Quantum Gravity Idea

Tomasz Kobierzycki
*To whom correspondence should be addressed; E-mail: shizykfizyk@gmail.com

Quantum gravity is one of biggest puzzle in modern physics. Idea behind it is that spacetime has quantum nature. General Theory of Relativity is a classical theory but from theoretical point of view quantum physics must apply to gravity (*1*). In this paper i will present possible model that describes wave function as tensor field in curved spacetime. Idea is that Planck's scale is fundamental and can't be lower scale than it, it means that smallest unit of time is Planck's time and so on, it means that energy can go to maximum of Planck's energy and in Planck's time there can't be more than one vibration of field. It connects cyclic nature of waves in scalar part of tensor field with geometry of spacetime. That cyclic nature of spacetime leads to loops that are closed surface in spacetime, it means that at inside of a black hole or any place where energy goes to Planck's energy there are loops in spacetime where events repeat.

Contents

1 Field equation

In this paper i will present wave function as a tensor field that depends on curved spacetime coordinates. I will write Laplacian with down indexes that means that it goes with respect to that indexes wirting it (*2*):

$$\Delta_{\mu\nu} = \frac{1}{\sqrt{\det g}} \frac{\partial}{\partial \zeta^{\mu}} \left(\sqrt{\det g}\, g^{\mu\nu} \frac{\partial}{\partial \zeta^{\nu}} \right) \tag{1}$$

In this form field equation takes a tensor form, where Einstein summation convention is used, where κ is some constant and T is energy tensor as:

$$\boxed{\Delta_{\mu\nu} \Psi_{a\mu}{}^{\mu\varphi} \Psi_{\varphi\nu}{}^{\nu\phi} g_{\phi b} = \kappa T_{a\varphi}{}^{\varphi\phi} g_{\phi b}}$$

This equation when summed over all indexes gives just a, b indexes left so it reduces to form of field equation that Laplacian goes with respect to μ, ν summed coordinates, and energy tensor has only two indexes, i can write that form where both wave field tensor and energy tensor are summed as in equation above as:

$$\boxed{\Delta_{\mu\nu} \Psi_{ab} = \kappa T_{ab}}$$

By it i can write equality of those two equations:

$$\Delta_{\mu\nu} \Psi_{a\mu}{}^{\mu\varphi} \Psi_{\varphi\nu}{}^{\nu\phi} g_{\phi b} = \Delta_{\mu\nu} \Psi_{ab} \tag{2}$$

$$\kappa T_{a\varphi}{}^{\varphi\phi} g_{\phi b} = \kappa T_{ab} \tag{3}$$

That are definitions of tensor field equation. Solution to that equation is some kind of cyclic functions in curved spacetime. Idea is that frequency can't be higher than Planck's frequency and mass in one Planck's length can't be more than Planck's mass that gives maximum energy per Planck's length equal to Planck's energy.

2 Solutions to field equation

In this chapter i will present wave equation solutions to field equations. Where i use standard spherical wave equation solution (*3*) (*4*),

$$\Psi(ct,r,\theta,\phi)=\sum_{l=0}^{\infty}\sum_{m=-1}^{l}\left(A_l r^l+\frac{B_l}{r^{l+1}}\right)Y_{lm}(\theta,\phi)\left(D_{ml}\sin(\omega_k t)+E_{ml}\cos(\omega_k t)\right) \tag{4}$$

I want to solve field equation for metric tensor, to do it first i rewrite it with additional term T^a_ϕ in both sides:

$$T^a_\phi \Delta_{\mu\nu}\Psi_{a\,\mu}^{\;\mu\,\varphi}\Psi_{\varphi\,\nu}^{\;\nu\,\phi}g_{\phi\,b}=\kappa T^a_\phi T_{a\,\varphi}^{\;\varphi\,\phi}g_{\phi\,b} \tag{5}$$

From right side of equation i just get metric tensor $\kappa g_{\phi b}$ from left side of equation i get:

$$T^a_\phi \Delta_{\mu\nu}\Psi_{a\,\mu}^{\;\mu\,\varphi}\Psi_{b\varphi\,\nu}^{\;\;\;\nu}=T^a_\phi\Delta_{\mu\nu}\Psi_{ab}=\left(T\otimes\Delta_{\mu\nu}\Psi\right)_{\phi b}=\kappa g_{\phi b} \tag{6}$$

Now i need to use Knocker delta to change indexes in equation to get metric tensor with indexes a,b so I get:

$$\left(T\otimes\Delta_{\mu\nu}\Psi\right)_{ab}=\kappa g_{\phi b}\delta^\phi_a=\kappa g_{ab} \tag{7}$$

Now only left is to choose constants, constants A_l,B_l will be equal to $A_l=\cos^2(\frac{\varphi_r}{2})r^{-l}c_l$ and B_l is equal to zero and $\cos^2(\frac{\varphi_r}{2})$, $\frac{1}{\cos^2(\frac{\varphi_r}{2})}$ it's part that says how much radius coordinate direction changes from it's normal direction- radius curves in direction of time but when it gets to half of full angle radius points in direction of time and time goes half angle to direction of radius, it's from point of view that is distant observer, for observer falling it's angle is zero it means term vanish. This term says about how much radius coordinate rotates and half of it is angle between normal radius and curved radius. For time coordinate it takes opposite sign than from space i will use notation $(-,+,+,+)$ (*5*). Constants D_{lm},E_{lm} are equal to $D_{lm}=0,E_{lm}=1$, frequency is equal to $\omega_k=ck$ where k is wave number and c is speed of light (subscript P letter means

Planck's unit). Where R is radius and mass and M is mass. Writing frequency I get:

$$\omega_k = \frac{Ml_P\omega}{Rm_P f_P} = ck \tag{8}$$

Where energy tensor with one index is equal to:

$$T_a = \begin{pmatrix} \frac{\epsilon_0}{\epsilon_P} & \frac{P_1}{p_P} & \frac{P_2}{p_P} & \frac{P_3}{p_P} \end{pmatrix} \tag{9}$$

Where ϵ_0 is energy density and P_a is momentum in a direction, ϵ_P means Planck's energy density and p_P means Planck's momentum , ρ is momentum density in a direction. General form of energy tensor is equal to:

$$T_{ab} = \begin{pmatrix} \frac{\epsilon_0^2}{\epsilon_P^2} & \frac{\rho_1\rho_1}{\rho_P^2} & \frac{\rho_1\rho_2}{\rho_P^2} & \frac{\rho_1\rho_3}{\rho_P^2} \\ \frac{\rho_1\rho_1}{\rho_P^2} & \frac{P_1^2}{p_P^2} & \frac{P_1P_2}{p_P^2} & \frac{P_1P_3}{p_P^2} \\ \frac{\rho_2\rho_1}{\rho_P^2} & \frac{P_1P_2}{p_P^2} & \frac{P_2^2}{p_P^2} & \frac{P_3P_2}{p_P^2} \\ \frac{\rho_3\rho_1}{\rho_P^2} & \frac{P_3P_1}{p_P^2} & \frac{P_3P_2}{p_P^2} & \frac{P_3^2}{p_P^2} \end{pmatrix} \tag{10}$$

Energy tensor limits energy to maximum energy of one unit (in Planck's unit) per one vibration of field. It means that energy of one Planck's length can't be more than Planck's energy. Energy tensor like metric tensor has only ten independent components it means wave function tensor has to be same- symmetric with respect to switching from indexes ab to indexes ba. It means that when system gains energy it can't gain more energy at one point of space than Planck's energy, it means that energy given to particle will eventually turn it into black hole when energy density gets to Planck's energy. Now i will write metric in full form, where constant $\kappa =$ is equal to Planck's length and $A_l = r^{-l}c_l$ that depends on wave function, where constant c_l times $Y_{lm}(\theta,\phi)$ can't give more than one, writing metric i get:

$$ds^2 = -\frac{1}{l_P}\frac{\epsilon_0}{\epsilon_P}\sum_{l=0}^{\infty}\sum_{m=-1}^{l}\cos^2(\frac{\varphi_r}{2})c_{l0}Y_{lm0}(\theta,\phi)\cos(\omega_k t)c^2dt^2 + \frac{1}{l_P}\frac{P_1}{p_P}\sum_{l=0}^{\infty}\sum_{m=-1}^{l}\frac{c_{l1}Y_{lm1}(\theta,\phi)\cos(\omega_k t)}{\cos^2(\frac{\varphi_r}{2})}dr^2$$
$$+\frac{1}{l_P}\frac{P_2}{p_P}\sum_{l=0}^{\infty}\sum_{m=-1}^{l}r^2c_{l2}Y_{lm,2}(\theta,\phi)\cos(\omega_k t)d\theta^2 + \frac{1}{l_P}\frac{P_3}{p_P}\sum_{l=0}^{\infty}\sum_{m=-1}^{l}r^2c_{l3}Y_{lm,3}(\theta,\phi)\cos(\omega_k t)d\phi^2$$

3 Many system field equation and energy tensor form

Field equation can be extend to many systems. First i write equation for two systems , that takes for of:

$$\Delta_{\mu\nu}\Psi^{r\mu}_{a\mu}\Psi^{\rho\nu}_{r\nu}\Psi^{k\varphi}_{k\rho}\Psi^{l\phi}_{l\varphi}g_{\phi b} = \kappa T^{r\rho}_{ar}T^{\phi\varphi}_{\rho\varphi}g_{\phi b} \tag{11}$$

To write general equation for many systems i first need to change how i label tensor indexes. I will use three letters $(q_1...q_{2n}, \phi_1...\phi_{2n}, r_1, r_2)$ where n is number of systems in field equation.

$$\Psi_{r_1r_2} = \Delta_{q_1q_2}\Psi^{q_1\phi_1}_{r_1q_1}\Psi^{q_2\phi_2}_{\phi_1q_2}\Psi^{q_3\phi_3}_{q_3\phi_2}\Psi^{q_4\phi_4}_{q_4\phi_3}...\Psi^{q_{2n-1}\phi_{2n-1}}_{q_{2n-1}\phi_{2n-2}}\Psi^{q_{2n}\phi_{2n}}_{q_{2n}\phi_{2n-1}}g_{\phi_{2n}r_2}$$

$$\kappa T_{r_1r_2} = \kappa T^{\phi_1\phi_2}_{\phi_1r_1}T^{\phi_3\phi_4}_{\phi_3\phi_2}..T^{\phi_{2n-1}\phi_{2n}}_{\phi_{2n-1}\phi_{2n-2}}g_{\phi_{2n}r_2}$$

Putting that equation into one part i get field equation for N systems:

$$\Psi_{r_1r_2} = \Delta_{q_1q_2}\Psi^{q_1\phi_1}_{r_1q_1}\Psi^{q_2\phi_2}_{\phi_1q_2}\Psi^{q_3\phi_3}_{q_3\phi_2}\Psi^{q_4\phi_4}_{q_4\phi_3}...\Psi^{q_{2n-1}\phi_{2n-1}}_{q_{2n-1}\phi_{2n-2}}\Psi^{q_{2n}\phi_{2n}}_{q_{2n}\phi_{2n-1}}g_{\phi_{2n}r_2} = \kappa T^{\phi_1\phi_2}_{\phi_1r_1}T^{\phi_3\phi_4}_{\phi_3\phi_2}..T^{\phi_{2n-1}\phi_{2n}}_{\phi_{2n-1}\phi_{2n-2}}g_{\phi_{2n}r_2} \tag{12}$$

Energy tensor showed before (equation 8), was in trivial form. It was without summing all part of field equation. Now i will present it's specific form that is not simplest case- it's general case:

$$T_{00} = \sum_{\phi_1...\phi_{2n}\in\Phi_{a_1},\Phi_{b_1}} \frac{\epsilon_{\Phi_{a_1}0}\epsilon_{\Phi_{b_1}0}}{E_P^2} \tag{13}$$

$$T_{a0} = \sum_{\phi_1...\phi_{2n}\in\Phi_{a_2},\Phi_{b_2}} \frac{\rho_{\Phi_{b_2}a}\rho_{\Phi_{a_2}1}}{\rho_P^2} \tag{14}$$

$$T_{ba} = \sum_{\phi_1...\phi_{2n}\in\Phi_{a_3},\Phi_{b_3}} \frac{P_{\Phi_{a_3}b}P_{\Phi_{b_3}a}}{p_P^2} \tag{15}$$

$$T_{ab} = \sum_{\phi_1...\phi_{2n}\in\Phi_{a_4},\Phi_{b_4}} \frac{P_{\Phi_{a_4}a}P_{\Phi_{b_4}b}}{p_P^2} \tag{16}$$

$$T_{0b} = \sum_{\phi_1...\phi_{2n}\in\Phi_{a_5},\Phi_{b_5}} \frac{\rho_{\Phi_{a_5}1}\rho_{\Phi_{b_5}b}}{\rho_P^2} \tag{17}$$

It's most general case where summation can change depending on case of system equation solution. It's possible for that tensor to be not symmetric $ab \neq ba$ but it only works with special cases. Indexes a, b go in energy tensor from one to three.

4 Measurement of wave function field

For each point of spacetime there has to be one wave function that has a probability of system being in that place and all of those wave function can interact before measuring it and make it change from all point wave functions to one wave function. It means to each point of spacetime i assign a wave function with normalization number. Let's start with simple one scalar wave function where Ψ^* is complex conjugate of wave function and A is normalization factor, I write probability in four dimensions spacetime as :

$$P = \sum_{\pm(ijkl)} A_{\pm(ijkl)} \int_F \Psi_{\pm(ijkl)}(\zeta_0 \pm i, \zeta_1 \pm j, \zeta_2 \pm k, \zeta_3 \pm l)\Psi^*_{\pm(ijkl)}(\zeta_0 \pm i, \zeta_1 \pm j, \zeta_2 \pm k, \zeta_3 \pm l)d^4\zeta_a = 1 \tag{18}$$

This equation is for scalar wave function but field equation is a tensor field so to get from scalar to tensor field i just read probability in directions of spacetime movement, each direction has it's own probability that can be understand as scalar part of tensor field i write it as:

$$P_{ab} = \sum_{\pm(i,j,k,l)} A_{\pm(ijkl),ab} \int_F \Psi_{\pm(ijkl),ab}\Psi^*_{\pm(ijkl)ab}d^4\zeta_a = 1 \tag{19}$$

It means that before measurement there is a wave function for each point of spacetime that means that for example particle can be in any place and it's gravitation field spreads from that point as curvature of spacetime. That interacts with every other point of spacetime and final gravity field is sum all waves. After measurement it changes and all wave functions go to zero in Planck's time and only one of wave functions stays - that means only one geometry of spacetime is left where particle is located mostly at one point of spacetime and from it wave propagates in case of particle that is located as point in spacetime, if system is located in many point's after measurement what is left is sum of point like waves:

$$P_{ab} = \sum_{\pm(ijkl)} \sum_{\pm(s_0s_1s_2s_3)} A_{\pm(ijkl)ab} \int_F \Psi_{\pm(s_0s_1s_2s_3ijkl)ab}\Psi^*_{\pm(s_0s_1s_2s_3ijkl)ab}d^4\zeta_a = 1 \tag{20}$$

5 Direct geometrical meaning of field equation

Field equation has direct geometrical meaning, it says for vector field (wave function) on any curved spacetime it's equal to energy vector field of that spacetime. And property of spacetime is that tensor field that is consequence of energy vector field and wave function vector field that means it some kind of cyclic change in curvilinear coordinates is equal to wave function vector field and energy vector field. It leads to closed curvilinear coordinates where cyclicity is equal to frequency of particular wave vector field. That means that for any given vector field that is cyclic (it obeys wave equation in curvilinear coordinates) change in curvilinear coordinates is equal to energy vector field of that change in curvilinear coordinates and it leads to direct change of curvilinear coordinates that is equal to tensor that comes from energy vector field and wave function vector field. I can solve field equation for any cyclic spacetime by special tensor let's say that we have a vector field X_i, X_j that depends on coordinates x_i, x_j, I can find surface of curved spactime. Where r_i, r_j is radius that starts from point $(0,0,0,0)$ and points to any point of a vector field. That tensor can be thought us 4-sphere drawing coordinates perpendicular to change in radius . I can write tensor that is given me a 4-sphere and it's radius coordinates where r_i, r_j component of radius in i, j direction:

$$\begin{aligned} S_{ij} &= \frac{\partial X_i}{\partial x_i}\frac{\partial X_j}{\partial x_j} dV_{\perp dr_i} dr_i \otimes dV_{\perp dr_j} dr_j + (\imath x_i)(\imath x_j) V_{\perp r_i} \otimes V_{\perp r_j} + (x_i)(x_j) V_{\perp r_i + dr_i} \otimes V_{\perp r_j + dr_j} \\ &= \frac{\partial r_i}{\partial \zeta_i}\frac{\partial r_j}{\partial \zeta_j} dV_{\perp dr_i} dr_i \otimes dV_{\perp dr_j} dr_j + (\imath x_i)(\imath x_j) V_{\perp r_i} \otimes V_{\perp r_j} + (x_i)(x_j) V_{\perp r_i + dr_i} \otimes V_{\perp r_j + dr_j} \end{aligned}$$

That radius has dependence on ζ_i, ζ_j 4-sphere components it means in curvilinear coordinates i can write tensor that is equal to S_{ij} tensor by changing vector field in vector space to vector field on 4-sphere. That tensor is equal to radius of 4-sphere in curved space. It means i can draw curved coordinates by tracking line perpendicular to radius vector. It gives curved spacetime where it curvature depends on change of vector field, thus radius of 4-sphere and for each radius component i get a coordinate perpendicular to where radius points.

6 Scale less than Planck's scale in field equation

Field equation has to be limited to match Planck's scale, it means time can't be lower than Planck's time same with measured distance. But it's possible to have a lower scale of spacetime, where speed of light is not the limit it means there can be less than Planck's unit of space and time. To make this idea work it still has to make local speed of light limit be preserved. If i have field equation i can write them in for example coordinates where speed of light is replaced with speed of light squared, and Planck's unit are squared , i can take limit to any speed of light limit and it gives Planck's unit to power of that limit. It's simple tensor transformation from one coordinate system to another, so i can rewrite field equation in coordinate system with letter n that means speed of light limit by:

$$\frac{\partial x_a}{\partial \chi_{a(n)}} \frac{\partial x_b}{\partial \chi_{b(n)}} \Delta_{\mu\nu} \Psi_{ab} = \kappa \frac{\partial x_a}{\partial \chi_{a(n)}} \frac{\partial x_b}{\partial \chi_{b(n)}} T_{ab} \tag{21}$$

It means more speed of light limit gets less wave function changes, so for example Planck's energy in our reference frame is maximum energy value but in reference frame where speed of light limit is speed of light squared it's energy of empty space. So maximum curvature in one reference frame can be empty spacetime with smallest possible curvature. It means massive particle can never jump from one level of speed of light to another- it will have less than minimum energy of higher speed of light reference frame so local limit of maximum speed is preserved. And even when it has Planck's energy from reference frame of higher speed of limit, it's just empty space energy, it means no information can still travel faster than speed of light. But from point of view of massless particle distance in spacetime is equal to zero, it means that massless particle can't tell what is speed of light limit in it's reference frame. It has energy same in any reference frame no matter what is speed of light limit. But in view of observer that is in reference frame that speed of light is higher , that massless particle still have less or equal energy to minimum energy of empty space so it can't send information.

References and Notes

1. *General Relativity and Quantum Gravity*

   https://en.wikipedia.org/wiki/Quantum_gravity

2. *Laplacian*

   https://en.wikipedia.org/wiki/Laplace_operator

3. *Wave equation in spherical polar coordinates*

   http://mathworld.wolfram.com/LaplacesEquationSphericalCoordinates.html

4. *Spherical Waves*

   https://www2.ph.ed.ac.uk/ paboyle/Teaching/PhysicalMaths/Notes_2010/notes_2010.pdf

5. *Sign Convention*

   https://en.wikipedia.org/wiki/Sign_convention