Oscillation Field of Space-Time

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In this paper i will present simplest model of spacetime that is derived from wave equation in any curved spacetime. General idea is that if I quantize wave by Planck scale units i will get metric that is dependent on wave field and leads to geometry of given spacetime. That geometry is understood as gravity and wave field is understood as quantum field that changes metric of spactime by it's energy. Solutions to field equation will always give some kind of cyclic functions that lead to loops in spacetime. Those loops are solutions to singularity problem and how mass-less particles see geometry of spacetime.

Field equation

Main idea of that idea is to connect geometry of wave that comes from wave equation with gravity- thus geometry of spacetime. For scalar wave- wave equation takes form $\Delta\psi = 0$, where Δ is Laplace operator, I add to it term with rest energy T i get equation: $\Delta\psi + T = 0$. Now I add metric tensor first component that has to be equal to left side of equation so i use Laplace operator on it $\Delta\psi_{00} + T_{00} = \Delta g_{00}$. Laplace operator goes from indexes $\varphi\phi$, rest of equation goes by indexes ab. I will write Laplace operator with subscript that means it indexes, so writing that equation again i get $\Delta_{\varphi\phi}\psi_{00} + T_{00} = \Delta_{\varphi\phi}g_{00}$. I can calculate rest of components of metric tensor same way so more generally for indexes ab i get: $\Delta_{\varphi\phi}\psi_{ab} + T_{ab} = \Delta_{\varphi\phi}g_{ab}$. But this equations misses indexes $\varphi\phi$ that are used in Laplace operator. Before i write whole equation let's stop and think about what it means- for now equations says that for metric tensor functions they are equal to wave with some defined energy. It means solutions to left side of equation are metric tensor components. Now only thing left is to make use of indexes to so both side have equality in physical meaning. Wave equation has for spacetime four dimensions it means it has four coordinates so energy tensor has to have four summation of energy components. Those summation components have to be other from Laplace operator components so i have to add three more summation indexes finally i get:

$$\boxed{\Delta_{\varphi\phi}\Psi^{\varphi\mu}_{a\varphi}\Psi^{\phi}_{\mu\phi b}+T^{\varphi\mu}_{a\varphi}T^{\phi}_{\mu\phi b}=\Delta_{\varphi\phi}g_{ab}}$$

That is field equation that says there is a direct connection between a wave equation and metric tensor. Einstein summation convention is used so it means i sum over indexes μ,φ,ϕ. Energy tensor has to have μ components with same sign as Laplace operator, for example in flat Minkowski spacetime with metric signature $(+---)$ zero component has plus sign and rest have minus one: $T^{\varphi 0}_{a\varphi}T^{\phi}_{0\phi b}, -T^{\varphi 1}_{a\varphi}T^{\phi}_{1\phi b}, -T^{\varphi 2}_{a\varphi}T^{\phi}_{2\phi b}, -T^{\varphi 3}_{a\varphi}T^{\phi}_{3\phi b}$ if i chose any other metric signature i will have to use it to in energy tensor, there are only two to chose for spacetime , either $(+---)$ i showed or $(-+++)$ where energy tensor components will be: $-T^{\varphi 0}_{a\varphi}T^{\phi}_{0\phi b}, T^{\varphi 1}_{a\varphi}T^{\phi}_{1\phi b}, T^{\varphi 2}_{a\varphi}T^{\phi}_{2\phi b}, T^{\varphi 3}_{a\varphi}T^{\phi}_{3\phi b}$. It means that in simplest case energy tensor is equal to either $T_{ab}=T^{\varphi 0}_{a\varphi}T^{\phi}_{0\phi b}-T^{\varphi 1}_{a\varphi}T^{\phi}_{1\phi b}-T^{\varphi 2}_{a\varphi}T^{\phi}_{2\phi b}-T^{\varphi 3}_{a\varphi}T^{\phi}_{3\phi b}$ where i skip rest of components (that come from Laplace operator components) or in another case $T_{ab}=-T^{\varphi 0}_{a\varphi}T^{\phi}_{0\phi b}+T^{\varphi 1}_{a\varphi}T^{\phi}_{1\phi b}+T^{\varphi 2}_{a\varphi}T^{\phi}_{2\phi b}+T^{\varphi 3}_{a\varphi}T^{\phi}_{3\phi b}$. By solving equation for any given wave field and energy tensor i will get metric tensor so from it i can get spacetime interval by known formula from general relativity ($ds^2=g_{ab}dx^a dx^b$), but i need to restrict energy to Planck units and it means frequency and wave number of wave function.

Planck Units

Planck units are key to energy scale that quantum gravity targets, energy is written as dived by Planck Energy so $E = \frac{E}{E_P}$ same with momentum: $p = \frac{p}{p_P}$ from those i can get frequency as $\omega = \omega t_P$ that comes from fact that i can have more than one vibration of field in Planck time, so wave number does obey same rule but with Planck length $k = kl_P$. Now i can use those units to construct energy tensor and wave field relations. First one is relation between wave number k and frequency ω and energy tensor so i get: $T^{\varphi 0}_{a\varphi} T^{\phi}_{0\phi b} - T^{\varphi 1}_{a\varphi} T^{\phi}_{1\phi b} - T^{\varphi 2}_{a\varphi} T^{\phi}_{2\phi b} - T^{\varphi 3}_{a\varphi} T^{\phi}_{3\phi b} = \omega^2 - c^2|k_1|^2 - c^2|k_2|^2 - c^2|k_3|^2$ for any given value of a and b this formula holds. But there will be multiple components each having those relation full filled. In simplest case where $a = b$ four of them for spacetime, to calculate rest energy i need to contract energy tensor with metric tensor $T^{ab}g_{ab} = (m_0c^2)^2$ *(1)* in simplest form it's sum of all four components times metric tensor so i get:

$$\begin{aligned}
T^{ab}g_{ab} = &\left(T^{0\varphi 0}_{\varphi} T^{\phi 0}_{\phi 0} - T^{0\varphi 1}_{\varphi} T^{\phi 0}_{1\phi} - T^{0\varphi 2}_{\varphi} T^{\phi 0}_{2\phi} - T^{0\varphi 3}_{\varphi} T^{\phi 0}_{3\phi}\right) g_{00} \\
&+ \left(T^{1\varphi}_{\varphi} T^{\phi 1}_{0\phi} - T^{1\varphi 1}_{\varphi} T^{\phi 1}_{\phi 1} - T^{1\varphi 2}_{\varphi} T^{1\phi}_{2\phi} - T^{1\varphi 3}_{\varphi} T^{1\phi}_{3\phi}\right) g_{11} \\
&+ \left(T^{2\varphi 0}_{\varphi} T^{\phi 2}_{0\phi} - T^{2\varphi 1}_{\varphi} T^{\phi 2}_{1\phi} - T^{2\varphi 2}_{\varphi} T^{\phi 2}_{2\phi} - T^{2\varphi 3}_{\varphi} T^{\phi 2}_{3\phi}\right) g_{22} \\
&+ \left(T^{3\varphi 0}_{\varphi} T^{\phi 3}_{0\phi} - T^{3\varphi 1}_{\varphi} T^{\phi 3}_{1\phi} + T^{3\varphi 2}_{\varphi} T^{\phi 3}_{2\phi} - T^{3\varphi 3}_{\varphi} T^{\phi 3}_{3\phi}\right) g_{33} = (m_0c^2)^2
\end{aligned}$$

Where tensor $T^{ab} = T^{a\mu\varphi}_{\varphi} T^{\phi b}_{\mu\phi}$, now i can write energy tensor components as with only using μ indexes: $T^{a\varphi 0}_{\varphi} T^{\phi b}_{0\phi} g_{ab} = E^2, \sum_{j=1}^{3} T^{a\varphi j}_{\varphi} T^{\phi b}_{j\phi} g_{ab} = p^2c^2$ its relation that makes mass invariant and it's need for this field to work. Now i can write whole thing in one equation:

$$T^{a\mu\varphi}_{\varphi} T^{\phi b}_{\mu\phi} g_{ab} = E^2 - p^2c^2 = (m_0c^2)^2$$

Gravity as wave field

From energy relations i can finally construct model of gravity. First i need to restrict energy limit to one in Planck units , same with momentum and frequency and wave number. It comes from fact that energy limit is connected to speed of light limit. If speed of light is limit then Planck energy is limit, if i would chose speed of light squared limit would be Planck energy squared. But locally it's always speed of light limit so i wont focus on extending this idea to faster than speed of light movement. Simplest form of solutions to wave field will be spherical one, i can write spherical scalar wave function as $\psi(r,\theta,\phi,t)$, so Laplace equation is $\Delta\psi(r,\theta,\phi,t) = 0$, where i can write it in full form as: $\nabla^2\psi(r,\theta,\phi,t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi(r,\theta,\phi,t)$ where nabla operator is just spacial components of wave function. Now simplest form of solution to this scalar wave equation is $\psi = \left(A' j_l(kr) + B' y_l(kr)\right) Y_l^m(\theta,\phi) T_0 e^{\pm i\omega t}$, where j_l, y_l are spherical Bessel functions and Y_l^m is spherical harmonic functions. From those solutions i can construct full field equation. I will only solve it for wave field where $a = b$ and i will use coordinates r,θ,ϕ,ct first i need real part of scalar wave equation (*3*) (*2*) $\Re(\psi) = \left(A' j_l(kr) + B' y_l(kr)\right)\Re(Y_l^m(\theta,\phi)) T_0 \cos(\omega t)$, next is to compute wave field by using of that it will be just that function solution squared with four indexes: $\Psi_{aa} = \Psi_{bb} = \Psi_a^2 = \Psi_b^2$ i can write it as: $\left(A' j_l(k_a r) + B' y_l(k_a r)\right)\Re(Y_{l_a}^{m_a}(\theta,\phi)) T_a \cos(\omega_a t) = \left(A' j_l(k_b r) + B' y_l(k_b r)\right)\Re(Y_{l_b}^{m_b}(\theta,\phi)) T_b \cos(\omega_b t)$ so wave field is :

$$\Psi_{ab} = \left(A' j_l(k_a r) + B' y_l(k_a r)\right)^2 \Re(Y_{l_a}^{m_a}(\theta,\phi))^2 T_a^2 \cos^2(\omega_a t)\Big|_{a=b}$$

Rest of wave field components are equal to zero, it means that $\Psi_{ab}|_{a\neq b} = 0$, from it i can get metric tensor that has two first part of a sphere next one is radius part and last part is time (*4*):

$$g_{ab} = \begin{pmatrix} \Psi_{00} & 0 & 0 & 0 \\ 0 & -\Psi_{11} & 0 & 0 \\ 0 & 0 & -r^2\Psi_{22} & 0 \\ 0 & 0 & 0 & -r^2\Psi_{33} \end{pmatrix}$$

From it i can get spacetime interval formula that is equal to $ds^2 = \Psi_{00}c^2dt^2 - \Psi_{11}dr^2 - r^2\Psi_{22}d\theta^2 - r^2\sin^2(\theta)\Psi_{33}d\phi^2$ where i use metric signature $(+---)$. Now energy tensor has to be equal to frequency and wave number i can write it for this solution as $\omega_a^2 - c^2|k_{1a}|^2 - c^2|k_{2a}|^2 - c^2|k_{3a}|^2 = T^{\varphi 0}_{a\varphi}T^{\phi}_{0\phi a} - T^{\varphi 1}_{a\varphi}T^{\phi}_{1\phi a} - T^{\varphi 2}_{a\varphi}T^{\phi}_{2\phi a} - T^{\varphi 3}_{a\varphi}T^{\phi}_{3\phi a}$ this is simplest solutions to wave field equation. From given wave field i can calculated metric - thus curvature of spacetime that is equal to gravity. In standard quantum mechanics probability of finding particle at some location is equal to $\int_A^B |\psi|^2 dx$ that can be understood as wave function times it's complex conjugate $\int_A^B \psi\psi^* dx$ if field equation i can get same result if i change second or first part of wave to complex conjugate so equation solutions becomes:$\Psi^{\varphi\mu}_{a\varphi}\Psi^{*\phi}_{\mu\phi b}, \Psi^{*\varphi\mu}_{a\varphi}\Psi^{\phi}_{\mu\phi b}$ for each a and b i need to calculate it by summing, finally get probability as integral of it:

$$P = \int \sum_{a=0}^{3}\sum_{b=0}^{3} \Psi^{\varphi\mu}_{a\varphi}\Psi^{*\phi}_{\mu\phi b} d^3x = \int \sum_{a=0}^{3}\sum_{b=0}^{3} \Psi^{*\varphi\mu}_{a\varphi}\Psi^{\phi}_{\mu\phi b} d^3x = 1$$

It can be thought as sum of tensor product between Ψ_a and Ψ_b if i skip the summation of wave field so final part is just summing of tensor product components: $\int \sum_a \sum_b \Psi_a \otimes \Psi_b^* d^3x = \int \sum_a \sum_b \Psi_a^* \otimes \Psi_b d^3x$, from field equation there comes always some kind of cyclic loop-like structure from a fact that for example in field spherical solutions there is term $\cos(\omega t)$ but ω is very small in most cases so it just goes to one, it is because it's multiplied by Planck time. With energy going to Planck energy this term creates loop that is frozen and gives one event that loops back to itself. Same thing happens if i take massless particle, if i write metric for massless particle it is equal to zero ($ds^2 = 0$) it means i can scale for time and distance so that i get a loop with energy lower than Planck energy $t = \frac{nt}{t_P}, r = \frac{nr}{l_P}$ but still spacetime interval has to be zero that's why i have to scale both time and space components i can write metric then as: $ds^2 = \Psi_{00}c^2\frac{n^2t^2}{t_P^2} - \Psi_{11}\frac{n^2r^2}{l_P^2} - \frac{n^2r^2}{l_P^2}\Psi_{22}d\theta^2 - \frac{n^2r^2}{l_P^2}\sin^2(\theta)\Psi_{33}d\phi^2\Big|_{n=1,2...} = 0$ so frequency is part with time and radius gives loop $l_p k_a \frac{nr}{l_P} = k_a r, t_P\omega\frac{nt}{t_P} = \omega_a t$ and because those particles have zero distance in spacetime it means they from their point of view move in loops.

References and Notes

1. *Energy Momentum Relation*

   https://en.wikipedia.org/wiki/Energy-momentum_relation

2. *Laplace in spherial coordinates*

   http://mathworld.wolfram.com/LaplacesEquationSphericalCoordinates.html

3. *Wave equation solutions*

   https://www.grandinetti.org/resources/Teaching/Chem4300/LectureCh8.pdf

4. *2-Sphere geometry*

   http://www.physics.usu.edu/Wheeler/GenRel/Lectures/2Sphere.pdf