Confirmation that impossible worlds mean nothing is necessary there but everything is possible.

Abstract: We confirm the definition that impossible worlds mean "nothing is necessary there but everything is possible". The truth axiom as given is *not* tautologous, but rather the logical value of truthity. The rules of extensionality and monotonicity as given are *not* tautologous. When the definition of impossible worlds is combined with monotonicity + T + 4 that conjecture is also tautologous. Therefore the failed equations are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Witczak, T. (2019). Generalized topological semantics for weak modal logics. arxiv.org/pdf/1904.06099.pdf tm.witczak@gmail.com

We shall work with the following list of axioms and rules: ...

$T : \Box\varphi \rightarrow \varphi$ (T.1)

LET p, q: φ, ψ.
#p>p ; TTTT TTTT TTTT TTTT (T.2)

$4 : \Box\varphi \rightarrow \Box\Box\varphi$ (4.1)

#p>##p ; TTTT TTTT TTTT TTTT (4.2)

$N : \Box\top$ (*truth axiom*) (N.1)

Remark N.1: We evaluate this expression as both the designated proof value and the designated truthity value:

#(p=p) = (p=p) ; NNNN NNNN NNNN NNNN (N.2)
#(%p>#p) = (p=p) ; NNNN NNNN NNNN NNNN (N.3)

Eqs. N.2 and N.3 are equivalent as the logical value for truthity, but are *not* tautologous.

$RE : \varphi \leftrightarrow \psi \vdash \Box\varphi \leftrightarrow \Box\psi$ (rule of extensionality) (RE.1)

(#p=#q)>(p=q) ; `TNNT TNNT TNNT TNNT` (RE.2)

$RM : \varphi \rightarrow \psi \vdash \Box\varphi \rightarrow \Box\psi$ (rule of monotonicity) (RM.1)

(#p>#q)>(p>q) ; `TNTT TNTT TNTT TNTT` (RM.2)

"[I]mpossible worlds … means that
nothing is necessary there but everything is possible". (1.1)

(~p=#p)\(#p=%p) ; `TTTT TTTT TTTT TTTT` (1.2)

Remark 1.2: The definition of impossible worlds as "nothing is necessary there but everything is possible" is a theorem.

"As for the monotonic system T 4,
probably it [was] not already investigated in the context of impossible worlds." (2.1)

((~p=#p)\(#p=%p))+(((#p>#q)>(p>q))+((#p>p)+(#p>##p))) ;
`TTTT TTTT TTTT TTTT` (2.2)

Axioms N, RE, and RM are *not* tautologous. However, Eq. 1.1 + monotonic + T + 4 is a theorem.