

# Solving Incompletely Predictable problems Polignac's and Twin Prime conjectures using Information-Complexity conservation

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**Abstract** Prime numbers are Incompletely Predictable numbers calculated using complex algorithm Sieve of Eratosthenes. Involving proposals that prime gaps and associated sets of prime numbers are infinite in magnitude, Twin prime conjecture deals with even prime gap 2 and is a subset of Polignac's conjecture which deals with all even prime gaps 2, 4, 6, 8, 10,.... Treated as Incompletely Predictable problems, we solve these conjectures with research method Information-Complexity conservation to get Plus Gap 2 Composite Number Continuous Law and Plus-Minus Gap 2 Composite Number Alternating Law.

**Keywords** Dimensional analysis, Incompletely Predictable problems, Information-Complexity conservation, Plus Gap 2 Composite Number Continuous Law, Plus-Minus Gap 2 Composite Number Alternating Law, Polignac's and Twin prime conjectures  
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## 1 Introduction

Uncountable complex numbers ( $\mathbb{C}$ ) include uncountable real numbers ( $\mathbf{R}$ ).  $\mathbf{R}$  = countable rational numbers ( $\mathbf{Q}$ ) + uncountable irrational numbers ( $\mathbf{R} - \mathbf{Q}$ ).  $\mathbf{R} - \mathbf{Q}$  = countable algebraic numbers + uncountable transcendental numbers.  $\mathbf{Q}$  include countable integers ( $\mathbf{Z}$ ) which include countable whole numbers ( $\mathbf{W}$ ) which in turn include countable natural numbers ( $\mathbf{N}$ ).  $\mathbf{N}$  is constituted by either countable even numbers ( $\mathbf{E}$ ) and countable odd numbers ( $\mathbf{O}$ ) or countable prime numbers ( $\mathbf{P}$ ), countable composite numbers ( $\mathbf{C}$ ) and Number '1'. Then (i) Set  $\mathbf{N} = \text{Set } \mathbf{E} + \text{Set } \mathbf{O}$ , (ii) Set  $\mathbf{N} = \text{Set } \mathbf{P} + \text{Set } \mathbf{C} + \text{Number '1'}$ , and (iii) Set  $\mathbf{N} \subset \text{Set } \mathbf{W} \subset \text{Set } \mathbf{Z} \subset \text{Set } \mathbf{Q} \subset \text{Set } \mathbf{R} \subset \text{Set } \mathbb{C}$ .

With increasing magnitude, arbitrary Set  $\mathbf{X}$  belongs to countable finite set (CFS), countable infinite set (CIS) or uncountable infinite set (UIS). Cardinality of Set  $\mathbf{X}$ ,  $|\mathbf{X}|$ , measures the "number of elements" in Set  $\mathbf{X}$ . E.g. Set **even P** has CFS of even  $\mathbf{P}$  with  $|\text{even } \mathbf{P}| = 1$ , Set  $\mathbf{N}$  has CIS of  $\mathbf{N}$  with  $|\mathbf{N}| = \aleph_0$ , and Set  $\mathbf{R}$  has UIS of  $\mathbf{R}$  with  $|\mathbf{R}| = \mathfrak{c}$  (cardinality of the continuum). Respectively, CIS of  $\mathbf{P}$  and  $\mathbf{C}$  are *Incompletely Predictable numbers* dependently

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calculated directly and indirectly from *complex algorithm* Sieve of Eratosthenes. Involving proposals that prime gaps and associated sets of prime numbers are infinite in magnitude, Twin prime conjecture deals with even prime gap 2 and is a subset of Polignac's conjecture dealing with all even prime gaps 2, 4, 6, 8, 10,..... Activities to prove these open problems in number theory equate to solving *Incompletely Predictable problems*.

All claims arising from these activities are made meaningful by providing definitions on above mentioned terms. Respectively, an Incompletely (Completely) Predictable number is locationally defined as a number whose position is *dependently (independently)* determined by complex (simple) calculations using complex (simple) equation or algorithm with (without) needing to know related positions of all preceding numbers in neighborhood. Simple properties are inferred from a phrase such as: "...simple equation or algorithm by itself will intrinsically incorporate actual location [and actual positions] of all Completely Predictable numbers". Solving Completely Predictable problems with simple properties amendable to *simple* treatments using *usual* mathematical tools such as Calculus result in 'Simple Elementary Fundamental Laws'-based solutions. Complex properties are inferred from a phrase such as: "...complex equation or algorithm by itself will intrinsically incorporate actual location [but not actual positions] of all Incompletely Predictable numbers". Solving Incompletely Predictable problems with complex properties amendable to *complex* treatments using *unusual* mathematical tools such as our novel research method Information-Complexity conservation as well as using *usual* mathematical tools such as Calculus result in 'Complex Elementary Fundamental Laws'-based solutions.

### 1.1 Dimensional analysis on Cardinality and "Dimensions"

For 'base quantities' such as *length, mass* and *time*; their fundamental SI 'units of measurement' are [respectively] given by meter (m), kilogram (kg) and second (s). The word 'dimension' is commonly used to denote 'units of measurement' in well-defined equations. Dimensional analysis (DA) is an analytic tool with resulting DA homogeneity and non-homogeneity (respectively) denoting valid and invalid equation when 'units of measurements' are "balanced" and "unbalanced" across both sides of the equation. E.g.  $2\text{ m} + 3\text{ m} = 5\text{ m}$  is a valid equation but  $2\text{ m} + 3\text{ kg} = 5\text{ mkg}$  is an invalid equation.

We use "Dimensions" to denote well-defined Incompletely Predictable entities obtained from Information-Complexity conservation. Relevant "Dimensions" *dependently* represent Number '1', **P** and **C**. Then *by default* any (sub)sets of **P** and **C** in well-defined equations can also be represented by their corresponding "Dimensions".

**Remark 1.1.** We can apply Dimensional analysis to "Dimensions" from Information-Complexity conservation and cardinality of relevant sets in certain well-defined equations.

Let **X** denote **E**, **O**, **N** [which are classified as Completely Predictable numbers], **P** and **C** [which are classified as Incompletely Predictable numbers]. For  $x = 1, 2, 3, 4, 5, \dots, \infty$ ; consider all  $\mathbf{X} \leq x$ . Then this "all  $\mathbf{X} \leq x$ " is definition for  $\mathbf{X}-\pi(x)$  [denoting "X counting function"] resulting in following two types of equations coined as (I) 'Exact' equation  $\mathbf{N}-\pi(x) = \mathbf{E}-\pi(x) + \mathbf{O}-\pi(x)$  with "non-varying" relationships  $\mathbf{E}-\pi(x) = \mathbf{O}-\pi(x)$  for all  $x = \mathbf{E}$  and  $\mathbf{E}-\pi(x) = \mathbf{O}-\pi(x) - 1$  for all  $x = \mathbf{O}$ , and (II) 'Inexact' equation  $\mathbf{N}-\pi(x) = 1 + \mathbf{P}-\pi(x) + \mathbf{C}-\pi(x)$  with "varying" relationships  $\mathbf{P}-\pi(x) > \mathbf{C}-\pi(x)$  for all  $x \leq 8$ ;  $\mathbf{P}-\pi(x) = \mathbf{C}-\pi(x)$  for  $x = 9, 11, \text{ and } 13$ ; and  $\mathbf{P}-\pi(x) < \mathbf{C}-\pi(x)$  for  $x = 10, 12, \text{ and all } x \geq 14$ .

Let "Dimensions" and different (sub)sets of **E**, **O**, **N**, **P** and **C** be 'base quantities'. Then exponent '1' of "Dimensions" and cardinality of these (sub)sets in well-defined equations

are corresponding 'units of measurement'. Performing DA on "Dimensions" for **PC** pairing are depicted later on. Performing DA on cardinality are depicted next.

For Set  $\mathbf{N} = \text{Set } \mathbf{E} + \text{Set } \mathbf{O}$ , then  $|\mathbf{N}| = |\mathbf{E}| + |\mathbf{O}| \implies \aleph_0 = \aleph_0 + \aleph_0$  thus conforming with DA homogeneity.

For Set  $\mathbf{N} = \text{Set } \mathbf{P} + \text{Set } \mathbf{C} + \text{Number '1'}$ , then Set  $\mathbf{N} - \text{Number '1'} = \text{Set } \mathbf{P} + \text{Set } \mathbf{C}$  and  $|\mathbf{N} - \text{Number '1'}| = |\mathbf{P}| + |\mathbf{C}| \implies \aleph_0 = \aleph_0 + \aleph_0$  thus conforming with DA homogeneity.

For Set  $\mathbf{N} - \text{Set even } \mathbf{P} - \text{Number '1'} = \text{Set odd } \mathbf{P} + \text{Set even } \mathbf{C} + \text{Set odd } \mathbf{C}$ , then  $|\mathbf{N} - \text{even } \mathbf{P} - \text{Number '1'}| = |\text{odd } \mathbf{P}| + |\text{even } \mathbf{C}| + |\text{odd } \mathbf{C}| \implies \aleph_0 = \aleph_0 + \aleph_0 + \aleph_0$  thus conforming with DA homogeneity. Symbolically represented by all available **O** prime gap = 1 and **E** prime gaps = 2, 4, 6, 8, 10, ...; **O** composite gap = 1 and **E** composite gap = 2; and **O** natural gap = 1; then  $|\text{Gap } 1 \mathbf{N} - \text{Gap } 1 \mathbf{P} - \text{Number '1'}| = |\text{Gap } 2 \mathbf{P}| + |\text{Gap } 4 \mathbf{P}| + |\text{Gap } 6 \mathbf{P}| + |\text{Gap } 8 \mathbf{P}| + |\text{Gap } 10 \mathbf{P}| + \dots + |\text{Gap } 1 \mathbf{C}| + |\text{Gap } 2 \mathbf{C}| \implies \aleph_0 = \aleph_0 + \aleph_0 + \aleph_0 + \aleph_0 + \aleph_0 + \dots \aleph_0 + \aleph_0$  thus conforming with DA homogeneity. It is known that  $|\text{Gap } 1 \mathbf{P}| = |\text{Number '1'}| = 1$  and  $|\text{Gap } 1 \mathbf{N}| = |\text{Gap } 1 \mathbf{C}| = |\text{Gap } 2 \mathbf{C}| = \aleph_0$ . Then solving Polignac's and Twin prime conjectures translate to successfully proving  $|\text{Gap } 2 \mathbf{P}| = |\text{Gap } 4 \mathbf{P}| = |\text{Gap } 6 \mathbf{P}| = |\text{Gap } 8 \mathbf{P}| = |\text{Gap } 10 \mathbf{P}| = \dots = \aleph_0$  with  $|\mathbf{E} \text{ prime gaps}| = \aleph_0$ .

**Outline of proof for Polignac's and Twin prime conjectures.** Requires simultaneously satisfying two mutually inclusive conditions: I. *With rigid manifestation of DA homogeneity*, quantitative<sup>1</sup> fulfillment by considering  $i \in \mathbf{E}$  for each Subset **odd P<sub>i</sub>** generated by **E** prime gap =  $i$  from Set **E prime gaps** occurs only if solitary cardinality value is present in equation

Set **odd P** =  $\sum_{i=2}^{\infty} \text{Subset odd } \mathbf{P}_i$  with  $|\text{odd } \mathbf{P}| = |\text{odd } \mathbf{P}_i| = |\mathbf{E} \text{ prime gaps}| = \aleph_0$ , and II. *With*

*rigid manifestation of DA non-homogeneity*, quantitative<sup>1</sup> fulfillment by considering  $i \in \mathbf{E}$  for each Subset **odd P<sub>i</sub>** generated by **E** prime gap =  $i$  from Set **E prime gaps** does not occur if

more than one cardinality values are present in equation Set **odd P** >  $\sum_{i=2}^{\infty} \text{Subset odd } \mathbf{P}_i$  with

$|\mathbf{E} \text{ prime gaps}| = \aleph_0$  having incorrect  $|\text{Subset(s) odd } \mathbf{P}| = \mathbf{N}$  (finite value) and/or Set **odd P** >  $\sum_{i=2}^{\mathbf{N}} \text{Subset odd } \mathbf{P}_i$  with  $|\text{odd } \mathbf{P}_i| = \aleph_0$  having incorrect  $|\mathbf{E} \text{ prime gaps}| = \mathbf{N}$  (finite value).

**Footnote 1:** Qualitative fulfillment of  $|\text{odd } \mathbf{P}| = |\text{odd } \mathbf{P}_i| = |\text{all } \mathbf{E} \text{ prime gaps}| = \aleph_0$  equates to Plus-Minus Gap 2 Composite Number Alternating Law being precisely obeyed by all **E** prime gaps apart from first **E** prime gap precisely obeying Plus Gap 2 Composite Number Continuous Law. Derived using Information-Complexity conservation, these Laws symbolize "end-result" proof on Polignac's and Twin prime conjectures. *Law of Continuity* is a heuristic principle *whatever succeed for the finite, also succeed for the infinite*. Then these Laws which inherently manifest 'Gap 2 Composite Number' on finite and infinite time scale should in principle "succeed for the finite, also succeed for the infinite".

**Polignac's and Twin prime conjectures mathematical foot-prints.** Six identifiable steps to prove these conjectures: *Step 1* Considering  $x \in \mathbf{N}$ , obtain Dimensions  $(2x - 2)^1, (2x - 4)^1, (2x - 5)^1, (2x - 7)^1, (2x - 8)^1, (2x - 9)^1, \dots, (2x - \infty)^1$  with specific groupings to constitute all elements of Set **P** [culminating in obtaining all prime gaps (= **E** prime gaps + Solitary **O** prime gap) with  $|\text{all prime gaps}| = \aleph_0$ ]. Note Dimension  $(2x - 2)^1$  represents  $x = 1$  (Number '1') which is neither **P** nor **C**. *Step 2* Considering  $i \in \mathbf{E}$ , confirm perpetual recurrences of individual **E** prime gap =  $i$  (associated with its unique odd **P<sub>i</sub>**) occur only when depicted as specific groupings of these Dimensions endowed with exponent '1' for all ranges of  $x$ . *Step 3* Perform DA on exponent '1' in these Dimensions. *Step 4* Perform DA on equation

Set **odd P** =  $\sum_{i=2}^{\infty} \text{Subset odd } \mathbf{P}_i$  to obtain  $|\text{odd } \mathbf{P}| = |\text{odd } \mathbf{P}_i| = \aleph_0$  whereby Subset **odd P<sub>i</sub>**

is derived from its associated unique **E** prime gap =  $i$  with  $|\mathbf{E} \text{ prime gaps}| = \aleph_0$ . *Step 5* Confirm 'Prime number' variable and 'Prime gap' variable complex algorithm "containing" all **P** with knowing their overall actual location [but not actual positions]<sup>2</sup>. *Step 6* Derive Plus-Minus Gap 2 Composite Number Alternating Law and Plus Gap 2 Composite Number Continuous Law using Information-Complexity conservation.

**Footnote 2:** This phrase implies all **P** (and **C**) are treated as Incompletely Predictable numbers. Actual positions will require using complex algorithm Sieve of Eratosthenes to *dependently* calculate positions of all preceding **P** (and **C**) in the neighborhood.

'Complex Elementary Fundamental Laws'-based solutions of Plus-Minus Gap 2 Composite Number Alternating Law and Plus Gap 2 Composite Number Continuous Law are obtained by undertaking the non-negotiable mathematical steps outlined above. These Laws are literally Completely Predictable meta-properties ('overall' *complex properties*) arising from "interactions" between **P** and **C** producing relevant patterns of Gap 2 Composite Number perpetual appearances [albeit with Incompletely Predictable timing]. We logically deduce explicit mathematical explanation for these meta-properties requires "complex" mathematical arguments. Attempts to give explicit mathematical explanation with "simple" mathematical arguments would intuitively mean Incompletely Predictable numbers **P** and **C** be (incorrectly and impossibly) treated as Completely Predictable numbers.

## 1.2 Brief overview of Polignac's and Twin prime conjectures

Occurring over 2000 years ago (c. 300 BC), ancient Euclid's proof on infinitude of **P** in totality [viz.  $|\mathbf{P}| = \aleph_0$  for Set **P**] predominantly by *reductio ad absurdum* (proof by contradiction) is earliest known but not the only proof for this simple problem in number theory. Since then dozens of proofs have been devised such as three chronologically listed: Goldbach's Proof using Fermat numbers (written in a letter to Swiss mathematician Leonhard Euler, July 1730), Furstenberg's Topological Proof in 1955[1], and Filip Saidak's Proof in 2006[2]. The strangest candidate is likely to be Furstenberg's Topological Proof.

In 2013, Yitang Zhang proved a landmark result showing some unknown even number ' $N$ ' < 70 million such that there are infinitely many pairs of **P** that differ by ' $N$ '[3]. By optimizing Zhang's bound, subsequent Polymath Project collaborative efforts using a new refinement of GPY sieve in 2013 lowered ' $N$ ' to 246; and assuming Elliott-Halberstam conjecture and its generalized form have further lower ' $N$ ' to 12 and 6, respectively. Then ' $N$ ' has intuitively more than one valid values such that there are infinitely many pairs of **P** that differ by each of those ' $N$ ' values [thus proving existence of more than one Subset **odd P<sub>i</sub>** with  $|\mathbf{odd P}_i| = \aleph_0$ ]. We can only theoretically lower ' $N$ ' to 2 (in regards to **P** with 'small gaps') but there are still an infinite number of **E** prime gaps (in regards to **P** with 'large gaps') that require "the proof that each will generate its unique set of infinite **P**".

**Remark 1.2.** Existence of maximal and non-maximal prime gaps supply crucial indirect evidence to intuitively support but does not prove proposition "Each even prime gap will generate an infinite magnitude of odd prime numbers on its own accord".

Comments relevant to Remark 1.2 are given in Section 2 below.

## 2 Supportive role of maximal and non-maximal prime gaps

We analyze data of all **P** obtained when extrapolated out over a wide range of  $x \geq 2$  integer values. As sequence of **P** carries on, **P** with ever larger prime gaps will appear. For given

**Table 1** First 17 prime gaps depicted in the format utilizing maximal prime gaps [depicted with asterisk symbol (\*)] and non-maximal prime gaps [depicted without this asterisk symbol].

Prime gap	Following the prime number	Prime gap	Following the prime number
1*	2	18*	523
2*	3	20*	887
4*	7	22*	1129
6*	23	24	1669
8*	89	26	2477
10	139	28	2971
12	199	30	4297
14*	113	32	5591
16	1831		

range of  $x$  integer values, prime gap =  $n_2$  is a 'maximal prime gap' if prime gap =  $n_1 <$  prime gap =  $n_2$  for all  $n_1 < n_2$ . In other words, the largest such prime gaps in this range are called maximal prime gaps. The term 'first occurrence prime gaps' refers to first occurrences of maximal prime gaps whereby maximal prime gaps are prime gaps of "at least of this length".

We use maximal prime gaps to denote 'first occurrence prime gaps'. CIS non-maximal prime gaps (endorsed with nickname 'slow jumpers') will always lag behind CIS maximal prime gaps for onset appearances in  $\mathbf{P}$  sequence. These are shown for first 17 prime gaps in Table 1. Apart from  $\mathbf{O}$  prime gap = 1 representing solitary even  $\mathbf{P}$  '2', remaining  $\mathbf{P}$  depicted in Table 1 consist of representative single odd  $\mathbf{P}$  for each  $\mathbf{E}$  prime gap. These odd  $\mathbf{P}$  will individually make one-off appearance in  $\mathbf{P}$  sequence in a *perpetual albeit Incompletely Predictable manner*. Initial seven of [majority] "missing" odd  $\mathbf{P}$  are 5, 11, 13, 17, 19, 29, 31,... belonging to Subset  $\mathbf{P}$  with 'residual' prime gaps are potential source of odd  $\mathbf{P}$  in relation to proposal that each  $\mathbf{E}$  prime gap from Set  $\mathbf{E}$  prime gaps will generate its specific Subset odd  $\mathbf{P}$ . Set all  $\mathbf{P}$  from all prime gaps = Subset  $\mathbf{P}$  from maximal prime gaps + Subset  $\mathbf{P}$  from non-maximal prime gaps + Subset  $\mathbf{P}$  from 'residual' prime gaps. Subset  $\mathbf{P}$  from 'residual' prime gaps with representation from all  $\mathbf{E}$  prime gaps must include all correctly selected "missing" odd  $\mathbf{P}$ . These observations support but does not prove proposition that each  $\mathbf{E}$  prime gap will generate its own Subset odd  $\mathbf{P}$  with  $|\text{odd } \mathbf{P}| = \aleph_0$ .

For  $i \in \mathbf{N}$ ; primordial  $P_i\#$  is analog of usual factorial for  $\mathbf{P} = 2, 3, 5, 7, 11, 13, \dots$ . Then  $P_1\# = 2, P_2\# = 2 \times 3 = 6, P_3\# = 2 \times 3 \times 5 = 30, P_4\# = 2 \times 3 \times 5 \times 7 = 210, P_5\# = 2 \times 3 \times 5 \times 7 \times 11 = 2310, P_6\# = 2 \times 3 \times 5 \times 7 \times 11 \times 13 = 30030$ , etc. English mathematician John Horton Conway coined the term 'jumping champion' in 1993. An integer  $n$  is a 'jumping champion' if  $n$  is the most frequently occurring difference (prime gap) between consecutive  $\mathbf{P} < x$  for some  $x$  integer values. Example: for any  $x$  with  $7 < x < 131$ ,  $n = 2$  (indicating twin  $\mathbf{P}$ ) is the 'jumping champion'. It has been conjectured that (i) the only 'jumping champions' are 1, 4 and primorials 2, 6, 30, 210, 2310, 30030,... and (ii) 'jumping champions' tend to infinity. Their required proofs will likely need proof of  $k$ -tuple conjecture.  $\mathbf{P}$  from 'jumping champion' prime gaps have their onset appearances in  $\mathbf{P}$  sequence in a *perpetual albeit Incompletely Predictable manner* [as another example to that outlined in previous paragraph].

### 3 Information-Complexity conservation

A formula, as equation or algorithm, is simply a Black Box generating necessary Output (with qualitative structural 'Complexity') when supplied with given Input (with quantitative

data 'Information'). This 'Information' and 'Complexity' are what is referred to in the term 'Information-Complexity conservation'.

**N (CIS):** 1, 2, 3, ..., +∞. Let  $x$  be from Set **X** such that  $x \in \mathbf{N}$ . Consider  $x$  for the upper boundary of interest in Set **X** whereby **X** is chosen from **N, E, O, P** or **C**.

**Lemma 3.1.** Natural counting function  $\mathbf{N}-\pi(x)$ , defined as  $|\mathbf{N} \leq x|$ , is Completely Predictable by independently using simple algorithm to be equal to  $x$ .

**Proof** Formula to generate **N** with 100% certainty is  $\mathbf{N}_i = i$  whereby  $\mathbf{N}_i$  is the  $i^{\text{th}}$  **N** and  $i = 1, 2, 3, \dots, \infty$ . For a given  $\mathbf{N}_i$ , its  $i^{\text{th}}$  position is simply  $i$ . Natural gap ( $G_{\mathbf{N}_i}$ ) =  $\mathbf{N}_{i+1} - \mathbf{N}_i$ , with  $G_{\mathbf{N}_i}$  always = 1. There are  $x \mathbf{N} \leq x$ . Thus  $\mathbf{N}-\pi(x) = |\mathbf{N} \leq x| = x$ . *The proof is now complete for Lemma 3.1* □.

**Lemma 3.2.** Even counting function  $\mathbf{E}-\pi(x)$ , defined as  $|\mathbf{E} \leq x|$ , is Completely Predictable by independently using simple algorithm to be equal to  $\text{floor}(x/2)$ .

**Proof.** Formula to generate **E** with 100% certainty is  $\mathbf{E}_i = iX2$  whereby  $\mathbf{E}_i$  is the  $i^{\text{th}}$  **E** and  $i = 1, 2, 3, \dots, \infty$  abiding to mathematical label "All **N** always ending with a digit 0, 2, 4, 6 or 8". For a given  $\mathbf{E}_i$ , its  $i^{\text{th}}$  position is calculated as  $i = \mathbf{E}_i/2$ . Even gap ( $G_{\mathbf{E}_i}$ ) =  $\mathbf{E}_{i+1} - \mathbf{E}_i$ , with  $G_{\mathbf{E}_i}$  always = 2. There are  $\lfloor \frac{x}{2} \rfloor \mathbf{E} \leq x$ . Thus  $\mathbf{E}-\pi(x) = |\mathbf{E} \leq x| = \text{floor}(x/2)$ . *The proof is now complete for Lemma 3.2* □.

**Lemma 3.3.** Odd counting function  $\mathbf{O}-\pi(x)$ , defined as  $|\mathbf{O} \leq x|$ , is Completely Predictable by independently using simple algorithm to be equal to  $\text{ceiling}(x/2)$ .

**Proof.** Formula to generate **O** with 100% certainty is  $\mathbf{O}_i = (iX2) - 1$  whereby  $\mathbf{O}_i$  is the  $i^{\text{th}}$  odd number and  $i = 1, 2, 3, \dots, \infty$  abiding to mathematical label "All **N** always ending with a digit 1, 3, 5, 7, or 9". For a given  $\mathbf{O}_i$  number, its  $i^{\text{th}}$  position is calculated as  $i = (\mathbf{O}_i + 1)/2$ . Odd gap ( $G_{\mathbf{O}_i}$ ) =  $\mathbf{O}_{i+1} - \mathbf{O}_i$ , with  $G_{\mathbf{O}_i}$  always = 2. There are  $\lceil \frac{x}{2} \rceil \mathbf{O} \leq x$ . Thus  $\mathbf{O}-\pi(x) = |\mathbf{O} \leq x| = \text{ceiling}(x/2)$ . *The proof is now complete for Lemma 3.3* □.

**Lemma 3.4.** Prime counting function  $\mathbf{P}-\pi(x)$ , defined as  $|\mathbf{P} \leq x|$ , is Incompletely Predictable with Set **P** dependently obtained using complex algorithm Sieve of Eratosthenes.

**Proof.** Algorithm to generate  $\mathbf{P}_i$  whereby  $\mathbf{P}_1 (= 2), \mathbf{P}_2 (= 3), \mathbf{P}_3 (= 5), \mathbf{P}_4 (= 7), \dots, \infty$  with 100% certainty is based on Sieve of Eratosthenes abiding to mathematical label "All **N** apart from 1 that are evenly divisible by itself and by 1". Although we can check primality of a given **O** by trial division, we can never determine its position without knowing positions of preceding **P**. Prime gap ( $G_{\mathbf{P}_i}$ ) =  $\mathbf{P}_{i+1} - \mathbf{P}_i$ , with  $G_{\mathbf{P}_i}$  constituted by all **E** except 1<sup>st</sup>  $G_{\mathbf{P}_1} = 3 - 2 = 1$ .  $\mathbf{P}-\pi(x) = |\mathbf{P} \leq x|$ . This is Incompletely Predictable and is calculated via mentioned algorithm. Using definition of prime gap, every **P** [represented here with aid of 'n' notation instead of usual 'i' notation] is written as  $\mathbf{P}_{n+1} = 2 + \sum_{i=1}^n G_{\mathbf{P}_i}$  with '2' denoting  $\mathbf{P}_1$ . Here  $i$  &  $n = 1, 2, 3, 4, 5, \dots, \infty$ . *The proof is now complete for Lemma 3.4* □.

**Lemma 3.5.** Composite counting function  $\mathbf{C}-\pi(x)$ , defined as  $|\mathbf{C} \leq x|$ , is Incompletely Predictable with Set **C** derived as Set **N** - Set **P** [dependently obtained using complex algorithm Sieve of Eratosthenes] - Number '1'.

**Proof.** Composite numbers abide to mathematical label "All **N** apart from 1 that are evenly divisible by numbers other than itself and 1". Algorithm to generate  $\mathbf{C}_i$  whereby  $\mathbf{C}_1 (= 4), \mathbf{C}_2 (= 6), \mathbf{C}_3 (= 8), \mathbf{C}_4 (= 9), \dots, \infty$  with 100% certainty is based [indirectly] on Sieve of Eratosthenes via selecting non-prime **N** to be **C**. We define Composite gap  $G_{\mathbf{C}_i}$  as  $\mathbf{C}_{i+1} - \mathbf{C}_i$  with  $G_{\mathbf{C}_i}$  constituted by 1 & 2.  $\mathbf{C}-\pi(x) = \mathbf{C} \leq x$ . This is Incompletely Predictable and always need to be calculated indirectly via mentioned algorithm. Using definition of composite gap, every **C** [represented here with aid of 'n' notation instead usual 'i' notation] is written as

$C_{n+1} = 4 + \sum_{i=1}^n G_{C_i}$  with '4' denoting  $C_1$ . Here  $i$  &  $n = 1, 2, 3, 4, 5, \dots, \infty$ . *The proof is now complete for Lemma 3.5*□.

Denote  $\mathbf{X}$  to be  $\mathbf{N}, \mathbf{E}, \mathbf{O}, \mathbf{P}$  or  $\mathbf{C}$ .  $X-\pi(x) = |\mathbf{X} \leq x|$  with  $x \in \mathbf{N}$ . We define and compute entity 'Grand-Total Gaps for  $\mathbf{X}$  at  $x$ ' (Grand-Total  $\Sigma X_x$ -Gaps).

**Proposition 3.6.** For any given  $x \geq 1$  values in Set  $\mathbf{N}$ , designated Complexity is represented by  $\Sigma \mathbf{N}_x$ -Gaps =  $x - \mathbf{N}$  with  $\mathbf{N} = 1$  being maximal.

**Proof.** Set  $\mathbf{N}$  (for  $x = 1$  to 12): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.  $\mathbf{N}-\pi(x) = 12$ . There are  $x - 1 = 11$   $\mathbf{N}$ -Gaps each of '1' magnitude: 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1.  $\Sigma \mathbf{N}_x$ -Gaps =  $11 \times 1 = 11$ . This equates to " $x - 1$ " – regarded as Complexity for  $\mathbf{N}$ . *The proof is now complete for Proposition 3.6*□.

**Proposition 3.7.** For any given  $x \geq 1$  values in constituent Set  $\mathbf{E}$  and Set  $\mathbf{O}$ , designated Complexity is represented by  $\Sigma \mathbf{EO}_x$ -Gaps =  $2x - \mathbf{N}$  with  $\mathbf{N} = 4$  being maximal.

**Proof.** Set  $\mathbf{E}$  and Set  $\mathbf{O}$  (for  $x = 1$  to 12): 2, 4, 6, 8, 10, 12 and 1, 3, 5, 7, 9, 11.  $\mathbf{E}-\pi(x) = 6$  and  $\mathbf{O}-\pi(x) = 6$ . There are  $\lfloor \frac{x}{2} \rfloor - 1 = 5$   $\mathbf{E}$ -Gaps each of '2' magnitude: 2, 2, 2, 2, 2.  $\Sigma \mathbf{E}_x$ -Gaps =  $5 \times 2 = 10$ , and  $\lfloor \frac{x}{2} \rfloor - 1 = 5$   $\mathbf{O}$ -Gaps each of '2' magnitude: 2, 2, 2, 2, 2.  $\Sigma \mathbf{O}_x$ -Gaps =  $5 \times 2 = 10$ . Grand-Total  $\Sigma \mathbf{EO}_x$ -Gaps =  $10 + 10 = 20$ . Depicted by Table 3 and Figure 2 in Appendix I,  $2x - \mathbf{N} = "2x - 4"$  [perpetual constant appearances of " $\mathbf{N} = 4$  being maximal"] is Complexity for  $\mathbf{E}$  and  $\mathbf{O}$ . *The proof is now complete for Proposition 3.7*□.

**Proposition 3.8.** For selected  $x \geq 2$  values in constituent Set  $\mathbf{P}$  and Set  $\mathbf{C}$ , designated Complexity is cyclically represented by  $\Sigma \mathbf{PC}_x$ -Gaps =  $2x - \mathbf{N}$  with  $\mathbf{N} = 7$  being minimal.

**Proof.** Set  $\mathbf{P}$  and Set  $\mathbf{C}$  (for  $x = 2$  to 12): 2, 3, 5, 7, 11 and 4, 6, 8, 9, 10, 12.  $\mathbf{P}-\pi(x) = 5$  and  $\mathbf{C}-\pi(x) = 6$ . There are four  $\mathbf{P}$ -Gaps of 1, 2, 2, 4 magnitude and five  $\mathbf{C}$ -Gaps of 2, 2, 1, 1, 2 magnitude.  $\Sigma \mathbf{P}_x$ -Gaps =  $1 + 2 + 2 + 4 = 9$ .  $\Sigma \mathbf{C}_x$ -Gaps =  $2 + 2 + 1 + 1 + 2 = 8$ . Grand-Total  $\Sigma \mathbf{PC}_x$ -Gaps =  $9 + 8 = 17$ . Depicted by Table 2 and Figure 1,  $2x - \mathbf{N} = "2x - 7"$  [perpetual intermittent and cyclical appearances of " $\mathbf{N} = 7$  being minimal"] is Complexity for  $\mathbf{P}$  and  $\mathbf{C}$ . *The proof is now complete for Proposition 3.8*□.

Designated Complexity is (i)  $x - \mathbf{N}$  with  $\mathbf{N} = 1$  (maximal) for Completely Predictable  $\mathbf{N}$ , (ii)  $2x - \mathbf{N}$  with  $\mathbf{N} = 7$  (minimal) for Incompletely Predictable  $\mathbf{P}$  &  $\mathbf{C}$ , and (iii)  $2x - \mathbf{N}$  with  $\mathbf{N} = 4$  (maximal) for Completely Predictable  $\mathbf{E}$  &  $\mathbf{O}$ . Interpretations:  $\mathbf{N}$  has minimal Complexity,  $\mathbf{E}$  &  $\mathbf{O}$  have intermediate Complexity, and  $\mathbf{P}$  &  $\mathbf{C}$  have maximal [varying] Complexity. Defacto baseline " $2x - 4$ " Grand-Total Gaps [minus 4 value] in  $\mathbf{E}-\mathbf{O}$  pairing > Defacto baseline " $2x - \geq 7$ " Grand-Total Gaps [minus  $\geq 7$  values] in  $\mathbf{P}-\mathbf{C}$  pairing.

Let both  $x$  &  $\mathbf{N} \in \mathbf{N}$ . We tabulate in Table 2 and graph in Figure 1 [Incompletely Predictable]  $\mathbf{P}-\mathbf{C}$  mathematical landscape for a relatively larger  $x = 2$  to 64 here (and ditto for [Completely Predictable]  $\mathbf{E}-\mathbf{O}$  mathematical landscape for relatively larger  $x = 1$  to 64 in Appendix I). The term "mathematical landscape" denotes specific mathematical patterns in tabulated and graphed data. "Dimension" contextually denotes Dimension  $2x - \mathbf{N}$  whereby (i) allocated [infinite]  $\mathbf{N}$  values result in Dimensions  $2x - 7, 2x - 8, 2x - 9, \dots, 2x - \infty$  for  $\mathbf{P}-\mathbf{C}$  finite scale mathematical landscape and (ii) allocated [finite]  $\mathbf{N}$  values for  $\mathbf{E}-\mathbf{O}$  finite scale mathematical landscape result in Dimension  $2x - 4$ . For  $\mathbf{P}-\mathbf{C}$  pairing, initial one-off Dimensions  $2x - 2, 2x - 4$  and  $2x - 5$  (in consecutive order) are exceptions [with Dimension  $2x - 2$  validly representing Number '1' which is neither  $\mathbf{P}$  nor  $\mathbf{C}$ ]. For  $\mathbf{E}-\mathbf{O}$  pairing, initial one-off Dimension  $2x - 2$  is an exception.  $\mathbf{P}-\mathbf{C}$  mathematical landscape consisting of Dimensions will intrinsically incorporate  $\mathbf{P}$  and  $\mathbf{C}$  in an integrated manner and there are infinite times whereby relevant Dimensions deviate away from 'baseline' Dimension  $2x - 7$  simply because  $\mathbf{P}$  [and, by default,  $\mathbf{C}$ ] in totality are rigorously proven to be infinite in magnitude. In contrast, there is complete lack of deviation away from 'baseline' Dimension  $2x - 4$  apart from one-off deviation by initial Dimension  $2x - 2$  in Appendix I.

**Table 2** Prime-Composite finite scale mathematical (tabulated) landscape using data obtained for  $x = 2$  to 64. The Number '1' is neither prime nor composite. Legend: C = composite, P = prime, Y = Dimension  $2x - 7$  (for visual clarity), N/A = Not Applicable.

x	P <sub>i</sub> or C <sub>i</sub> , Gaps	ΣPC <sub>x</sub> -Gaps	Dimension	x	P <sub>i</sub> or C <sub>i</sub> , Gaps	ΣPC <sub>x</sub> -Gaps	Dimension
1	N/A	0	2x-2	33	C21, 1	58	2x-8
2	P1, 1	0	2x-4	34	C22, 1	59	2x-9
3	P2, 2	1	2x-5	35	C23, 1	60	2x-10
4	C1, 2	1	Y	36	C24, 2	61	2x-11
5	P3, 2	3	Y	37	P12, 4	67	Y
6	C2, 2	5	Y	38	C25, 1	69	Y
7	P4, 4	7	Y	39	C26, 1	70	2x-8
8	C3, 1	9	Y	40	C27, 1	71	2x-9
9	C4, 1	10	2x-8	41	P13, 2	75	Y
10	C5, 2	11	2x-9	42	C28, 2	77	Y
11	P5, 2	15	Y	43	P14, 4	79	Y
12	C6, 2	17	Y	44	C29, 1	81	Y
13	P6, 4	19	Y	45	C30, 1	82	2x-8
14	C7, 1	21	Y	46	C31, 2	83	2x-9
15	C8, 1	22	2x-8	47	P15, 6	87	Y
16	C9, 1	23	2x-9	48	C32, 1	89	Y
17	P7, 2	27	Y	49	C33, 1	90	2x-8
18	C10, 2	29	Y	50	C34, 1	91	2x-9
19	P8, 4	31	Y	51	C35, 1	92	2x-10
20	C11, 1	33	Y	52	C36, 1	93	2x-11
21	C12, 1	34	2x-8	53	P16, 6	99	Y
22	C13, 2	35	2x-9	54	C37, 1	101	Y
23	P9, 6	39	Y	55	C38, 1	102	2x-8
24	C14, 1	41	Y	56	C39, 1	103	2x-9
25	C15, 1	42	2x-8	57	C40, 1	104	2x-10
26	C16, 1	43	2x-9	58	C41, 1	105	2x-11
27	C17, 1	44	2x-10	59	P17, 2	111	Y
28	C18, 2	45	2x-11	60	C42, 2	113	Y
29	P10, 2	51	Y	61	P18, 6	115	Y
30	C19, 2	53	Y	62	C43, 1	117	Y
31	P11, 6	55	Y	63	C44, 1	118	2x-8
32	C20, 1	57	Y	64	C45, 1	119	2x-9

In Figure 1, Dimensions  $2x - 7$ ,  $2x - 8$ ,  $2x - 9$ , ...,  $2x - \infty$  are symbolically represented by  $-7$ ,  $-8$ ,  $-9$ , ...,  $\infty$  with  $2x - 7$  displayed as 'baseline' Dimension whereby Dimension trend (Cumulative Sum Gaps) must repeatedly reset itself onto this 'baseline' Dimension on a perpetual basis. Dimensions symbolically represented by ever larger negative integers will correspond to **P** associated with ever larger prime gaps and this phenomenon will generally happen at ever larger  $x$  values (with complete presence of Chaos and Fractals being manifested in our graph). At ever larger  $x$  values,  $\mathbf{P}-\pi(x)$  will overall become larger but with a *decelerating* trend whereas  $\mathbf{C}-\pi(x)$  will overall become larger but with an *accelerating* trend. This support ever larger prime gaps appearing at ever larger  $x$  values.

Definitive derivation of data in Table 2 is illustrated by two examples for position  $x = 31$  & 32. For  $i$  &  $x \in \mathbf{N}$ ;  $\Sigma\text{PC}_x\text{-Gap} = \Sigma\text{PC}_{x-1}\text{-Gap} + \text{Gap value at } \mathbf{P}_{i-1} \text{ or Gap value at } \mathbf{C}_{i-1}$  whereby (i)  $\mathbf{P}_i$  or  $\mathbf{C}_i$  at position  $x$  is determined by whether relevant  $x$  value belongs to a **P** or **C**, and (ii) both  $\Sigma\text{PC}_1\text{-Gap}$  and  $\Sigma\text{PC}_2\text{-Gap} = 0$ . Example, for position  $x = 31$ : 31 is **P** (**P11**). Desired Gap value at **P10** = 2. Thus  $\Sigma\text{PC}_{31}\text{-Gap} (55) = \Sigma\text{PC}_{30}\text{-Gap} (53) + \text{Gap value at } \mathbf{P10} (2)$ . Example, for position  $x = 32$ : 32 is **C** (**C20**). Desired Gap value at **C19** = 2. Thus  $\Sigma\text{PC}_{32}\text{-Gap} (57) = \Sigma\text{PC}_{31}\text{-Gap} (55) + \text{Gap value at } \mathbf{C20} (2)$ . 'Overall magnitude of



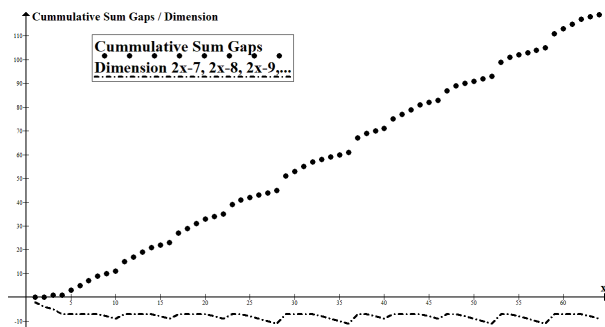


Fig. 1 Prime-Composite finite scale mathematical (graphed) landscape using data obtained for  $x = 2$  to 64. Bottom graph symbolically represent "Dimensions" using ever larger negative integers.

C will always be greater than that of P' will hold true from  $x = 14$  onwards. For instance, position  $x = 61$  corresponds to P 61 which is 18<sup>th</sup> P, whereas [the one lower] position  $x = 60$  corresponding to C 60 is the [much higher] 42<sup>nd</sup> C.

#### 4 Polignac's and Twin prime conjectures

Previous section alludes to P-C finite scale mathematical landscape. This section alludes to P-C infinite scale mathematical landscape. Let 'Y' symbolizes (baseline) Dimension  $2x - 7$ . Let prime gap at  $P_i = P_{i+1} - P_i$  with  $P_i$  &  $P_{i+1}$  respectively symbolizes consecutive "first" & "second" P in any  $P_i$ - $P_{i+1}$  pairings. We denote (i) Dimensions YY grouping [depicted by  $2x - 7$  initially appearing twice in (iii)] to represent signal for appearances of P pairings other than twin P such as cousin P, sexy P, etc; (ii) Dimension YYYYY grouping to represent signal for appearances of P pairings as twin P; and (iii) Dimension  $(2x - \geq 7)$ -Progressive-Grouping allocated to  $2x - 7, 2x - 7, 2x - 8, 2x - 9, 2x - 10, 2x - 11, \dots, 2x - \infty$  as elements of precise and proportionate CFS Dimensions representation of an individual  $P_i$  with its associated prime gap namely, Dimensions  $2x - 7$  &  $2x - 7$  pairing = twin P (with both its prime gap & CFS cardinality = 2);  $2x - 7, 2x - 7, 2x - 8$  &  $2x - 9$  pairing = cousin P (with both its prime gap & CFS cardinality = 4);  $2x - 7, 2x - 7, 2x - 8, 2x - 9, 2x - 10$  &  $2x - 11$  pairing = sexy P (with both its prime gap & CFS cardinality = 6); and so on. The higher order [traditionally defined as closest possible] prime groupings of three P as P triplets, of four P numbers as prime quadruplets, of five P numbers as prime quintuplets, etc consist of relevant serendipitous groupings abiding to mathematical rule: With exception of three 'outlier' P 3, 5, & 7; groupings of any three P as P, P+2, P+4 combination (viz. manifesting two consecutive twin P) is a mathematical impossibility. The 'anomaly' one of every three consecutive O is a multiple of three, and hence this particular number cannot be P, explains this impossibility. Then closest possible P grouping [viz. for prime triplet] must be either P, P+2, P+6 format or P, P+4, P+6 format.

P groupings not respecting traditional closest-possible-prime groupings are also the norm occurring infinitely often, indicating continual presence of prime gaps  $\geq 6$ . As P become sparser at larger range, perpetual presence of (i) prime gaps  $\geq 6$  [which we propose to arbitrarily represent 'large gaps'] and (ii) prime gaps 2 & 4 [which we propose to arbitrarily represent 'small gaps'] with progressive greater magnitude will cumulatively occur for each prime gap but always in a decelerating manner. With permanent requirement at larger range of intermittently resetting to baseline Dimension  $2x - 7$  occurring [either two

or] four times in a row, nature seems to dictate, at the very least, perpetual twin **P** or one other non-twin **P** occurrences is inevitable.

We dissect Dimension **YYYY** unique signal for twin **P** appearances. Initial two CFS Dimensions **YY** components of **YYYY** represent "first" **P** component of twin **P** pairing. Last two Dimensions **YY** components of **YYYY** signifying appearance of "second" **P** component of twin **P** pairing is also the initial first-two-element component of full CFS Dimensions representation for "first" **P** component of following non-twin **P** pairing. Twin **P** are uniquely represented by repeating *single* type Dimension  $2x - 7$ . In all other 'higher order' **P** pairings (with prime gaps  $\geq 4$ ), they require *multiple* types Dimension representation. There is qualitative aspect association of *single* type Dimension representation for twin **P** resulting in "less colorful" Plus Gap 2 Composite Number *Continuous Law* as opposed to *multiple* types Dimension representation for all other 'higher order' **P** pairings resulting in "more colorful" Plus-Minus Gap 2 Composite Number *Alternating Law*. 'Gap 2 Composite Number' occurrences in both Laws on finite scale are (directly) observed in Figure 1 & Table 2 for  $x = 2$  to 64, and on infinite scale are (indirectly) deduced using logical arguments for all  $x$  values.

We endow all "Dimensions" with exponent of '1' for perusal in on-going mathematical arguments.  $P_1 = 2$  is represented by CFS as Dimension  $(2x - 4)^1$  (with both prime gap & CFS cardinality = 1);  $P_2 = 3$  is represented by CFS as Dimensions  $(2x - 5)^1$  &  $(2x - 7)^1$  (with both prime gap & CFS cardinality = 2);  $P_3 = 5$  is represented by CFS Dimension  $(2x - 7)^1$  &  $(2x - 7)^1$  (with both prime gap & CFS cardinality = 2), etc.

**Proposition 4.1.** Let Case 1 be Completely Predictable **E** & **O** pairing and Case 2 be Incompletely Predictable **P** & **C** pairing. Furthermore, let Case 1 and Case 2 be independent of each other. Then for any given  $x$  value, there exist grand total number of Dimensions [Complexity] such that it exactly equal to either two combined subtotal number of Dimensions [Complexity] to precisely represent **E** & **O** in Case 1, or combined subtotal number of Dimensions [Complexity] to precisely represent **P** & **C** & Number '1' in Case 2.

**Proof.** **N** is directly constituted from either combined **E** & **O** in Case 1 or combined **P** & **C** & Number '1' in Case 2 – Number '1' is neither **P** nor **C**. Correctly designated infinitely many CFS of Dimensions used to represent combined **E** & **O** in Case 1 and combined **P** & **C** & Number '1' in Case 2 must also directly and proportionately be representative of relevant **N** arising from combined subtotal of **E** & **O** in Case 1 and from combined subtotal of **P** & **C** & Number '1' in Case 2. *The proof is now complete for Proposition 4.1*  $\square$ .

**Proposition 4.2.** Let Case 1 be Completely Predictable **E** & **O** pairing and Case 2 be Incompletely Predictable **P** & **C** pairing. Furthermore, let Case 1 and Case 2 be independent of each other. Part I: For any given  $x$  value apart from  $x = 1$  value in Case 1 and  $x = 1, 2,$  and 3 values in Case 2; Dimension  $(2x - N)^1$  [Complexity] representations of all Completely Predictable **E** & **O** in Case 1 and all Incompletely Predictable **P** & **C** & Number '1' in Case 2 are such that they are given by  $N = 4$  in Case 1 and by  $N \geq 7$  in Case 2. Part II: Odd **P** obeys 'Plus-Minus Composite Gap 2 Number Alternating Law' for prime gaps  $\geq 4$  and 'Plus Composite Gap 2 Number Continuous Law' for prime gap = 2.

**Proof.** Apart from first Dimension  $(2x - 2)^1$  representation in **E** & **O** pairing in Case 1 and first three Dimension  $(2x - 2)^1$ , Dimension  $(2x - 4)^1$  and Dimension  $(2x - 5)^1$  representations in **P** & **C** pairing in Case 2; possible  $N$  value in Dimension  $(2x - N)^1$  representation has been shown to be (constantly) maximal 4 for Case 1 and (variably) minimal 7 for Case 2. For Case 2, we again note Dimension  $(2x - 2)^1$  to (validly) represent Number '1' which is neither **P** nor **C**. These nominated Dimensions simply represent possible (constant) baseline "2x - 4" Grand-Total Gaps as per Proposition 3.7 for Case 1 & (variable) baseline "2x - 7" Grand-Total Gaps as per Proposition 3.8 for Case 2. Note that all CFS of Dimensions that can be used to precisely represent combined **E** & **O** in Case 1 will persistently consist

of same [solitary] Dimension  $(2x - 4)^1$  after first Dimension  $(2x - 2)^1$ . Perpetual repeated deviation of  $N$  values away from  $N = 7$  (minimum) in Case 2 is simply representing infinite magnitude of  $\mathbf{P}$  &  $\mathbf{C}$ . *The proof is now complete for Part I of Proposition 4.2*□.

Derived Dimensions will comply with Incompletely Predictable property as explained using  $\mathbf{P}$  '61'. At Position  $x = 61$  equating to  $\mathbf{P}_{18} = 61$ , it is represented by CFS Dimensions  $(2x - 7)^1, (2x - 7)^1, (2x - 8)^1, (2x - 9)^1, (2x - 10)^1$  &  $(2x - 11)^1$  (with both prime gap & CFS cardinality = 6). This representation indicates an "unknown but correct"  $\mathbf{P}$  with prime gap = 6 when we intentionally conceal full information '61' =  $31^{st}$   $\mathbf{O} = 18^{th}$   $\mathbf{P}$  with prime gap = 6. But to arrive at this representation requires calculations of all preceding CFS Dimensions thus manifesting hallmark Incompletely Predictable property of CFS Dimensions.

Overall sum total of individual CFS Dimensions required to represent every  $\mathbf{P}$  is infinite in magnitude as  $|\text{all } \mathbf{P}| = \aleph_0$ . Standalone Dimensions  $\mathbf{YY}$  groupings [representing signals for "higher order" non-twin  $\mathbf{P}$  appearances] &/or as front Dimensions  $\mathbf{YY}$  (sub)groupings [which by itself is fully representative of twin  $\mathbf{P}$  as Dimensions  $\mathbf{YYYY}$  appearances] need to recur on an indefinite basis. Then twin  $\mathbf{P}$  and "higher order" cousin  $\mathbf{P}$ , sexy  $\mathbf{P}$ , etc should aesthetically all be infinite in magnitude because (respectively) they regularly and universally arise as part of Dimension  $\mathbf{YYYY}$  and Dimension  $\mathbf{YY}$  appearances. An isolated  $\mathbf{P}$  is defined as a  $\mathbf{P}$  such that neither  $\mathbf{P} - 2$  nor  $\mathbf{P} + 2$  is  $\mathbf{P}$ . In other words, isolated  $\mathbf{P}$  is not part of a twin  $\mathbf{P}$  pair. Example 23 is an isolated  $\mathbf{P}$  since 21 and 25 are both  $\mathbf{C}$ . Then repeated inevitable presence of Dimension  $\mathbf{YY}$  grouping is nothing more than indicating repeated occurrences of isolated  $\mathbf{P}$ . This constitutes another view on Dimension  $\mathbf{YY}$ .

CIS of Gap 1 Composite Numbers are fully associated with non-twin  $\mathbf{P}$  as they eternally occur in between any two consecutive non-twin  $\mathbf{P}$ . CIS of Gap 2 Composite Numbers are (i) fully associated with twin  $\mathbf{P}$  as they are eternally present in between any twin  $\mathbf{P}$  pair, and (ii) partially associated with non-twin  $\mathbf{P}$  as they are eternally present alternatingly or intermittently in between any two consecutive non-twin  $\mathbf{P}$ . Then (i) Gap 1 Composite Numbers do not have valid representation by  $\mathbf{E}$  prime gap = 2, and (ii) Gap 2 Composite Numbers have valid representations by all  $\mathbf{E}$  prime gaps = ["consistently" only for] 2, ["inconsistently" for each of] 4, 6, 8, 10,.... This is an alternative view on  $\mathbf{P}$  from perspective of CFS composite gaps [instead of CIS prime gaps] with intrinsic patterns having *alternating presence and absence* of Gap 2 Composite Numbers associated with every CFS Dimensions representations of  $\mathbf{P}$  with prime gaps  $\geq 4$ , viz. 'Plus-Minus Gap 2 Composite Number Alternating Law'. CFS Dimensions representations of Twin  $\mathbf{P}$  are always associated with Gap 2 Composite Numbers, viz. 'Plus Gap 2 Composite Number Continuous Law'.

Examples for both Laws: A twin  $\mathbf{P}$  (prime gap = 2) in its unique CFS Dimensions format always has Gap 2 Composite Numbers in a [constant] pattern. A cousin  $\mathbf{P}$  (prime gap = 4) in its unique CFS Dimensions format always has two Gap 1 Composite Numbers & then one Gap 2 Composite Number [combined] pattern *alternating* with three consecutive Gap 1 Composite Numbers [non-combined] pattern. From this simple observation alone, we deduce we can generate an infinite magnitude of  $\mathbf{C}$  from each composite gaps 1 & 2. Gap 2 Composite Numbers *alternating* pattern behavior in cousin  $\mathbf{P}$  will not hold true unless twin  $\mathbf{P}$  & all other non-cousin  $\mathbf{P}$  are infinite in magnitude and integratedly supplying essential "driving mechanism" to eternally sustain this Gap 2 Composite Numbers *alternating* pattern behavior in cousin  $\mathbf{P}$ . Thus we establish twin  $\mathbf{P}$  and cousin  $\mathbf{P}$  in their CFS Dimensions formats are CIS intertwined together when depicted using  $\mathbf{C}$  with composite gaps = 1 & 2 with each supplying their own peculiar (infinite) share of associated Gap 2 Composite Numbers [thus contributing to overall pool of Gap 2 Composite Numbers].

An inevitable statement in relation to "Gap 2 Composite Numbers pool contribution" based on above reasoning: At the bare minimum, *either* twin  $\mathbf{P}$  *or* at least one of non-twin

$\mathbf{P}$  must be infinite in magnitude. An inevitable impression: All generated subsets of  $\mathbf{P}$  from 'small gaps' [of 2 & 4] and 'large gaps' [of  $\geq 6$ ] alike should each be CIS thus allowing true uniformity in  $\mathbf{P}$  distribution. Again we see in Table 2 above depicting  $\mathbf{P}\text{-C}$  data for  $x = 2$  to 64 that, for instance,  $\mathbf{P}$  with prime gap = 6 must also persistently have this 'last-place' Gap 2 Composite Numbers intermittently appearing in certain rhythmic *alternating* patterns, thus complying with Plus-Minus Gap 2 Composite Number Alternating Law. This CFS Dimensions representation for  $\mathbf{P}$  with prime gaps = 6 will again generate their infinite share of associated Gap 2 Composite Numbers to contribute to this pool. The presence of this last-place Gap 2 Composite Numbers in various alternating pattern in appearances & non-appearances must *self-generatingly* be similarly extended in a mathematically consistent fashion *ad infinitum* to all other remaining infinite number of prime gaps [which were not discussed in details above]. *The proof is now complete for Part II of Proposition 4.2*□.

## 5 Rigorous Proofs now named as Polignac's and Twin prime hypotheses

The proofs on lemmas and propositions from previous section supply all necessary evidences to fully support Theorem Polignac-Twin prime I to IV below thus depicting proofs for Polignac's and Twin prime conjectures in a rigorous manner. Gap 1 Composite Numbers do not have valid representation by  $\mathbf{E}$  prime gap = 2, and Gap 2 Composite Numbers have valid representations by all  $\mathbf{E}$  prime gaps = [“consistently” only for] 2, [“inconsistently” for each of] 4, 6, 8, 10,.... Plus-Minus Gap 2 Composite Number Alternating Law confirms that Gap 2 Composite Numbers present in each  $\mathbf{P}$  with prime gaps  $\geq 4$  situation must appear as some sort of “rhythmic patterns of alternating presence and absence” for Gap 2 Composite Numbers. Twin  $\mathbf{P}$  with prime gap = 2 obeying Plus Gap 2 Composite Number Continuous Law can be understood as special situation of “(non-)rhythmic patterns with continual presence” for relevant Gap 2 Composite Numbers.

In 1849 when French mathematician Alphonse de Polignac (1826 - 1863) was admitted to Polytechnique, he made what is known as Polignac's conjecture which relates complete set of odd  $\mathbf{P}$  to all  $\mathbf{E}$  prime gaps. Twin prime conjecture, which relates twin prime numbers to prime gap = 2, is nothing more than a subset of Polignac's conjecture.

**Theorem Polignac-Twin prime I.** Incompletely Predictable prime numbers  $\mathbf{P}_n = 2, 3, 5, 7, 11, \dots, \infty$  or composite numbers  $\mathbf{C}_n = 4, 6, 8, 9, 10, \dots, \infty$  are CIS with overall actual location [but not actual positions] of all prime or composite numbers accurately represented by complex algorithm involving prime gaps  $G_{P_i}$  viz.  $\mathbf{P}_{n+1} = 2 + \sum_{i=1}^n G_{P_i}$  or involving composite

gaps  $G_{C_i}$  viz.  $\mathbf{C}_{n+1} = 4 + \sum_{i=1}^n G_{C_i}$  whereby prime & composite numbers are symbolically represented here with aid of 'n' notation instead of usual 'i' notation; and  $i$  &  $n = 1, 2, 3, 4, 5, \dots, \infty$ . Number '2' in first algorithm represents  $\mathbf{P}_1$ , the very first (and only even)  $\mathbf{P}$ . Number '4' in second algorithm represent  $\mathbf{C}_1$ , the very first (and even)  $\mathbf{C}$ .

**Proof.** We treat above algorithms as unique mathematical objects looking for key intrinsic properties and behaviors. Each  $\mathbf{P}$  or  $\mathbf{C}$  is assigned a unique prime or composite gap. Absolute number of  $\mathbf{P}$  or  $\mathbf{C}$  and (thus) prime or composite gaps are infinite in magnitude. As original formulae containing all  $\mathbf{P}$  or  $\mathbf{C}$  by themselves (viz. without supplying prime or composite gaps as “input information” to generate  $\mathbf{P}$  or  $\mathbf{C}$  as “output complexity”), these algorithms intrinsically incorporate overall actual location [but not actual positions] of all  $\mathbf{P}$  or  $\mathbf{C}$ . *The proof is now complete for Theorem Polignac-Twin prime I*□.

**Theorem Polignac-Twin prime II.** Set of prime gaps  $G_{P_i} = 2, 4, 6, 8, 10, \dots, \infty$  is infinite in magnitude whereby these prime gaps accurately and completely represented by Dimensions  $(2x - 7)^1, (2x - 8)^1, (2x - 9)^1, \dots, (2x - \infty)^1$  must satisfy Information-Complexity conservation in a consistent manner.

**Proof.** Part I of Proposition 4.2 proved all **P** are represented by Dimension  $(2x - N)^1$  with  $N \geq 7$  for any given  $x$  value (except for  $x = 2$  &  $3$  values). Note that although  $x = 1$  is neither **P** nor **C**, it is validly represented by Dimension  $(2x - 2)^1$ . If each **P** is endowed with a specific prime gap value, then each such prime gap must [via logical mathematical deduction] be represented by Dimension  $(2x - N)^1$ . We advocate this nominated method of prime gap representation using Dimensions be [purportedly] the only way to achieve Information-Complexity conservation. The preceding mathematical statements are correct as there is a unique prime gap value associated with each **P**. Proposition 5.1 below based on principles from Set theory provides further supporting materials that prime gaps are infinite in magnitude. *The proof is now complete for Theorem Polignac-Twin prime II*□.

**Theorem Polignac-Twin prime III.** To maintain Dimensional analysis (DA) homogeneity, those Dimensions  $(2x - N)^1$  from Theorem Polignac-Twin prime II must contain eternal repetitions of well-ordered sets constituted by Dimensions  $(2x - 7)^1, (2x - 8)^1, (2x - 9)^1, (2x - 10)^1, (2x - 11)^1, \dots, (2x - \infty)^1$ .

**Proof.** This Theorem is stated in greater details as "To maintain DA homogeneity, those aforementioned [endowed with exponent 1] Dimensions  $(2x - N)^1$  from Theorem Polignac-Twin prime II must repeat themselves indefinitely in following specific combinations – (i) Dimension  $(2x - 7)^1$  only appearing as twin [two-times-in-a-row] and quadruplet [four-times-in-a-row] sequences, and (ii) Dimensions  $(2x - 8)^1, (2x - 9)^1, (2x - 10)^1, (2x - 11)^1, \dots, (2x - \infty)^1$  appearing as progressive groupings of **E** 2, 4, 6, 8, 10, ...,  $\infty$ ." To accommodate the only even **P** '2', exceptions to this DA homogeneity compliance will expectedly occur right at beginning of **P** sequence – (i) one-off appearance of Dimensions  $(2x - 2)^1, (2x - 4)^1$  and  $(2x - 5)^1$  and (ii) one-off appearance of Dimension  $(2x - 7)^1$  as a quintuplet [five-times-in-a-row] sequence which is equivalent to (eternal) non-appearance of Dimension  $(2x - 6)^1$  at  $x = 4$ . [We again note Dimension  $(2x - 2)^1$  validly represent Number '1' which is neither **P** nor **C**.] These sequentially arranged sets are CFS whereby from  $x = 11$  onwards, each set always commence initially as 'baseline' Dimension  $(2x - 7)^1$  at  $x = \mathbf{O}$  values and always end with its last Dimension at  $x = \mathbf{E}$  values. Each set also have varying cardinality with values derived from all **E**; and correctly combined sets always intrinsically generate two infinite sets of **P** and, by default, **C** in an integrated manner. Our Theorem Polignac-Twin prime III simply represent a mathematical summary derived from Section 3 & 4 of all expressed characteristics of Dimension  $(2x - N)^1$  when used to represent **P** with intrinsic display of DA homogeneity. See Proposition 5.2 below for further details on DA aspect. *The proof is now complete for Theorem Polignac-Twin prime III*□.

**Theorem Polignac-Twin prime IV.** Aspect 1. The "quantitative" aspect to existence of both prime gaps and their associated prime numbers as sets of infinite magnitude will be shown to be correct by utilizing principles from Set theory. Aspect 2. The "qualitative" aspect to existence of both prime gaps and their associated prime numbers as sets of infinite magnitude will be shown to be correct by 'Plus-Minus Gap 2 Composite Number Alternating Law' and 'Plus Gap 2 Composite Number Continuous Law'.

**Proof.** Required concepts from Set theory involve cardinality of a set with its 'well-ordering principle' application. Supporting materials for these concepts based on 'pigeon-hole principle' in relation to Aspect 1 are outlined in Proposition 5.1 below. 'Plus-Minus Gap 2 Composite Number Alternating Law' is applicable to all **E** prime gaps [apart from first **E** prime gap = 2 for twin primes]. The prime gap = 2 situation will obey 'Plus Gap 2

Composite Number Continuous Law'. These Laws are essentially Laws of Continuity inferring underlying intrinsic driving mechanisms that enables infinity magnitude association for both prime gaps & prime numbers to co-exist. By the same token, these Laws have the important implication that they must be applicable to those relevant prime gaps on an perpetual time scale. Supporting materials in relation to Aspect 2 are found in Proposition 4.2 above. *The proof is now complete for Theorem Polignac-Twin prime IV*□.

We note two mutually inclusive conditions: Condition 1. Presence of all Dimensions that repeat themselves on an indefinite basis and with exponent of '1' will give rise to complete sets of **P** & **C** ["DA-wise one & only one mathematical possibility argument" associated with inevitable *de novo* DA homogeneity], and Condition 2. Presence of any Dimension(s) that do not repeat itself (themselves) on an indefinite basis or with exponent other than '1' will give rise to incomplete set of **P** & **C** or incorrect set of non-**P** & non-**C** ["DA-wise mathematical impossibility argument" associated with inevitable *de novo* DA non-homogeneity]. When met, these two conditions will fully support the point that CFS Dimensions representations of **P** & **C** [with respective prime & composite gaps] are totally accurate. Condition 1 reflect proof from Theorem Polignac-Twin prime III above as all **P** & **C** are associated with DA homogeneity when their Dimensions are endowed with exponent of '1'. Condition 2 invoke corollary on inevitable appearance of incomplete **P** or **C** or non-**P** or non-**C** [associated with DA non-homogeneity] being tightly incorporated into this mathematical framework. See Propositions 5.1 and 5.2, and Corollary 5.3 below for supporting materials on DA homogeneity & non-homogeneity.

We analyze **P** (& **C**) in terms of (i) measurements based on cardinality of CIS and (ii) pigeonhole principle which states that if  $n$  items are put into  $m$  containers, with  $n > m$ , then at least one container must contain more than one item. We note that ordinality of all infinite **P** (& **C**) is "fixed" implying that each one of the infinite well-ordered Dimension sets conforming to CFS type as constituted by Dimensions  $(2x - 7)^1, (2x - 8)^1, (2x - 9)^1, (2x - 10)^1, (2x - 11)^1, \dots, (2x - \infty)^1$  on respective gaps for **P** (& **C**) must also be "fixed".

**Proposition 5.1.** "Even number prime gaps are infinite in magnitude with each even number prime gap generating odd prime numbers which are again infinite in magnitude" is supported by principles from Set theory and two Laws based on Gap 2 Composite Number.

**Proof.** We validly exclude even **P** '2' here. Let (i) cardinality  $T = \aleph_0$  for Set **all odd P** derived from **E** prime gaps 2, 4, 6, ...,  $\infty$ , (ii) cardinality  $T_2 = \aleph_0$  for Subset **odd P** derived from **E** prime gap 2, cardinality  $T_4 = \aleph_0$  for Subset **odd P** derived from **E** prime gap 4, cardinality  $T_6 = \aleph_0$  for Subset **odd P** derived from **E** prime gap 6, etc. Paradoxically  $T = T_2 + T_4 + T_6 + \dots + T_\infty$  equation is valid despite  $T = T_2 = T_4 = T_6 = \dots = T_\infty$  [with well-ordering principle "stating that every non-empty set of positive integers contains a least element" fulfilled by each (sub)set]. But if Subset **odd P** derived from one or more **E** prime gap(s) are finite in magnitude, this will breach  $\aleph_0$  'uniformity' resulting in (i) DA non-homogeneity and (ii) inequality  $T > T_2 + T_4 + T_6 + \dots + T_\infty$ . In language of pigeonhole principle "stating that if  $n$  items are put into  $m$  containers with  $n > m$ , then at least one container must contain more than one item", residual **odd P** (still CIS in magnitude) not accounted for by CFS-type **E** prime gap(s) will have to be [incorrectly] contained in one (or more) of composite gap(s). These arguments using cardinality constitute proof that (i) **E** prime gaps and (ii) odd **P** generated from each **E** prime gap, must all be CIS. *The proof [on "quantitative" aspect] is now complete for Proposition 5.1*□.

Complete set of **P** is represented by Dimensions  $(2x - N)^1$ . Table 2 & Figure 1 on **PC** finite scale mathematical landscape depict perpetual repeating features used in "qualitative" statements supporting (i) Plus-Minus Gap 2 Composite Number Alternating Law (stated as **C** with composite gaps = 2 present in each of **P** with prime gaps  $\geq 4$  situation must be

observed to appear as some sort of rhythmic patterns of alternating presence and absence of this type of **C**), and (ii) Plus Gap 2 Composite Number Continuous Law (stated as **C** with composite gaps = 2 continual appearances in each of (twin) **P** with prime gap = 2 situation). Plus-Minus Gap 2 Composite Number Alternating Law has intrinsic mechanism to automatically generate all prime gaps  $\geq 4$  in a mathematically consistent *ad infinitum* manner. Plus Gap 2 Composite Number Continuous Law has built-in intrinsic mechanism to further generate prime gap = 2 appearances in a mathematically consistent *ad infinitum* manner. *The proof [on "qualitative" aspect] is now complete for Proposition 5.1*□.

**Proposition 5.2.** The presence of Dimensional analysis homogeneity will always result in correct and complete set of prime (and composite) numbers.

**Proof.** DA homogeneity is completely dependent on all Dimensions being consistently endowed with exponent '1'. As all **P** (& **C**) are "fixed", we deduce from Figure 1 & Table 2 that there is one (& only one) way to represent Information-Complexity conservation using our defined Dimensions. Thus, there is one (& only one) way to depict all **P** (& **C**) using these Dimensions in a self-consistent manner and this is achieved with the one (& only one) DA homogeneity possibility. *The proof is now complete for Proposition 5.2*□.

**Corollary 5.3.** The presence of Dimensional analysis non-homogeneity will always result in incorrect and/or incomplete set of prime (and composite) numbers.

**Proof.** For optimal clarity, we endow all Dimensions with exponent '1' depicted as  $(2x - 7)^1, (2x - 8)^1, (2x - 9)^1, (2x - 10)^1, (2x - 11)^1, \dots, (2x - \infty)^1$ . Proposition 5.2 equates DA homogeneity with correct & complete set of **P** (& **C**). There are "more than one" DA possibilities when, for instance, a particular [first] term from  $(2x - 7)^0, (2x - 8)^1, (2x - 9)^1, \dots, (2x - \infty)^1$  "terminates" prematurely and does not perpetually repeat [with loss of continuity]. There are intuitively two 'broad' DA possibilities here; namely, (one) DA homogeneity possibility and (one) DA non-homogeneity possibility – Dimension  $(2x - 7)^0$  [= 1] with its exponent arbitrarily set as '0' against-all-trend in this case. Thus Dimension  $(2x - 7)^1$  that stop recurring at some point in **P** (or **C**) sequence may cause well-ordered CFS sets from progressive groupings of [**E**] 2, 4, 6, 8, 10, ...,  $\infty$  for Dimensions  $(2x - 8)^1, (2x - 9)^1, (2x - 10)^1, (2x - 11)^1, \dots, (2x - \infty)^1$  to stop existing (and ultimately for sequential **P** (or **C**) to stop appearing) at that point with ensuing outcome that **P** (or **C**) may overall be incorrectly finite or incomplete in magnitude. Finally also manifesting DA non-homogeneity, a Dimension endowed with fractional exponent values other than '1' such as ' $\frac{2}{5}$ ' or ' $\frac{3}{5}$ ' will result in non-**P** (or non-**C**) [fractional] numbers. *The proof is now complete for Corollary 5.3*□.

Each [fixed] finite scale mathematical landscape "page" as part of [fixed] infinite scale mathematical landscape "pages" for **P** & **C** display Chaos [sensitivity to initial conditions viz. positions of subsequent **P** & **C** are "sensitive" to positions of initial **P** & **C**] and Fractals [manifesting fractal dimensions with self-similarity viz. those aforementioned Dimensions for **P** & **C** are always present, albeit in non-identical manner, for all ranges of  $x \geq 2$ ]. Advocated in another manner, Chaos and Fractals phenomena of those Dimensions for **P** & **C** are always present signifying accurate composition of **P** & **C** in different [predetermined] finite scale mathematical landscape "(snapshot) pages" for **P** & **C** that are self-similar but never identical – and there are an infinite number of these finite scale mathematical landscape "(snapshot) pages". The crucial mathematical step in representing all **P** (& **C**) and prime (& composite) gaps with "Dimensions" based on Information-Complexity conservation allows us to obtain the two Laws based on Gap 2 Composite Numbers and perform DA on these entities. The 'strong' principle argument is DA homogeneity equates to complete set of **P** (& **C**) whereas DA non-homogeneity does not equate to complete set of **P** (& **C**). We could also advocate for a 'weak' principle argument supporting DA homogeneity for **P** (& **C**) in that nature should not "favor" any particular Dimension(s) to terminate and therefore DA

non-homogeneity does not, and cannot, exist for **P** (& **C**). Abiding to our advocated convention that 'conjecture' be termed 'hypothesis' once proven; we now call Polignac's & Twin prime conjectures as Polignac's & Twin prime hypotheses.

## 6 Conclusions

Harnessed property: CIS of [Completely Predictable] natural numbers 1, 2, 3, 4, 5, 6, 7,... having CIS of [Completely Predictable] natural gaps 1, 1, 1, 1, 1, 1,... are constituted by three dependent sets of numbers: (i) CIS of [Incompletely Predictable] odd prime numbers 3, 5, 7, 11, 13, 17,... having CIS of [Incompletely Predictable] prime gaps 2, 2, 4, 2, 4,... plus CFS of solitary [Incompletely Predictable] even prime number 2 having CFS of [Incompletely Predictable] prime gap 1 (ii) CIS of [Incompletely Predictable] even and odd composite numbers 4, 6, 8, 9, 10, 12,... having CIS of [Incompletely Predictable] composite gaps 2, 2, 1, 1, 2, 2,... and (iii) CFS of solitary odd number '1' [neither prime nor composite]. Treated as Incompletely Predictable problems endowed with "meta-properties", we gave relatively elementary proofs on Polignac's and Twin prime conjectures based on this harnessed property by performing Dimensional analysis on (sub)sets and "Dimensions" of prime and composite numbers, and obtaining 'Plus-Minus Gap 2 Composite Number Alternating Law' and 'Plus Gap 2 Composite Number Continuous Law'.

Prime number theorem describes asymptotic distribution of prime numbers among positive integers by formalizing intuitive idea that prime numbers become less common as they become larger through precisely quantifying rate at which this occurs using probability. Nontrivial zeros [from 'Axes intercept relationship interface' relevant to Riemann hypothesis] and prime numbers [from 'Numerical relationship interface' relevant to prime number theorem] are Incompletely Predictable entities and numbers. Deep-seated connections exist between Riemann hypothesis and prime number theorem (which is fully delineated by prime counting function [denoted here with  $\pi(x)$ ]). Solving Incompletely Predictable problem Riemann hypothesis is instrumental in proving efficacy of techniques that estimate  $\pi(x)$  efficiently. This should now confirm "best possible" bound for error ("smallest possible" error) of prime number theorem.

In mathematics, logarithmic integral function or integral logarithm  $\text{li}(x)$  is a special function. Relevant to problems of physics and with number theoretic significance, it occurs in prime number theorem as an estimate of  $\pi(x)$  whereby the form of this special function is defined so that  $\text{li}(2) = 0$ ; viz.  $\text{li}(x) \equiv \int_2^x \frac{du}{\ln u} = \text{li}(x) - \text{li}(2)$ . There are less accurate ways of estimating  $\pi(x)$  such as conjectured by Gauss and Legendre at end of 18th century. This is approximately  $x/\ln x$  in the sense  $\lim_{x \rightarrow \infty} \frac{\pi(x)}{x/\ln x} = 1$ . Skewes' number is any of several extremely large numbers used by South African mathematician Stanley Skewes as upper bounds for smallest natural number  $x$  for which  $\text{li}(x) < \pi(x)$ . These bounds have since been improved by others: there is a crossing near  $e^{727.95133}$  but it is not known whether this is the smallest. John Edensor Littlewood, who was Skewes' research supervisor, proved in 1914[4] that there is such a [first] number; and found that sign of difference  $\pi(x) - \text{li}(x)$  changes infinitely often. This refute all prior numerical evidence that seem to suggest  $\text{li}(x)$  was always more than  $\pi(x)$ . The key point is [100% accurate]  $\pi(x)$  mathematical tool being "wrapped around" by [less-than-100% accurate] approximate mathematical tool  $\text{li}(x)$  infinitely often via this 'sign of difference' changes meant that  $\text{li}(x)$  is the most efficient approximate mathematical tool. Contrast this with "crude"  $x/\ln x$  approximate mathematical tool where values



**Table 3** Even-Odd mathematical (tabulated) landscape using data obtained for  $x = 1$  to 64. Legend: **E**=even, **O**=odd, **Y**=Dimension  $2x-4$ .

x	E <sub>i</sub> or O <sub>i</sub> , Gaps	ΣEO <sub>x</sub> -Gaps	Dimension	x	E <sub>i</sub> or O <sub>i</sub> , Gaps	ΣEO <sub>x</sub> -Gaps	Dimension
1	O1, 2	0	2x-2	33	O17, 2	62	Y
2	E1, 2	0	Y	34	O17, 2	64	Y
3	O2, 2	2	Y	35	O17, 2	66	Y
4	E2, 2	4	Y	36	O17, 2	68	Y
5	O3, 2	6	Y	37	O17, 2	70	Y
6	E3, 2	8	Y	38	O17, 2	72	Y
7	O4, 2	10	Y	39	O17, 2	74	Y
8	E4, 2	12	Y	40	O17, 2	76	Y
9	O5, 2	14	Y	41	O17, 2	78	Y
10	E5, 2	16	Y	42	O17, 2	80	Y
11	O6, 2	18	Y	43	O17, 2	82	Y
12	E6, 2	20	Y	44	O17, 2	84	Y
13	O7, 2	22	Y	45	O17, 2	86	Y
14	E7, 2	24	Y	46	O17, 2	88	Y
15	O8, 2	26	Y	47	O17, 2	90	Y
16	E8, 2	28	Y	48	O17, 2	92	Y
17	O9, 2	30	Y	49	O17, 2	94	Y
18	E9, 2	32	Y	50	O17, 2	96	Y
19	O10, 2	34	Y	51	O17, 2	98	Y
20	E10, 2	36	Y	52	O17, 2	100	Y
21	O11, 2	38	Y	53	O17, 2	102	Y
22	E11, 2	40	Y	54	O17, 2	104	Y
23	O12, 2	42	Y	55	O17, 2	106	Y
24	E12, 2	44	Y	56	O17, 2	108	Y
25	O13, 2	46	Y	57	O17, 2	110	Y
26	E13, 2	48	Y	58	O17, 2	112	Y
27	O14, 2	50	Y	59	O17, 2	114	Y
28	E14, 2	52	Y	60	O17, 2	116	Y
29	O15, 2	54	Y	61	O17, 2	118	Y
30	E15, 2	56	Y	62	O17, 2	120	Y
31	O16, 2	58	Y	63	O17, 2	122	Y
32	E16, 2	60	Y	64	O17, 2	124	Y

obtained diverge away from  $\pi(x)$  at increasingly greater rate when larger range of prime numbers are studied.

**Appendix I: Tabulated and graphical data on Even-Odd mathematical landscape**

We tabulate (in Table 3) and graph (in Figure 2) [Completely Predictable] **E-O** mathematical landscape for  $x = 1$  to 64. Involved Dimensions are  $2x - 2$  &  $2x - 4$  with **Y** denoting Dimension  $2x - 4$  for visual clarity. This mathematical landscape of Dimension  $2x - 4$  (except for first and only Dimension  $2x - 2$ ) will intrinsically incorporate **E** & **O** in an integrated manner. Except for first **O**, all Completely Predictable **E** & **O** and all their associated gaps are represented by countable finite set of [single] Dimension  $2x - 4$ . Dimensions  $2x - 2$  &  $2x - 4$  are symbolically represented by  $-2$  &  $-4$  with  $2x - 4$  displayed as 'baseline' Dimension whereby Dimension trend (Cumulative Sum Gaps) must reset itself onto this (Grand-Total Gaps) 'baseline' Dimension after initial Dimension  $2x - 2$  on a permanent basis. Graphical appearances of Dimensions symbolically represented by two negative integers are Completely Predictable with both Even- $\pi(x)$  and Odd- $\pi(x)$  becoming larger at a constant rate. There is a complete absence of Chaos and Fractals phenomena.

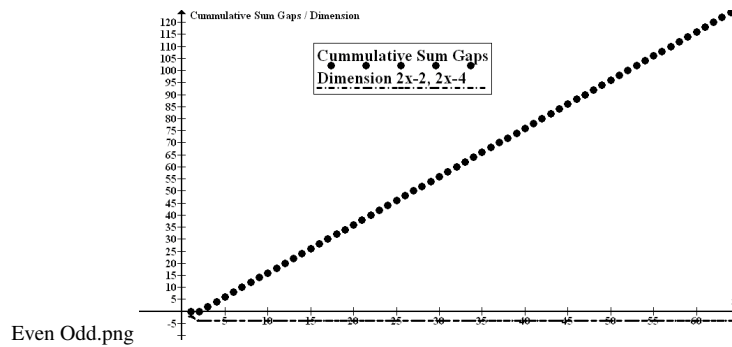


Fig. 2 Even-Odd mathematical (graphed) landscape using data obtained for  $x = 1$  to 64.

Definitive derivation of data in Table 3 is illustrated by two examples for position  $x = 31$  & 32. For  $i$  &  $x \in 1, 2, 3, \dots, \infty$ ;  $\Sigma EO_x\text{-Gap} = \Sigma EO_{x-1}\text{-Gap} + \text{Gap value at } E_{i-1} \text{ or Gap value at } O_{i-1}$  whereby (i)  $E_i$  or  $O_i$  at position  $x$  is determined by whether relevant  $x$  value belongs to **E** or **O**, and (ii) both  $\Sigma EO_1\text{-Gap}$  and  $\Sigma EO_2\text{-Gap} = 0$ . Example, for position  $x = 31$ : 31 is **O** (O16). Our desired Gap value at O15 = 2. Thus  $\Sigma EO_{31}\text{-Gap}$  (58) =  $\Sigma EO_{30}\text{-Gap}$  (56) + Gap value at O15 (2). Example, for position  $x = 32$ : 32 is **E** (E16). Our desired Gap value at E15 = 2. Thus  $\Sigma EO_{32}\text{-Gap}$  (60) =  $\Sigma EO_{31}\text{-Gap}$  (58) + Gap value at E15 (2).

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## References

1. Furstenberg, H. (1955). On the infinitude of primes. *Amer. Math. Monthly*, 62, (5) 353, <http://dx.doi.org/10.2307/2307043>
2. Saidak, F. (2006), A New Proof of Euclid's theorem, *Amer. Math. Monthly*, 113, (10) 937, <http://dx.doi.org/10.2307/27642094>
3. Zhang, Y. (2014), Bounded gaps between primes, *Ann. Math.* 179(3) (2014) 1121 – 1174, <http://dx.doi.org/10.4007/annals.2014.179.3.7>
4. Littlewood, J. E. (1914), Sur la distribution des nombres premiers. *Comptes Rendus de l'Acad. Sci. Paris*, 158, 1869 – 1872