggest goal of quantum gravity theory. In this paper i will present idea of quantum spacetime that can be thought as gravity in quantum scale, this spacetime is fixed it means this idea is not background independent it lives on specific modified spherical spacetime. That spherical spactime does not break at inside of black hole and works in low energy very close to general relativity- it changes mostly after passing event horizon. Key idea is to use Planck energy units of energy and momentum as measure of curvature of spacetime. Model predicts that if energy goes to Planck energy time stops-all light cones are frozen and it happens from point of view of observer falling into black hole. From field equation there is calculated wave function vector that represents state of quantum system and thus leads to it's gravity effects.

Field Equation

Energy tensor is extension of Einstein energy momentum relation (*1*) to sixteen parts where only ten of them are independent. Tensor itself has four indexes but i use contraction of one index to match metric tensor. I can write energy tensor components as, where indexes i, j, l go from one to three and k goes from zero to three :

$$T^k_{k00} = \left(E^0_0\right)^2 - \left(E^1_1\right)^2 - \left(E^2_2\right)^2 - \left(E^3_3\right)^2$$

$$T^0_{0ij} = -p_i p_j$$

$$T^k_{kij} = -p^0_{0i} p^0_{0j} + p^l_{li} p^l_{lj}$$

$$T^k_{k0j} = T^k_{kj0} = -E^0_0 p^0_{0j} + E^l_l p^l_{lj}$$

Energy and momentum are dived by Planck units, it means energy is equal to energy divided by Planck energy and same with momentum- it is so maximum value of energy and momentum can be one. Field equation with wave function vector ψ_μ energy tensor and metric tensor is equal to :

$$\begin{cases} \Delta\psi_\mu - T^\gamma_{\gamma\mu\nu}\psi^\nu = 0 \\ T^\gamma_{\gamma\mu\nu}\psi^\nu - g_{\mu\nu}\psi^\nu = 0 \end{cases}$$

Where Δ (*2*) is Laplace operator, metric tensor then is equal to (*3*):

$$g_{\mu\nu} = s(\mu,\nu)\left[\left(1 - \left|T^0_{0\mu\nu}\right|\right) - \frac{r^2}{dx^\mu dx^\nu}\left[(d\phi_3^2 + \sin^2(\phi_3)\left(d\phi_2^2 + \sin^2(\phi_2) d\phi_1^2\right)\right]_{\mu\nu}\right]$$

Function $s(\mu,\nu)$ is a sign function (*4*), for zero part is has plus sign $s(0,0) = 1$ for rest part it has minus sign $s(\mu,\nu) = -1$. Radius is equal to: $r^2 = \frac{dx^\mu dx^\nu}{\left|T^0_{0\mu\nu}\right|}$, Differential of angle is defined by $d\theta \to \frac{\theta_1 - \theta_0}{2\pi}$, that means that for angle 2π its equal to one, where θ_0 means begin angle and θ_1 final angle. When i want to calculate relative change in length in one frame of reference to another it's equal to:

$$dx^{\mu'} dx^{\nu'} = dx^\mu dx^\nu \sec\left(\left|T^0_{0\mu\nu}\right|\pi\right)\left[\sec\left(\left|T^{0'}_{0\mu\nu}\right|\pi\right)\right]^{-1}$$

Simplest solutions

Simplest solutions to this equations are plane waves. First i write down energy tensor, where zero component is changing with radius, i can write component $T^0_{000} = \frac{Ml_P}{m_P R}$, M means mass, R is radius and l_p, m_P is Planck length and mass. Rest of the components follow same rule just with momentum, so i can write energy tensor solutions as:

$$T_{00} = \left(\frac{Ml_P}{m_P R}\right)^2 - \left(\frac{p^0_{01} l_P}{p_P R}\right)^2 - \left(\frac{p^0_{02} l_P}{p_P R}\right)^2 - \left(\frac{p^0_{03} l_P}{p_P R}\right)^2$$

$$T_{11} = -\left(\frac{p^1_{10} l_P}{p_P R}\right)^2 + \left(\frac{p^1_{11} l_P}{p_P R}\right)^2 + \left(\frac{p^1_{12} l_P}{p_P R}\right)^2 + \left(\frac{p^1_{13} l_P}{p_P R}\right)^2$$

$$T_{22} = -\left(\frac{p^2_{20} l_P}{p_P R}\right)^2 + \left(\frac{p^2_{21} l_P}{p_P R}\right)^2 + \left(\frac{p^2_{22} l_P}{p_P R}\right)^2 + \left(\frac{p^2_{23} l_P}{p_P R}\right)^2$$

$$T_{33} = -\left(\frac{p^3_{30} l_P}{p_P R}\right)^2 + \left(\frac{p^3_{31} l_P}{p_P R}\right)^2 + \left(\frac{p^3_{32} l_P}{p_P R}\right)^2 + \left(\frac{p^3_{33} l_P}{p_P R}\right)^2$$

From it follows plane wave solutions so i get wave function (*5*) vector as (*6*):

$$\psi_\mu = \begin{pmatrix} -\Psi_0 e^{ix^a k_{a0} - i\omega_0 t} & \Psi_1 e^{ix^a k_{a1} - i\omega_1 t} & \Psi_2 e^{ix^a k_{a2} - i\omega_2 t} & \Psi_3 e^{ix^a k_{a3} - i\omega_3 t} \end{pmatrix}$$

There is relation between energy tensor and wave numbers of wave function that has to be fulfilled, they have to be equal:

$$T_{00} = \omega_0^2 - k_{10}^2 - k_{20}^2 - k_{30}^2$$

$$T_{11} = -\omega_1^2 + k_{11}^2 + k_{21}^2 + k_{31}^2$$

$$T_{22} = -\omega_2^2 + k_{12}^2 + k_{22}^2 + k_{32}^2$$

$$T_{33} = -\omega_3^2 + k_{13}^2 + k_{23}^2 + k_{33}^2$$

Those are simplest solutions to wave equation, from them i can calculate metric tensor. thus geometry of spacetime for given wave function vector.

Many system equation and measurement

If i have one system equation is in really simple form as expressed in first chapter. But it can be extended to many system using tensor product. First i write probability for one system, it's just sum of vector wave function components with it's complex conjugate (*7*):

$$P = \int_{x_1,t_1}^{x_2,t_2} \sum_{\mu=0}^{3} \psi_\mu \psi_\mu^* d^4x$$

Probability tells what is change of particle being in position of spactime x_1, t_1 to x_2, t_2 where x has three components and d^4x means spacial and time components. Whole probability has to be equal to one so that integral for whole spacetime is one:

$$P = \int_X \sum_{\mu=0}^{3} \psi_\mu \psi_\mu^* d^4x = 1$$

For many system i use tensor product and change Laplace operator to be sum for many coordinates, first operator is sum of operators for each particle so that i can write first part of equation as:

$$\left(\Delta_1 + \Delta_2 ... + \Delta_n\right)\left(\psi_{\mu_1} \otimes \psi_{\mu_2} ... \otimes \psi_{\mu_n}\right) - T^{\gamma_1...\gamma_n}_{\gamma_1...\gamma_n \mu_1...\mu_n \nu_1...\nu_n}\left(\psi^{\nu_1} \otimes \psi^{\nu_2} ... \otimes \psi^{\nu_n}\right) = 0$$

From it i can write second part of field equation that is equality between metric tensor and energy tensor by:

$$T^{\gamma_1...\gamma_n}_{\gamma_1...\gamma_n \mu_1...\mu_n \nu_1...\nu_n}\left(\psi^{\nu_1} \otimes \psi^{\nu_2} ... \otimes \psi^{\nu_n}\right) - g_{\mu_1 \nu_1} ... g_{\mu_n \nu_n}\left(\psi^{\nu_1} \otimes \psi^{\nu_2} ... \otimes \psi^{\nu_n}\right) = 0$$

Those are many systems field equations, now i can write probability for n system state by just extending vector sum to many indexes:

$$P = \int_{x_1,t_1}^{x_2,t_2} ... \int_{x_{1n},t_{1n}}^{x_{2n},t_{2n}} \sum_{\mu=0}^{3} \psi_\mu \psi_\mu^* ... \sum_{\mu_n=0}^{3} \psi_{\mu_n} \psi_{\mu_n}^* d^4x_1 ... dx_n^4$$

Cosmological Model

Field equation solutions can be extended to universe as whole, to do it i just need to calculate balance between expansion and contraction of spacetime. Key idea is that universe can expand or contract or stay static it depends on energy of matter in universe and radius of visible universe. Universe is understood as all casual events that we can get information from so it means our light cone can reach it. Universe is understood only local, from our frame of reference. First i write Λ_D that is equal to:

$$\Lambda_D = 1 - \frac{2Ml_P}{Rm_P}$$

If Λ_D is equal to zero i get static universe that is equal to black hole event horizon, for positive value i get expanding universe because expansion has more energy than matter, for negative value i get contracting universe (energy of matter is bigger than expansion). Now i can write radius dependent on time as:

$$R(t) = R_0 + ct\left(1 - \frac{2Ml_P}{ctm_P}\right)$$

If mass stay constant i can write whole Λ_D as function of time:

$$\Lambda_D(t) = 1 - \frac{2Ml_P}{\left[R_0 + ct\left(1 - \frac{2Ml_P}{ctm_P}\right)\right]m_P}$$

If mass does not stay constant i need to make it a function of time. From those i can calculate energy density of expansion by:

$$\rho = \frac{Mc^2\left(1 - \frac{Ml_P}{Rm_P}\right)}{\frac{4}{3}\pi R^2 \frac{Ml_P}{m_P}}$$

From this model i can calculate universe as whole futures, this model focus only on energy tensor first component T_{00} it means it does not have momentum in it and rest of field equation components. But for low momentum universe it works as good approximation, for direct cosmological model all part of equation has to be taken into account like in this chapter.

Black holes model

This model does not break at Planck energy level and in approximation it's very close to Einstein solutions, main change is when those equations are applied to black holes. First from a view point of falling observer into a black hole- there is nothing strange at event horizon (like in Einstein solutions) but when observer gets to half distance of event horizon time does stop. It comes from a fact that energy of time and one spacial components are equal to maximum so time itself and space itself are not moving they are frozen. But for observer far outside a black hole space and time freezes at event horizon, it is because relative change in coordinate gives infinity- that means from geometry point of view that space is pointing in direction that is perpendicular to normal space direction same with time, so it takes infinite amount of time to reach observer outside of that event horizon. After crossing event horizon time starts to run backwards from point of view of falling observer, till it gets to minus time of event- light cones are falling inside a black hole to a limit of Planck length size. Using math i can write it first as from point of view of falling observer (momentum goes in x direction for simplicity) $\left(1-\left|T^0_{000}\right|\right)=\left(1-\left|T^0_{011}\right|\right)=0$:

$$ds^2=-c^2dt^2\left[(d\phi_3^2+\sin^2(\phi_3)\left(d\phi_2^2+\sin^2(\phi_2)d\phi_1^2\right)\right]_{00}+dx^2\left[(d\phi_3^2+\sin^2(\phi_3)\left(d\phi_2^2+\sin^2(\phi_2)d\phi_1^2\right)\right]_{11}$$

And for observer outside a black hole where i use transform or relative length (first at event horizon then at half ot it's length), it's equal to:

$$c^2dt^{2'}=c^2dt^2\sec\left(\left|\frac{1}{2}\right|\pi\right)\left[\sec\left(\left|T^{0'}_{0\mu\nu}\right|\pi\right)\right]^{-1}=\infty$$

$$c^2dt^{2'}=c^2dt^2\sec(\pi)\left[\sec\left(\left|T^{0'}_{0\mu\nu}\right|\pi\right)\right]^{-1}=-c^2dt^2$$

$$ds^2=c^2dt^2\left[(d\phi_3^2+\sin^2(\phi_3)\left(d\phi_2^2+\sin^2(\phi_2)d\phi_1^2\right)\right]_{00}-dx^2\left[(d\phi_3^2+\sin^2(\phi_3)\left(d\phi_2^2+\sin^2(\phi_2)d\phi_1^2\right)\right]_{11}$$

General field equation and symmetries

Field equation explains gravity only- but there is a way to make it explain any force by changing second part of field equation. Gravity has symmetry between energy and curvature of spacetime- they are both equal. But for other forces there can be more energy system has than it goes to curvature of spacetime. First i re-write field equation and add term to second part of it this term:

$$\begin{cases} \Delta\psi_\mu - T^\gamma_{\gamma\mu\nu}\psi^\nu = 0 \\ T^\gamma_{\gamma\mu\nu}\psi^\nu - g_{\mu\nu}\psi^\nu = K^\gamma_{\gamma\mu\nu}\psi^\nu \end{cases}$$

It means that for any other force than gravity there can be more energy than it goes to curvature of spacetime or there can be less energy than curvature of spacetime- it depends of the force. This equation leads to two symmetries that come from extending field equation. First symmetry says that system is massless so in first equation both Laplace operator times wave function vector gives zero and energy tensor gives zero. Second symmetry says that tensor K is equal to zero or more. Both of those symmetries can be fulfilled or not fulfilled . I can write those symmetries as:

$$\begin{cases} S_1 \Leftrightarrow \Delta\psi_\mu = 0 \rightarrow T^\gamma_{\gamma\mu\nu}\psi^\nu = 0 \\ S_2 \Leftrightarrow T^\gamma_{\gamma\mu\nu}\psi^\nu - g_{\mu\nu}\psi^\nu \geq 0 \rightarrow K^\gamma_{\gamma\mu\nu}\psi^\nu \geq 0 \end{cases}$$

From those symmetries i can create a combination of all possible symmetries, that will gives four pair $(+S_1, +S_2), (-S_1, +S_2), (+S_1, -S_2), (-S_1, -S_2)$ each of those pair has a specific wave function solution, so i need to modify field equation to match all possible symmetry states system does obey. I can think of symmetry pairs as an operator that does change wave function, matrix is a (1,1) tensor so i can write field equation as:

$$\begin{cases} \Delta\psi_\nu S^\nu_\mu - \partial_\mu T^\gamma_{\gamma\mu\nu}\psi^\nu = 0 \\ \partial_\mu T^\gamma_{\gamma\mu\nu}\psi^\nu - \partial_\mu g_{\mu\nu}\psi^\nu = \partial_\mu K^\gamma_{\gamma\mu\nu}\psi^\nu \end{cases}$$

Where ∂_μ is equal to $\frac{\partial}{\partial x_\mu}$, symmetry matrix has four components of summation (ν index) each of them represents one symmetry pair. So first one is first symmetry and so on, it gives four possible wave vector solutions that are summed. For equation to work i need to take derivative with respect to vector wave function with index μ, it just states that any change in of wave function vector due symmetry state is equal it's change in energy. It leads to change in metric and K tensor to rest part of equation. Idea behind symmetries is that sum of symmetries for any given system states constant- it does not change. I can write symmetries for any system as sum of vector components, that vector N is equal to:

$$N_\nu = \begin{pmatrix} S_1 + S_2 & -S_1 + S_2 & S_1 - S_2 & -S_1 - S_2 \end{pmatrix}$$

Where S can have value of one, minus one or zero. For each of this value there is assign matrix S_μ^ν components. So it means that if $\nu = 1$ o get first symmetry from N_ν matrix and it leads to first S_μ^ν component. Because symmetries stay constant for any system i can write that for any given wave function vector derivative with respect to ν index gives zero:

$$\partial_\nu \psi_\mu N_\nu^\nu = 0$$

Where i use sum of vector N components that stays constant. First symmetry states that system does move with speed of light (it's massless), second symmetry states that energy of system is equal or greater than it's effect on curvature of spacetime. Those two symmetries are natural ways to take basics properties of field equation and they lead to general conservation of those symmetries. Idea behind how system changes is that when it breaks symmetry it leads to energy change , for example electron emitting photon breaks symmetry and creates a photon (virtual one) then it absorbs it and gets extra energy that did come from symmetry breaking, that's general approach to how interaction of other than gravity system works, there is a symmetry breaking that generates force particle carrier and it has to give zero in symmetry change so it has to be absorbed or create another particle that is absorbed to match symmetry change equal to zero. Symmetry states are natural propeties of field equation so extension to general field equation is just grouping field equation solutions.

Summary and meaning of spherical spacetime

This simple idea of background dependent spacetime that is extension of 4-sphere comes from idea of creating spacetime geometry from rotation of arrows. That rotation of arrows comes from wave function and idea is that each direction that arrow from wave function can rotate is equal to spacetime geometry, that's why this idea is based on background dependent spacetime. Because wave function has to be in vector form and each part of wave function is dependent on four scalar components x, y, z, ct each part of metric tensor has to match all four components to make an equality. That's why each component of metric tensor has one component coming from vector part of wave function dx, dy, dz, cdt and three components that come from scalar part of wave function. It leads to energy tensor that for each wave function component has four components that are summed - that come from scalar parts of wave function. In general case there are ten independent wave function components and ten independent metric tensor components. Field equation says that for a given wave function vector there is sum of four metric tensor components and sixteen energy tensor components that lead to one wave function vector, in presented case there are only one metric tensor component summation and thus it lead to four energy tensor component summation. Another idea is that energy is measured in Planck's units, so one is equal to Planck unit of energy, that leads to very weak gravity effects on single particle level but bigger when there is a lot of mass and energy coming from many particles or gravity get stronger at Planck energy level, where quantum effects come to play and spacetime freezes. This model does not have any point where it leads to singularity of spacetime, that stop of light cone movement is key idea of solving black hole and begin of universe mystery. Cosmological model i presented is simplest solution to expansion or contraction of spacetime depending on it's energy and radius, it leads to expansion of universe till energy of expansion will turn universe into black hole (*8*).

References and Notes

1. *Energy Momentum Relation*

   https://en.wikipedia.org/wiki/Energy-momentum_relation

2. *Laplacian*

   http://mathworld.wolfram.com/Laplacian.html

3. *3-sphere metric tensor*

   http://mathworld.wolfram.com/Hypersphere.html

4. *It's just written this way to make it more compact it comes from fact that used metric signature is* $(+,-,-,-)$*, so it gives only plus sign to time component rest is understood us multiplication of metric sign so it gives minus.*

5. *Wave equation*

   http://mathworld.wolfram.com/WaveEquation.html

6. *Wave function sign changes in covariant components for time to minus sign. It does not change in contravariant components.*

7. *Wave function for many systems*

   https://en.wikipedia.org/wiki/Wave_function

8. *Photon travels from one Planck length to another goes back to start if expansion gets even bigger it goes one Planck length backwards and it reaches singularity, it means universe will be some kind of cyclic model. Expansion will lead to black hole global state from which universe did come out.*