Nature works the way Number works

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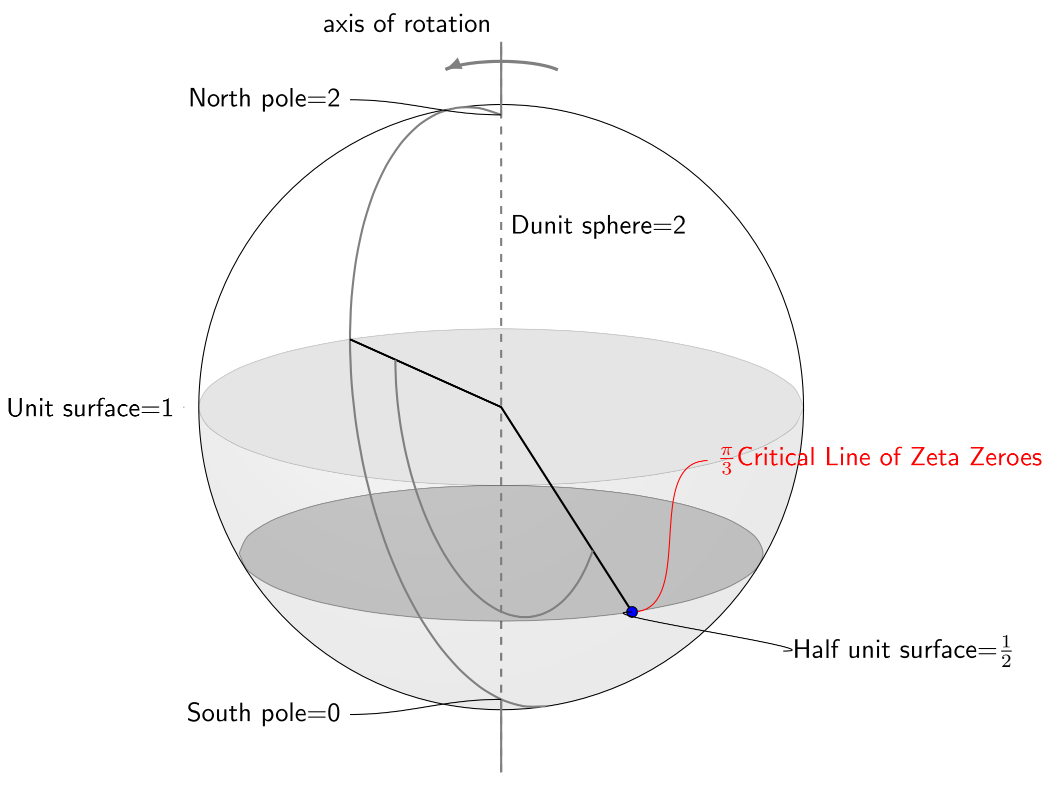
Abstract

Based on Euler 's formula a concept of duality unit or dunit circle is discovered. Continuing with Riemann hypothesis is proved from different angles, zeta values are renormalised to remove the poles of zeta function and discover relationships between numbers and primes. Imaginary number i can be defined such a way that it eases the complex logarithm and accounts for the scale difference between very big and very small. Pi can also be a base to natural logarithm and complement the scale gap. Other unsolved prime conjectures are also proved with the help of theorems of numbers and number theory.



Contents

1 Eulers formula, the unit circle, the unit sphere

$z = r(\cos x + i \sin x)$ is the trigonometric form of complex numbers. Using Eulers formula $e^{ix} = \cos x + i \sin x$ we can write $z = re^{ix}$. Putting $x = \pi$ in Eulers formula we get , $e^{i\pi} = -1$.Putting $x = \frac{\pi}{2}$ we get $e^{i\pi} = 1$. So the equation of the points lying on unit circle $z = e^{ix} = 1$. But that's not all. If $x = \frac{\pi}{3}$ in trigonometric form then $z = cos(\frac{\pi}{3}) + i.sin(\frac{\pi}{3}) = \frac{1}{2}(\sqrt{3} + i)$.So $|z| = r = \sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2} = \frac{1}{2}.\sqrt{4} = \frac{1}{2}.2 = 1$.So another equation of the points lying on unit circle $\mathbf{z = \frac{1}{2}e^{ix} = 1}$. Although both the equation are of unit circle, usefulness of $\mathbf{z = \frac{1}{2}e^{ix} = 1}$ is greater than $z = e^{ix} = 1$ as $\mathbf{z = \frac{1}{2}e^{ix} = 1}$ bifurcates mathematical singularity and introduces unavoidable mathematical duality particularly in studies of zeta function. $\mathbf{z = \frac{1}{2}e^{ix} = 1}$ can be regarded as dunit circle. When Unit circle in complex plane is stereo-graphically projected to unit sphere the points within the area of unit circle gets mapped to southern hemisphere, the points on the unit circle gets mapped to equatorial plane, the points outside the unit circle gets mapped to northern hemisphere. Dunit circle can be also be easily projected to Riemann sphere.

2 Euler the Grandfather of Zeta function

[1]In 1737, Leonard Euler published a paper where he derived a tricky formula that pointed to a wonderful connection between the infinite sum of the reciprocals of all natural integers (zeta function in its simplest form) and all prime numbers.

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + ... = \frac{2.3.5.7.11....}{1.2.4.6.8....}$$

Now:

$$1 + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 ... = \frac{2}{1}$$

$$1 + \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4 ... = \frac{3}{2}$$

⁝Euler product form of Zeta function when $s > 1$:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(1 + \frac{1}{p^s} + \frac{1}{p^{2s}} + \frac{1}{p^{3s}} + \frac{1}{p^{4s}} ...\right)...$$

Equivalent to:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \frac{1}{1 - P^{-s}}...$$

To carry out the multiplication on the right, we need to pick up exactly one term from every sum that is a factor in the product and, since every integer admits a unique prime factorization, the reciprocal of every integer will be obtained in this manner - each exactly once.

In the year of 1896 - Jacques Hadamard and Charles Jean de la Valle-Poussin independently prove the prime number theorem which essentially says that if there exists a limit to the ratio of primes upto a given number and that numbers natural logarithm, that should be equal to 1. When I started reading about number theory I wondered that if number theory is proved then what is left? the biggest job is done.I questioned myself why zeta function can not be defined at 1. Calculus has got set of rules for checking convergence of any infinite series, sometime especially when we are enclosing infinities to unity, those rules falls short to check the convergence of infinite series. In spite of that Euler was successful proving sum to product form and calculated zeta values for some numbers by hand only. Leopold Kronecker proved and interpreted Euler's formulas is the outcome of passing to the right-sided limit as $s \to 1^+$. I decided I will stick to Grandpa Eulers approach in attacking the problem.

3 Riemann the father of Zeta function

[2]Riemann showed that zeta function have the property of analytic continuation in the whole complex plane except for s=1 where the zeta function has its pole. Riemann Hypothesis is all about non trivial zeros of zeta function. There are trivial zeros which occur at every negative even integer. There are no zeros for $s > 1$. All other zeros lies at a critical strip $0 < s < 1$. In this critical strip he conjectured that all non trivial zeros lies on a critical line of the form of $z = \frac{1}{2} \pm iy$ i.e. the real part of all those complex numbers equals $\frac{1}{2}$.The zeta function satisfies Riemann's functional equation :

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s)$$

4 Proof of Riemann Hypothesis

4.1 Proof using Riemanns functional equation

Multiplying both side of functional equation by $(s-1)$ we get

$$(1-s)\zeta(s) = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)(1-s)\Gamma(1-s)\zeta(1-s).....(6)$$

Putting $(1-s)\Gamma(1-s) = \Gamma(2-s)$ we get:

$$\zeta(1-s) = \frac{(1-s)\zeta(s)}{2^1\pi^{(1-1)}\sin\left(\frac{\pi}{2}\right)\Gamma(2-s)}$$

$s \to 1$we get: $\because \lim_{s\to 1}(s-1)\zeta(s) = 1 \therefore (1-s)\zeta(s) = -1$ and $\Gamma(2-1) = \Gamma(1) = 1$

$$\zeta(0) = \frac{-1}{2^1\pi^0\sin\left(\frac{\pi}{2}\right)} = -\frac{1}{2}$$

Examining the functional equation we shall observe that the pole of zeta function at $Re(s) = 1$ is solely attributable to the pole of gamma function. In the critical strip $0 < s < 1$ Gauss's Pi function or the factorial function holds equally good if not better in Mellin transformation of exponential function. We can remove the pole of zeta function by way of removing the pole of gamma function. Using Gauss's Pi function instead of Gamma function we can rewrite the functional equation as follows:

$$\zeta(s) = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Pi(-s)\zeta(1-s).....(7)$$

Putting $s = 1$we get:

$$\zeta(1) = 2^1\pi^{(1-1)}\sin\left(\frac{\pi}{2}\right)\Pi(-1)\zeta(0) = 1$$

The zeta function is now defined on entire $\mathbb{C}$, and as such it becomes an entire function.In complex analysis, Liouville's theorem states that every bounded entire function must be constant. That is, every holomorphic function f for which there exists a positive number M such that $|f(z)| \leq M$ for all z in $\mathbb{C}$ is constant.Entire zeta function is constant as none of the values of zeta function do not exceed $M = \zeta(2) = \frac{\pi^2}{6}$.Maximum modulus principle further requires that non constant holomorphic functions attain maximum modulus on the boundary of the unit circle. Constant Entire zeta function duly complies with Maximum modulus principle as it reaches Maximum modulus $\frac{\pi^2}{6}$ out side the unit circle i.e.

on the boundary of the double unit circle. Gauss's Mean Value Theorem requires that in case a function is bounded in some neighborhood, then its mean value shall occur at the center of the unit circle drawn on the neighborhood. $|\zeta(0)| = \frac{1}{2}$ is the mean modulus of entire zeta function. Inverse of maximum modulus principle implies points on half unit circle give the minimum modulus or zeros of zeta function. Minimum modulus principle requires holomorphic functions having all non zero values shall attain minimum modulus on the boundary of the unit circle. Having lots of zero values holomorphic zeta function do not attain minimum modulus on the boundary of the unit circle rather points on half unit circle gives the minimum modulus or zeros of zeta function. Everything put together it implies that points on the half unit circle will mostly be the zeros of the zeta function which all have $\pm\frac{1}{2}$ as real part as Riemann rightly hypothesized. Putting $s = \frac{1}{2}$

$$\zeta\left(\frac{1}{2}\right) = 2^{\frac{1}{2}}\pi^{(1-\frac{1}{2})}\sin\left(\frac{\pi}{2.2}\right)\Pi\left(-\frac{1}{2}\right)\zeta\left(\frac{1}{2}\right)$$

$$\zeta\left(\frac{1}{2}\right)\left(1+\frac{\sqrt{2.\pi.\pi}}{2.\sqrt{2}}\right) = 0$$

$$\zeta\left(\frac{1}{2}\right)\left(1+\frac{\pi}{2}\right) = 0$$

$$\zeta\left(\frac{1}{2}\right) = 0$$

Therefore principal value of $\zeta(\frac{1}{2})$ is zero and Riemann Hypothesis holds good.

4.2 Proof using Eulers original product form

Eulers Product form of Zeta Function in Eulers exponential form of complex numbers is as follows:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(1 + re^{i\theta} + r^2 e^{i2\theta} + r^3 e^{i3\theta} ... \right) ....(4)$$

Now any such factor $\left(1 + re^{i\theta} + r^2 e^{i2\theta} + r^3 e^{i3\theta} ... \right)$ will be zero if

$$\left(re^{i\theta} + r^2 e^{i2\theta} + r^3 e^{i3\theta} ... \right) = -1 = e^{i\pi} .....(5)$$

Applying Euler's sum to product to unity rule infinite sum of infinite number of points on the unit circle can be shown to follow:

$$\theta + 2\theta + 3\theta + 4\theta ... = \pi$$

$$r + r^2 + r^3 + r^4 .... = 1$$

We can solve θ and r as follows:

$$\begin{aligned} \theta + 2\theta + 3\theta + 4\theta ... &= \pi \\ \theta(1 + 2 + 3 + 4 ...) &= \pi \\ \theta . \zeta(-1) &= \pi \\ \theta . \frac{-1}{12} &= \pi \\ \theta &= -12\pi \end{aligned}$$

$$\begin{aligned} r + r^2 + r^3 + r^4 .... &= 1 \\ r(1 + r + r^2 + r^3 + r^4 ....) &= 1 \\ r \frac{1}{1-r} &= 1 \\ r &= 1 - r \\ r &= \frac{1}{2} \end{aligned}$$

We can determine the real part of the non trivial zeros of zeta function as follows:

$$r \cos\theta = \frac{1}{2} \cos(-12\pi) = \frac{1}{2}$$

Therefore Principal value of $\zeta(\frac{1}{2})$ will be zero, hence Riemann Hypothesis is proved.

4.3 Proof using alternate product form

Eulers alternate Product form of Zeta Function in Eulers exponential form of complex numbers is as follows:

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(\frac{1}{1 - \frac{1}{re^{i\theta}}} \right) ... = \prod_p \left(\frac{re^{i\theta}}{re^{i\theta} - 1} \right) ....$$

Multiplying both numerator and denominator by $re^{i\theta} + 1$we get:

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(\frac{re^{i\theta}(re^{i\theta} + 1)}{(re^{i\theta} - 1)(re^{i\theta} + 1)} \right) ....$$

Now any such factor $\left(\frac{re^{i\theta}(re^{i\theta}+1)}{(re^{i\theta}-1)(re^{i\theta}+1)} \right)$ will be zero if $re^{i\theta}(re^{i\theta} + 1)$ is zero:

$$\begin{aligned} re^{i\theta}(re^{i\theta} + 1) &= 0 \\ re^{i\theta}(re^{i\theta} - e^{i\pi}) &= 0 \\ r^2 e^{i2\theta} - re^{i(\pi-\theta)*} &= 0 \\ r^2 e^{i2\theta} &= re^{i(\pi-\theta)} \end{aligned}$$

We can solve θ and r as follows:

$$\begin{aligned} 2\theta &= (\pi - \theta) & r^2 &= r \\ 3\theta &= \pi & \frac{r^2}{r} &= \frac{r}{r} \\ \theta &= \frac{\pi}{3} & r &= 1 \end{aligned}$$

*As there is -1 and we have started with unit circle the sign of θ get a minus.

We can determine the real part of the non trivial zeros of zeta function as follows:

$$r \cos\theta = 1. \cos(\frac{\pi}{3}) = \frac{1}{2}$$

Therefore Principal value of $\zeta(\frac{1}{2})$ will be zero, and Riemann Hypothesis is proved.

5 Infinite product or sum of Zeta values

5.1 Infinite product of positive Zeta values converges

$$\zeta(1) = 1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1} ... = \left(1 + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} ...\right)\left(1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3} ...\right)\left(1 + \frac{1}{5^1} + \frac{1}{5^2} ...\right) ...$$

$$\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} ... = \left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} ...\right)\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} ...\right)\left(1 + \frac{1}{5^2} + \frac{1}{5^4} ...\right) ...$$

$$\zeta(3) = 1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} ... = \left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9} ...\right)\left(1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9} ...\right)\left(1 + \frac{1}{5^3} + \frac{1}{5^6} ...\right) ...$$

$\vdots$

From the side of infinite sum of negative exponents of all natural integers:

$$\begin{aligned}
&\zeta(1)\zeta(2)\zeta(3)...\\
&= \left(1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1} ...\right)\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} ...\right)\left(1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} ...\right) ...\\
&= 1 + \left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} ...\right) + \left(\frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3} ...\right) + \left(\frac{1}{4^1} + \frac{1}{4^2} + \frac{1}{4^3} ...\right) ...\\
&= 1 + 1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1} + \frac{1}{5^1} + \frac{1}{6^1} + \frac{1}{7^1} + \frac{1}{8^1} + \frac{1}{9^1} ...\\
&= 1 + \zeta(1)
\end{aligned}$$

$\vdots$

From the side of infinite product of sum of negative exponents of all primes:

$$\zeta(1)\zeta(2)\zeta(3)... =$$

$$\left(1 + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} ...\right)\left(1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3} ...\right)\left(1 + \frac{1}{5^1} + \frac{1}{5^2} + \frac{1}{5^3} ...\right) ...$$

$$\left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} ...\right)\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} ...\right)\left(1 + \frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6} ...\right) ...$$

$$\left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9} ...\right)\left(1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9} ...\right)\left(1 + \frac{1}{5^3} + \frac{1}{5^6} + \frac{1}{5^9} ...\right) ...$$

$\vdots$

$$= \left(1 + 1\right)\left(1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3} ...\right)\left(1 + \frac{1}{5^1} + \frac{1}{5^2} + \frac{1}{5^3} ...\right) ...$$

$$\left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} ...\right)\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} ...\right)\left(1 + \frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6} ...\right) ...$$

$$\left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9} ...\right)\left(1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9} ...\right)\left(1 + \frac{1}{5^3} + \frac{1}{5^6} + \frac{1}{5^9} ...\right) ...$$

$\vdots$
continued to next page....

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Simultaneously halfing and doubling each factor and writing it sum of two equivalent forms

$$= 2\left(\frac{1}{2}\left(1+\frac{\frac{1}{3}}{1-\frac{1}{3}}+1+\frac{1}{3^1}+\frac{1}{3^2}+\frac{1}{3^3}...\right)\right)\left(\frac{1}{2}\left(1+\frac{\frac{1}{5}}{1-\frac{1}{5}}+1+\frac{1}{5^1}+\frac{1}{5^2}+\frac{1}{5^3}...\right)\right)...$$

$$\left(\frac{1}{2}\left(1+\frac{\frac{1}{4}}{1-\frac{1}{4}}+1+\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}...\right)\right)\left(\frac{1}{2}\left(1+\frac{\frac{1}{9}}{1-\frac{1}{9}}+1+\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}...\right)..)$$

$$\left(\frac{1}{2}\left(1+\frac{\frac{1}{8}}{1-\frac{1}{8}}+1+\frac{1}{2^3}+\frac{1}{2^6}+\frac{1}{2^9}...\right)\left(\frac{1}{2}\left(1+\frac{\frac{1}{27}}{1-\frac{1}{27}}+1+\frac{1}{3^3}+\frac{1}{3^6}+\frac{1}{3^9}...\right)\right)...$$

$$\vdots$$

$$= 2\left(\frac{1}{2}\left(1+\frac{1}{2}+1+\frac{1}{3^1}+\frac{1}{3^2}+\frac{1}{3^3}...\right)\right)\left(\frac{1}{2}\left(1+\frac{1}{4}+1+\frac{1}{5^1}+\frac{1}{5^2}+\frac{1}{5^3}...\right)\right)...$$

$$\left(\frac{1}{2}\left(1+\frac{1}{3}+1+\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}...\right)\left(\frac{1}{2}\left(1+\frac{1}{8}+1+\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}...\right)\right)...$$

$$\left(\frac{1}{2}\left(1+\frac{1}{7}+1+\frac{1}{2^3}+\frac{1}{2^6}+\frac{1}{2^9}...\right)\left(\frac{1}{2}\left(1+\frac{1}{26}+1+\frac{1}{3^3}+\frac{1}{3^6}+\frac{1}{3^9}...\right)\right)...$$

$$\vdots$$

$$= 2\left(1+\frac{1}{2}\left(\frac{1}{2}+\frac{1}{3^1}+\frac{1}{3^2}+\frac{1}{3^3}...\right)\right)\left(1+\frac{1}{2}\left(\frac{1}{4}+\frac{1}{5^1}+\frac{1}{5^2}+\frac{1}{5^3}...\right)\right)...$$

$$\left(1+\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}...\right)\right)\left(1+\frac{1}{2}\left(\frac{1}{8}+\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}...\right)\right)...$$

$$\left(1+\frac{1}{2}\left(\frac{1}{7}+\frac{1}{2^3}+\frac{1}{2^6}+\frac{1}{2^9}...\right)\right)\left(1+\frac{1}{2}\left(\frac{1}{26}+\frac{1}{3^3}+\frac{1}{3^6}+\frac{1}{3^9}...\right)\right)...$$

$$\vdots$$

$$= 2\left(1+\frac{1}{2}\left(\frac{1}{2^1}+\frac{1}{3^1}+\frac{1}{4^1}...+\frac{1}{2^1}+\frac{1}{3^1}+\frac{1}{4^1}...\right)\right)$$

$$= 2\left(1+\frac{1}{2}\left(2\zeta(1)-2\right)\right)$$

$$= 2(1-1+\zeta(1))$$

$$= 2\zeta(1)$$

Now comparing two identities:

$$\boxed{1+\zeta(1)=2\zeta(1))}$$

$$\boxed{\zeta(1)=1}$$

Hence Infinite product of positive Zeta values converges to 2

5.2 Infinite product of negative Zeta values converges

$$\zeta(-1) = 1 + 2^1 + 3^1 + 4^1 + 5^1... = \left(1 + 2 + 2^2 + 2^3...\right)\left(1 + 3 + 3^2 + 3^3...\right)\left(1 + 5 + 5^2 + 5^3...\right)...$$

$$\zeta(-2) = 1 + 2^2 + 3^2 + 4^2 + 5^2... = \left(1 + 2^2 + 2^4 + 2^6...\right)\left(1 + 3^2 + 3^4 + 3^6...\right)\left(1 + 5^2 + 5^4 + 5^6...\right)...$$

$$\zeta(-3) = 1 + 2^3 + 3^3 + 4^3 + 5^3... = \left(1 + 2^3 + 2^6 + 2^9...\right)\left(1 + 3^3 + 3^6 + 3^9...\right)\left(1 + 5^3 + 5^6 + 5^9...\right)...$$

$\vdots$

From the side of infinite sum of negative exponents of all natural integers:

$$\begin{aligned}
&\zeta(-1)\zeta(-2)\zeta(-3)...\\
&= \left(1 + 2^1 + 3^1 + 4^1 + 5^1...\right)\left(1 + 2^2 + 3^2 + 4^2 + 5^2...\right)\left(1 + 2^3 + 3^3 + 4^3 + 5^3...\right)...\\
&= 1 + \left(2 + 2^2 + 2^3...\right) + \left(3 + 3^2 + 3^3...\right) + \left(4 + 4^2 + 4^3...\right)...\\
&= 1 + \left(1 + 2 + 2^2 + 2^3... - 1\right) + \left(1 + 3 + 3^2 + 3^3... - 1\right) + \left(1 + 4 + 4^2 + 4^3... - 1\right)...\\
&= 1 + \left(-\frac{1}{1} - 1\right) + \left(-\frac{1}{2} - 1\right) + \left(-\frac{1}{3} - 1\right) + \left(-\frac{1}{4} - 1\right)...\\
&= 1 - \left(\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}...\right) + 1 + 1 + 1 + 1...\right)\\
&= 1 - \left(\zeta(1) + \zeta(0)\right)\\
&= 1 - \left(1 - \frac{1}{2}\right)\\
&= \frac{1}{2}
\end{aligned}$$

From the side of infinite product of sum of negative exponents of all primes:

$$\begin{aligned}
&\zeta(-1)\zeta(-2)\zeta(-3)... =\\
&\left(1 + 2 + 2^2 + 2^3...\right)\left(1 + 3 + 3^2 + 3^3...\right)\left(1 + 5 + 5^2 + 5^3...\right)...\\
&\left(1 + 2^2 + 2^4 + 2^6...\right)\left(1 + 3^2 + 3^4 + 3^6...\right)\left(1 + 5^2 + 5^4 + 5^6...\right)...\\
&\left(1 + 2^3 + 2^6 + 2^9...\right)\left(1 + 3^3 + 3^6 + 3^9...\right)\left(1 + 5^3 + 5^6 + 5^9...\right)...\\
&\vdots\\
&= 1 + 2^1 + 3^1 + 4^1 + 5^1...\\
&= \zeta(-1)
\end{aligned}$$

Therefore $\boxed{\zeta(-1) = \frac{1}{2}}$ must be the second root of $\zeta(-1)$ apart from the known one $\zeta(-1) = \frac{-1}{12}$.

Using Gauss's Pi function instead of Gamma function on the dunit circle we can rewrite the functional equation as follows:

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Pi(s-2)\zeta(1-s)$$

Putting $s = -1$we get:

$$\zeta(-1) = 2^{-1} \pi^{(-1-1)} \sin\left(\frac{-\pi}{2}\right) \Pi(-3)\zeta(2) = \frac{1}{2}$$

To proof Ramanujans Way

$$\sigma = \boxed{1+2+3+4+5+6+7+8+9.....}$$
$$2\sigma = \boxed{0+1+2+3+4+5+6+7+8+9...} + 1+1+1+1+1+1+1...*$$

Subtracting the bottom from the top one we get:

$$-\sigma = 0+1+1+1+1+1+1+1+1...+1+1+1+1+1+1+1...$$
$$\sigma = -(1+1+1+1+1+1+1+1+1+1........)$$
$$\sigma = \frac{1}{2}$$

*The second part is calculated subtracting bottom from the top before doubling.

5.3 Infinite product of All Zeta values converges

$$\boxed{\zeta(-1)\zeta(-2)\zeta(-3)...\zeta(1)\zeta(2)\zeta(3)... = \zeta(-1).2.\zeta(1) = \frac{1}{2}.2.1 = 1}$$

5.4 Infinite sum of Positive Zeta values converges

$$\zeta(1) = 1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1}...$$
$$\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}...$$
$$\zeta(3) = 1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3}...$$
$$\vdots$$
$$\zeta(1) + \zeta(2) + \zeta(3)...$$
$$= \left(1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1}...\right) + \left(1 + 1 + 1 + 1 + ...\right)$$
$$= \zeta(1) + \zeta(0) = 1 - \frac{1}{2} = \frac{1}{2}$$
$$\boxed{\zeta(1) + \zeta(2) + \zeta(3)... = \frac{1}{2}}$$

5.5 Infinite sum of Negative Zeta values converges

$$\zeta(-1) = 1 + 2^1 + 3^1 + 4^1 + 5^1...$$
$$\zeta(-2) = 1 + 2^2 + 3^2 + 4^2 + 5^2...$$
$$\zeta(-3) = 1 + 2^3 + 3^3 + 4^3 + 5^3...$$
$$\vdots$$
$$\zeta(-1) + \zeta(-2) + \zeta(-3)...$$
$$= \left(1 + 2^1 + 3^1 + 4^1 + 5^1...\right) + \left(1 + 1 + 1 + 1 + ...\right)$$
$$= \zeta(-1) + \zeta(0) = \frac{1}{2} - \frac{1}{2} = 0$$
$$\boxed{\zeta(-1) + \zeta(-2) + \zeta(-3)... = \zeta(-1) + \zeta(0) = 0}$$

5.6 Infinite sum of All Zeta values converges

$$\boxed{\zeta(-1) + \zeta(-2) + \zeta(-3)...\zeta(1) + \zeta(2) + \zeta(3)... = 0 + \frac{1}{2} = \frac{1}{2}}$$

5.7 $\zeta(-1)$ is responsible for trivial zeroes

$$
\begin{aligned}
\zeta(-1) &= \left(1+2+2^2+2^3...\right)\left(1+3+3^2+3^3...\right)\left(1+5+5^2+5^3...\right)...\\
&= \left(1+\frac{1}{1-2}\right)\left(1+3+3^2+3^3...\right)\left(1+5+5^2+5^3...\right)...\\
&= 0
\end{aligned}
$$

These are the trivial zeros get reflected to negative even numbers via critical line on $\zeta(-1) = \frac{1}{2}$

6 Primes product = 2.Sum of numbers

We know :

$\zeta(-1) = \zeta(1) + \zeta(0)$

$$or\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}...\right)+\left(1+1+1+1+...\right)=\frac{1}{2}$$

$$or\left(1+1\right)+\left(1+\frac{1}{2}\right)+\left(1+\frac{1}{3}\right)+\left(1+\frac{1}{4}\right)+...=\frac{1}{2}$$

$$or\left(\frac{2}{1}+\frac{3}{2}+\frac{4}{3}+\frac{5}{4}+\frac{6}{5}...\right)=\frac{1}{2}$$

LCM of the denominators can be shown to equal the square root of primes product.

Reversing the numerator sequence can shown to equal the sum of integers.

$$or\left(\frac{1+2+3+4+5+6+7...*}{2.3.5.7.11...**}\right)=\frac{1}{2}$$

$$or 2.\sum_{N=1}^{\infty} N = \prod_{i=1}^{\infty} P_i$$

*Series of terms written in reverse order.

**Product of All numbers can be written as 2 series of infinite product of all prime powers

**One arises from individual numbers and another from the number series.

$$LCM = \prod_{i=1}^{\infty} P_i^1.P_i^2.P_i^3.P_i^4.P_i^5.P_i^6...P_i^1.P_i^2.P_i^3.P_i^4.P_i^5.P_i^6...$$

$$LCM = \prod_{i=1}^{\infty} P_i^{(1+2+3+4+5+6+7...)+(1+2+3+4+5+6+7...)}...$$

$$LCM = \prod_{i=1}^{\infty} P_i^{\frac{1}{2}+\frac{1}{2}}...$$

$LCM = 2.3.5.7.11...$

7 Complex number and complex logarithm

Thanks to Roger coats who first time used i in complex logarithm. Thanks to euler who extended it to exponential function and tied i, pi and exponential function to unity in his famous formula. Now taking lead from both of their work and applying results of zeta function which are simultaneously continuous logarithmic function and continuous exponential function we can redefine Complex number and complex logarithm as follows.

7.1 First root of i

In dunit circle we have seen $\mathbf{z = \frac{1}{2}e^{ix} = 1}$ is another form of unit circle. We can rewrite :

$$z = \frac{1}{2}e^{ix} = 1 = \frac{1}{2}e^{\ln 2}$$

we can say :

$$e^{ix} = e^{ln2}$$

taking logarithm both side :

$$ix = ln(2)$$

setting x=1 :

$$\boxed{ln(2) = e^{ln(ln(2))} = e^{ln(i)} = i \approx e^{-\frac{1}{e}} \approx 2^{-\frac{1}{2}} \approx e - 2*}$$

or

$$\boxed{ln(2)^{\frac{1}{ln(ln(2))}} = i^{\frac{1}{ln(i)}} = e \approx -\frac{1}{ln(i)} \approx 2 + i*}$$

we get two more identity like $e^{i\pi} + 1 = 0$:

$$\boxed{\frac{1}{e} + ln(i) = 0 = e + \frac{1}{ln(i)}}$$

again we know $i^2 = -1$, taking log both side

$$\boxed{\ln(-1) = 2\ln i = 2ln(ln(2))}$$

* computed value(even wolfram alpha can't be that match accurate as the nature, there may be slight difference based on the devices capabilities) therefore matches our definition. Bingo! we have hit the bull's eye. Let us understand the logic. As we are forcing x to take value of 1, i is getting forced to reveal the proportion of constant e is dependent upon itself.

Example 1 *Find natural logarithm of -5 using first root of i*

$\ln(-5) = \ln(-1)+\ln(5) = 2ln(ln(2))+\ln(5) = 2ln(e-2)+\ln(5) = 0.876412071(approx)$

Example 2 *Find natural logarithm of -5i using first root of i*

$\ln(-5i) = \ln(-1)+\ln(5)+\ln(i) = 2ln(ln(2))+\ln(5)+ln(ln(2)) = 0.509899151(approx)$

Example 3 *Find natural logarithm of -5i using first root of i Transform the complex number 2+9i using first root of i.*

$$(\text{a})e^{2+9i} = e^{2+9X0.693147181} = e^{8.238324625} = 3783.196723(approx)$$

7.2 Middle scale constants

Puting the value of i in Eulers identity we get constants of the middle scale.

Constant 1

$$\boxed{e^{i\pi} = e^{ln(2).\pi} = 8.824977827 = e^{2.17758609}...(approx)}$$

Constant 2

$$\boxed{e^{i\frac{\pi}{2}} = e^{\frac{ln(2).\pi}{2}} = 2.970686424 = e^{1.088793045}...(approx)}$$

Constant 3

$$\boxed{e^{i\frac{\pi}{3}} = e^{\frac{ln(2).\pi}{3}} = 2.066511728 = e^{0.72586203}...(approx)}$$

Constant 4

$$\boxed{e^{i\frac{\pi}{4}} = e^{\frac{ln(2).\pi}{4}} = 1.723567934 = e^{0.544396523}...(approx)}$$

Constant 5

$$\boxed{\frac{1}{e^{i\pi}} = \frac{1}{e^{ln(2).\pi}} = 0.113314732 = e^{-2.17758609}...(approx)}$$

Constant 6

$$\boxed{\frac{1}{e^{i\frac{\pi}{2}}} = \frac{1}{e^{\frac{ln(2).\pi}{2}}} = 0.336622537 = e^{-1.088793045}...(approx)}$$

Constant 7

$$\boxed{\frac{1}{e^{i\frac{\pi}{3}}} = \frac{1}{e^{\frac{ln(2).\pi}{3}}} = 0.483907246 = e^{-0.72586203}...(approx)}$$

Constant 8

$$\boxed{\frac{1}{e^{i\frac{\pi}{4}}} = \frac{1}{e^{\frac{ln(2).\pi}{4}}} = 0.58019181 = e^{-0.544396523}...(approx)}$$

7.3 Second root of i

From $i^2 = -1$ we know that i shall have atleast two roots or values, one we have already defined, another we need to find out. We know that at $\frac{\pi}{3}$ zeta function (which is bijectively holomorphic and deals with both complex exponential and its inverse i.e. complex logarithm) attains zero. Let us use Eulers formula to define another possible value of i as Eulers formula deals with unity which comes from the product of exponential and its inverse i.e. logarithm.

Lets assume:

$e^{i\frac{\pi}{3}} = z$

taking natural log both side :

$\frac{i\pi}{3} = ln(z)$

Lets set:$ln(z) = i + \frac{1}{3}$

$i\pi = 1 + 3i$

$i(\pi - 3) = 1$

$$\boxed{i = \frac{1}{\pi - 3}}$$

$$\boxed{\pi * = 3 + \frac{1}{i}}$$

we get two more identity like $e^{i\pi} + 1 = 0$:

$$\boxed{ln(i) - 2 = 0 = \frac{1}{ln(i)} - \frac{1}{2}}$$

again we know $i^2 = -1$, taking log both side

$$\boxed{\ln(-1) = 2\ln i = 2ln\left(\frac{1}{\pi - 3}\right)}$$

* computed value (even wolfram alpha can't be that match accurate as the nature, there may be slight difference based on the devices capabilities) therefore matches our definition. Bingo! We have hit the bull's eye.

Example 4 *Find natural logarithm of -5 using second root of i*

$$\ln(-5) = \ln(-1) + \ln(5) = 2ln\left(\frac{1}{\pi - 3}\right) + \ln(5) = 5.519039873(approx)$$

Example 5 *Find natural logarithm of -5i using second root of i*

$$\ln(-5i) = \ln(-1) + \ln(5) + \ln(i) = 2ln\left(\frac{1}{\pi - 3}\right) + \ln(5) + ln\left(\frac{1}{\pi - 3}\right) = 7.473840854(approx)$$

Example 6 *Transform the complex number 3+i using second root of i.*

$$e^{3+i} = e^{3+1X7.06251330593105} = e^{10.0625133059311} = 23447.3627750323(approx)$$

7.4 The scale of very large and very small numbers

Putting the value of i in Eulers identity we get large constants applicable for cosmic scale and their reciprocals are useful constants to deal with quantum world.

Constant 9

$$\boxed{e^{i\pi} = e^{\frac{\pi}{\pi-3}} = 4,324,402,934 = e^{22.18753992}...(approx)}*$$

Constant 10

$$\boxed{e^{i\frac{\pi}{2}} = e^{\frac{\pi}{2(\pi-3)}} = 65,760 = e^{11.09376703}...(approx)}$$

Constant 11

$$\boxed{e^{i\frac{\pi}{3}} = e^{\frac{\pi}{3(\pi-3)}} = 1,629 = e^{7.395721609}...(approx)}$$

Constant 12

$$\boxed{e^{i\frac{\pi}{4}} = e^{\frac{\pi}{4(\pi-3)}} = 256.4375 = e^{5.54688497}...(approx)}$$

Constant 13

$$\boxed{\frac{1}{e^{i\pi}} = \frac{1}{e^{\frac{\pi}{\pi-3}}} = 2.31E-10 = e^{-22.18753992}...(approx)}$$

Constant 14

$$\boxed{\frac{1}{e^{i\frac{\pi}{2}}} = \frac{1}{e^{\frac{\pi}{2(\pi-3)}}} = 1.52E-05 = e^{-11.09376703}...(approx)}$$

Constant 15

$$\boxed{\frac{1}{e^{i\frac{\pi}{3}}} = \frac{1}{e^{\frac{\pi}{3(\pi-3)}}} = 6.14E-04 = e^{-7.395721609}...(approx)}$$

Constant 16

$$\boxed{\frac{1}{e^{i\frac{\pi}{4}}} = \frac{1}{e^{\frac{\pi}{4(\pi-3)}}} = 0.0039 = e^{-5.54688497}...(approx)}$$

* This constant is a product of physical constants (dimensionless) as follows:

$$\frac{\text{2.mass of electron.speed of light squared.charles ideal gas constant}}{\text{boltzman constant}} \approx e^{\frac{\pi}{(\pi-3)}}$$

7.5 Third and final root of i

Can there be a third root of i, why not? Three sides of a triangle can enclose a circle, value of pi is just little more than 3, we see 3 generation of stars in the universe, there are 3 generation of matter in the standard model, spatially we cannot imagine more than three dimensions. Let us use Eulers formula to define another possible value of i as Eulers formula deals with unity which comes from the product of exponential and its inverse i.e. logarithm.

Lets assume:

$$e^{i\frac{\pi}{3}} = z$$

taking natural log both side :

$$\frac{i\pi}{3} = ln(z)$$

Lets set: $ln(z) = \frac{2\pi}{3(\pi-3)} - \frac{1}{2(\pi-3)}$

$$i\pi = \frac{4\pi-3}{2(\pi-3)}$$

$$\boxed{i = \frac{4\pi-3}{2\pi(\pi-3)}}$$

$$\boxed{\pi* = 3 + \frac{9}{6i}}$$

we get two more identity like $e^{i\pi}+1=0$:

$$\boxed{ln(i) - \frac{19}{8} = 0 = \frac{1}{ln(i)} - \frac{8}{19}}$$

again we know $i^2 = -1$, taking log both side

$$\boxed{\ln(-1) = 2\ln i = 2ln\left(\frac{4\pi-3}{2\pi(\pi-3)}\right)}$$

* computed value (even wolfram alpha can't be that match accurate as the nature, there may be slight difference based on the devices capabilities) therefore matches our definition. Bingo! We have hit the bull's eye.

Example 7 *Find natural logarithm of -5 using third root of i*

$$\ln(-5) = \ln(-1) + \ln(5) = 2ln\left(\frac{4\pi-3}{2\pi(\pi-3)}\right) + \ln(5) = 6.359793515(approx)$$

Example 8 *Find natural logarithm of -5i using third root of i*

$$\ln(-5i) = \ln(-1)+\ln(5)+\ln(i) = 2ln\left(\frac{4\pi-3}{2\pi(\pi-3)}\right)+\ln(5)+ln\left(\frac{4\pi-3}{2\pi(\pi-3)}\right) = 8.734971317(approx)$$

Example 9 *Transform the complex number 3+i using third root of i.*

$$e^{3+i} = e^{3+1X10.7529249} = e^{13.7529249} = 939332.598(approx)$$

7.6 The scale of very large and very small continues....

Putting the value of i in Eulers identity we get large constants applicable for cosmic scale and their reciprocals are useful constants to deal with quantum world.

Constant 17

$$\boxed{e^{i\pi} = e^{\frac{4\pi-3}{2(\pi-3)}} = 4.68853E+14 = e^{33.7813104}...(approx)}$$

Constant 18

$$\boxed{e^{i\frac{\pi}{2}} = e^{\frac{4\pi-3}{4(\pi-3)}} = 2.34426E+14 = e^{33.08816109}...(approx)}$$

Constant 19

$$\boxed{e^{i\frac{\pi}{3}} = e^{\frac{4\pi-3}{6(\pi-3)}} = 1.56284E+14 = e^{32.68269598}...(approx)}$$

Constant 20

$$\boxed{e^{i\frac{\pi}{4}} = e^{\frac{4\pi-3}{8(\pi-3)}} = 1.17213E+14 = e^{32.39501391}...(approx)}$$

Constant 21

$$\boxed{\frac{1}{e^{i\pi}} = \frac{1}{e^{\frac{\pi}{\pi-3}}} = 2.13287E-15 = e^{-33.7813104}...(approx)}$$

Constant 22

$$\boxed{\frac{1}{e^{i\frac{\pi}{2}}} = \frac{1}{e^{\frac{\pi}{2(\pi-3)}}} = 4.26573E-15 = e^{-33.08816109}...(approx)}$$

Constant 23

$$\boxed{\frac{1}{e^{i\frac{\pi}{3}}} = \frac{1}{e^{\frac{\pi}{3(\pi-3)}}} = 6.3986E-15 = e^{-32.68269598}...(approx)}$$

Constant 24

$$\boxed{\frac{1}{e^{i\frac{\pi}{3}}} = \frac{1}{e^{\frac{\pi}{4(\pi-3)}}} = 8.53146E-15 = e^{-32.39501391}...(approx)}$$

8 Pi based logarithm

One thing to notice is that pi is intricately associated with e. We view pi mostly associated to circles, what it has to do with logarithm? Can it also be a base to complex logarithm? Although base pi logarithm are not common but this can be handy in complex logarithm. We know:

$$ln(2).\frac{\pi}{4}$$
$$= \ln e^{\frac{ln(2)}{4}}.\pi$$
$$= \left(\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots\right)\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{11} - \frac{1}{13} + \cdots\right)$$
$$= \left(1 + \frac{1}{\lfloor 3} - \frac{1}{\lfloor 5} + \frac{1}{\lfloor 7} - \cdots\right) + \left(1 + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 4} + \frac{1}{\lfloor 6} + \cdots\right) - \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots\right)$$
$$= \left(1 - \frac{i^3}{\lfloor 3} + \frac{i^5}{\lfloor 5} - \frac{i^7}{\lfloor 7} - \cdots\right) + \left(1 - \frac{i^2}{\lfloor 2} + \frac{i^4}{\lfloor 4} - \frac{i^6}{\lfloor 6} + \cdots\right) - \frac{1}{1 - \frac{1}{2}}$$
$$= \sin(i) + \cos(i) - 2$$

Lets set:$\pi = \sin(i) + \cos(i)$ and replacing $\pi - 2 = \ln(\pi)$ we can write

$$\frac{\ln\left(e^{\frac{ln(2)}{4}}\right)}{\ln(\pi)} = \frac{1}{\pi} = \pi^{-1}\text{Lets set:}e^{\frac{ln(2)}{4}} = \pi^{\pi^{je}} \text{ we can write } \boxed{\pi^{je} = -1}$$

8.1 3 roots of j and its complementary scales

Similar to 3 roots of i, there can be 3 roots of j which will give another complex logarithmic scales. I will not make the readers crazy anymore with this nasty mind-boggling staff. I am writing directly the scales because it feel so boring writing the same staff again and again. Do not ask me how i got it ? Same head spinning and mind twisting algorithms. Better I will ask my readers to undergo this processes themselves little bit to have a better feel of my work on complex logarithm. Remember that you will get always hints from the transcendental parts of pi to set i similar to transcendental parts of e. My inspiration for trying this was recursive nature of zeta function, Mandelbrot fractal, Rogers Ramanujan continued fraction, Ramanujan's infinite radicals.

Constant 25

$$\boxed{\pi^{je} = \pi^{ln(2\pi).e} = 304.5755639 = e^{5.718919214}...(approx)}$$

Constant 26

$$\boxed{\pi^{je} = \pi^{\frac{e}{e-2}.e} = 130089.9289 = e^{11.77598125}...(approx)}$$

Constant 27

$$\boxed{\pi^{je} = \pi^{\frac{1}{\ln(2)}.e} = 199385.9251 = e^{12.20299755}...(approx)}$$

9 Grand integrated invariant scale

In nature around us we see things grow or decay exponentially. In calculus e is the magic number whose derivative and integration is itself. Thats why we took e as the base of natural logarithm and we analyze very big data somehow related to nature in natural logarithmic scale. But doesn't that sound that nature is scale variant? How can immensely big numbers can be scaled down to that small number e. Here nature plays number theory.Wherever infinitely big as well as infinitesimally small numbers are involved nature do not follow natural logarithmic scale i.e. $e^1, e^2, e^3, e^4, e^5, e^6, e^7$ or inversely $\frac{1}{e^1}, \frac{1}{e^2}, \frac{1}{e^3}, \frac{1}{e^4}, \frac{1}{e^5}, \frac{1}{e^6}, \frac{1}{e^7}$ will not give us 5 sigma answer,rather we will be off by 4 sigma.Howsoever strange it may sound it is real and it is logical too. Believe it or not a truly invariant scale will be approximately $e^1, e^5, e^{10}, e^{15}, e^{20}, e^{25}, e^{30}$ or inversely $\frac{1}{e^1}, \frac{1}{e^5}, \frac{1}{e^{10}}, \frac{1}{e^{15}}, \frac{1}{e^{20}}, \frac{1}{e^{25}}, \frac{1}{e^{30}}$. I propose that, we shall call it Grand integrated invariant scale.Who will second me? For even better results Mr. Perfectionists can always take the exact scale.

10 Proof of other unsolved problems

In the light of identities derived from zeta function most of the unsolved prime conjectures turn obvious as follows:

10.1 Fundamental formula of integers

Primes product = 2.Sum of numbers can be generalized to all even numbers as zeta function all the poles being removed now shows bijectively holomorphic property and as such become absolutely analytic or literally an entire function. $2.\sum_{N=1}^{\infty} N = \prod_{i=1}^{\infty} P_i$ is open ended and self replicating.Similarly $\sum_{N=1}^{\infty} N = \prod_{i=2}^{\infty} P_i$ is self sufficient. We can pick partial series, truncate series to get even and odd numbers.

* $2.\sum_{N=1}^{\infty} N = \prod_{i=1}^{\infty} P_i$ can be regarded as Fundamental formula of all even numbers.
* $\sum_{N=1}^{\infty} N = \prod_{i=2}^{\infty} P_i$ can be regarded as Fundamental formula of all odd numbers excluding primes.
* $\sum_{N=1}^{\infty} N = P_i$ can be regarded as Fundamental formula of all primes.

10.2 Goldbach Binary/Even Conjecture

If we take two odd prime in the left hand side of fundamental formula of even numbers then retaining the fundamental pattern both the side have a highest common factor of 2. That means all the even numbers can be expressed as sum of at least 1 pair of primes i.e. 2 primes and if we multiply both side by 2 then some even numbers can be expressed as sum of 2 pair of primes i.e. 4 primes. Overall an even number can be expressed as sum of maximum 2+2+2=6 primes one pair each in half unit, unit and dunit circle. However immediately after 3 pairs of prime one pi rotation completes (* perhaps that is the reason we are not allowed to think more than 3 spatial and one temporal dimension) and the prime partition sequence breaks, i.e. beyond dunit circle it starts over and over again cyclically along the number line. Ramanujans derived value $2\zeta(-1) = -\frac{2}{12} = -\frac{1}{6}$ actually indicates that limit (*perhaps this is the reason we see mostly 6 electrons in the outermost shell although in the electron cloud it can pop in and out from and to half unit, unit and dunit circle. Perhaps this is the reason we see maximum 6 generation of particles either quarks, leptons, bosons although all 6 are not yet discovered).

$$2(p_1 + p_2) = 2.p_3$$
$$4(p_1 + p_2 + p_3 + p_4) = 2.2.p_5.p_6$$
$$8(p_1 + p_2 + p_3 + p_4 + p_5 + p_6) = 2.2.2.p_7.p_8.p_9$$

*This part of the document do neither form part of the proof nor the authors personal views to be considered seriously. Its just an addendum to the document.

10.3 Goldbach Tarnary/Odd Conjecture

In case of odd number also we can have combination of 3 and 6 primes. Beyond that no more prime partition is possible.

$$(p_1 + p_2 + p_3) = 3.p_4$$
$$(p_1 + p_2 + p_3 + p_4 + p_5 + p_6) = 3.p_7.p_8$$

10.4 Polignac prime conjectures

10.4.1 Twin prime conjecture

Lets test whether prime gap of 2 preserves the fundamental formula of numbers.

$p^2 + 2p = p(p + 2)$

adding 2 both side will turn both side into prime as $p^2 + 2p + 2$ cannot be factorised.

$p^2 + 2p + 2 = p(p + 2) + 2$

$p^2 + 2p + 2 = p_1.p_2$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall be infinite number of twin primes.

10.4.2 Cousin prime conjecture

Lets test whether prime gap of 4 preserves the fundamental formula of numbers.

$p^2 + 4p = p(p + 4)$

adding 1 both side will turn both side into prime as $p^2 + 4p + 1$ cannot be factorised.

$p^2 + 4p + 1 = p(p + 4) + 1$

$p^2 + 4p + 1 = p_1.p_2$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall be infinite number of cousin primes.

10.4.3 Sexy prime conjecture

Lets test whether prime gap of 6 preserves the fundamental formula of numbers.

$p^2 + 6p = p(p + 6)$

adding 1 both side will turn both side into prime as $p^2 + 6p + 1$ cannot be factorised.

$p^2 + 6p + 1 = p(p + 6) + 1$

$p^2 + 6p + 1 = p_1.p_2$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall be infinite number of cousin primes.

10.4.4 Other Polignac prime conjectures

Similarly all other polignac primes of the form of p+2n shall be there infinitely.

10.5 Sophie Germain prime conjecture

Lets test whether prime gap of 2p preserves the fundamental formula of numbers which will generate sophie germain prime pairs.

$2p^2 = p(2p)$

adding 1 both side will turn both side into prime as $2p^2 + 1$ cannot be factorised.

$2p^2 + 1 = p(2p) + 1$

$2p^2 + 1 = p_1.p_2$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall be infinite number of Sophie Germain primes.

10.6 Landau's prime conjecture

We need to check whether there shall always be infinite number of N^2+1 primes.

$N^2 + 1 = N^2 + 1$

Adding N and multiplying both side by 2 will turn both side into an even number.

$2(N^2 + N + 1) = 2.(N^2 + N + 1)$

dividing by 2 both side will turn it into prime as $N^2 + N + 1$ cannot be factorised.

$(N^2 + N + 1) = P$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall always be infinite number of $N^2 + 1$ primes.

10.7 Legendre's prime conjecture

Lets take sum of two successive numbers square and test whether they conform to the fundamental formula of numbers.

$N^2 + N^2 + 2N + 1 = N^2 + (N + 1)^2$

adding 1 both side will turn both side into an even number.

$2(N^2 + N + 1) = 2.P$

dividing by 2 both side will turn it into prime as $N^2 + N + 1$ cannot be factorised.

$(N^2 + N + 1) = P$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall always be a prime between two successive numbers square.

10.8 Brocard's prime conjecture

As square of a prime and square of its successor both have identical powers they shall have a highest common factor of 4 in $4(p_1 + p_2 + p_3 + p_4...) = 2.2.p_5.p_6...$, and there shall be at least four primes between them as Brocard conjectured.

10.9 Opperman's prime conjecture

Lets test whether gap of N between $N(N-1)$ and N^2 preserves the fundamental formula of numbers which will give us the count of primes between the pairs.

$N^2 - N + N^2 = N(N-1) + N^2$

adding 3N+1 both side will turn both side into an even number.

$2(N^2 + N + 1) = 2.(N^2 + N + 1)$

dividing by 2 both side will turn it into prime as $N^2 + N + 1$ cannot be factorised.

$(N^2 + N + 1) = P$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall be atleast one prime between $N(N-1)$ and N^2 as Opperman conjectured.

Lets test whether gap of N between $N(N+1)$ and N^2 preserves the fundamental formula of numbers which will give us the count of primes between the pairs.

$N^2 + N + N^2 = N(N+1) + N^2$

adding N+2 both side will turn both side into an even number.

$2(N^2 + N + 1) = 2.(N^2 + N + 1)$

dividing by 2 both side will turn it into prime as $N^2 + N + 1$ cannot be factorised.

$(N^2 + N + 1) = P$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall be atleast one prime between N^2 and $N(N+1)$ as Opperman conjectured.

10.10 Collatz conjecture

As fundamental formula of numbers is proved to be continuous, collatz conjectured operations on any number shall always end 1 being the singularity. Hence collatz conjecture is proved to be trivial.

References

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