Einstein's Notational Equation of Electro-Magnetic Field Equation in Rindler spacetime

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ABSTRACT

We find Einstein's notational equation of the electro-magnetic field equation and the electro-magnetic field in Rindler space-time. Because, electromagnetic fields of the accelerated frame include in general relativity theory.

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1. Introduction

Our article's aim is that we find Einstein notation's equations in general relativity theory instead of the electro-magnetic field equations in Rindler space-time.

Rindler coordinate are

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})$$

$$x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0} \quad , y = \xi^2, z = \xi^3 \tag{1}$$

The electro-magnetic field equation is in Rindler space-time [1].

$$\vec{\nabla}_\xi \cdot \vec{E}_\xi = 4\pi\rho_\xi(1 + \frac{a_0\xi^1}{c^2}) \tag{2-i}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi \times \{\vec{B}_\xi(1+\frac{a_0\xi^1}{c^2})\} = \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{E}_\xi}{c\partial\xi^0} + \frac{4\pi}{c}\vec{j}_\xi \tag{2-ii}$$

$$\vec{\nabla}_\xi \cdot \vec{B}_\xi = 0 \tag{2-iii}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi \times \{\vec{E}_\xi(1+\frac{a_0\xi^1}{c^2})\} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{B}_\xi}{c\partial\xi^0} \tag{2-iv}$$

$$\vec{E}_\xi = (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}), \vec{B}_\xi = (B_{\xi^1}, B_{\xi^2}, B_{\xi^3}), \vec{\nabla}_\xi = (\frac{\partial}{\partial\xi^1}, \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial\xi^3})$$

The Electro-magnetic field $(\vec{E}_\xi, \vec{B}_\xi)$ is defined in Rindler spacetime [1].

$$\vec{E}_\xi = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_\xi}{c\partial\xi^0}$$

$$\vec{B}_\xi = \vec{\nabla}_\xi \times \vec{A}_\xi$$

In this time, $\vec{\nabla}_\xi = (\frac{\partial}{\partial\xi^1}, \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial\xi^3}), \vec{A}_\xi = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3})$ (3)

2. Einstein's Notational Equation in General Relativity theory

Electromagnetic field tensor $F_{\xi}^{\ \mu\upsilon}$ is in Rindler space-time,

$$F_{\xi}^{\ \mu\upsilon} = \begin{pmatrix} 0 & E_{\xi^1} & E_{\xi^2} & E_{\xi^3} \\ -E_{\xi^1} & 0 & (1+\frac{a_0\xi^1}{c^2})B_{\xi^3} & -(1+\frac{a_0\xi^1}{c^2})B_{\xi^2} \\ -E_{\xi^2} & -(1+\frac{a_0\xi^1}{c^2})B_{\xi^3} & 0 & (1+\frac{a_0\xi^1}{c^2})B_{\xi^1} \\ -E_{\xi^3} & (1+\frac{a_0\xi^1}{c^2})B_{\xi^2} & -(1+\frac{a_0\xi^1}{c^2})B_{\xi^1} & 0 \end{pmatrix} \quad (4)$$

Electromagnetic field tensor $F_{\xi\,\mu\upsilon}$ is in Rindler space-time,

$$F_{\xi\,\mu\upsilon} = \begin{pmatrix} 0 & -(1+\frac{a_0\xi^1}{c^2})E_{\xi^1} & -(1+\frac{a_0\xi^1}{c^2})E_{\xi^2} & -(1+\frac{a_0\xi^1}{c^2})E_{\xi^3} \\ (1+\frac{a_0\xi^1}{c^2})E_{\xi^1} & 0 & B_{\xi^3} & -B_{\xi^2} \\ (1+\frac{a_0\xi^1}{c^2})E_{\xi^2} & -B_{\xi^3} & 0 & B_{\xi^1} \\ (1+\frac{a_0\xi^1}{c^2})E_{\xi^3} & B_{\xi^2} & -B_{\xi^1} & 0 \end{pmatrix} \quad (5)$$

Hence, Eq(3) is

$$F_{\xi\,\mu\upsilon} = \frac{\partial A_{\xi^\upsilon}}{\partial \xi^\mu} - \frac{\partial A_{\xi^\mu}}{\partial \xi^\upsilon}, \quad A_{\xi^\mu} = ((1+\frac{a_0\xi^1}{c^2})^2\phi_\xi, \vec{A}_\xi) \quad (6)$$

Eq(2-i),Eq(2-ii),Eq(2-iii),Eq(2-iv) are

$$F_{\xi}^{\ \mu\upsilon}{}_{,\upsilon} = \frac{4\pi}{c} j^\mu (1+\frac{a_0\xi^1}{c^2}) \quad (7\text{-i})$$

$$F_{\xi\,\mu\upsilon,\lambda} + F_{\xi\,\upsilon\lambda,\mu} + F_{\xi\,\lambda\mu,\upsilon} = 0 \quad (7\text{-ii})$$

Hence, the Lagrangian L_ξ of electromagnetic field in Rindller space-time is,

$$\begin{aligned} L_\xi &= -\frac{1}{4} F_{\xi}^{\ \mu\upsilon} F_{\xi\,\mu\upsilon} \\ &= -\frac{1}{2}(1+\frac{a_0\xi^1}{c^2})(B_\xi^{\ 2} - E_\xi^{\ 2}), \end{aligned}$$

$$\vec{E}_\xi = (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}), \ \vec{B}_\xi = (B_{\xi^1}, B_{\xi^2}, B_{\xi^3}), \ \left|\vec{E}_\xi\right| = E_\xi, \left|\vec{B}_\xi\right| = B_\xi \quad (8)$$

3. Conclusion

We find Einstein's notational equations of the electro-magnetic field equation in uniformly accelerated frame.

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