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Solving Incompletely Predictable problems Polignac's and Twin Prime conjectures with research method Information-Complexity conservation

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Abstract Prime numbers are Incompletely Predictable numbers calculated using complex algorithm Sieve of Eratosthenes. Involving proposals that prime gaps and associated sets of prime numbers are infinite in magnitude, Twin prime conjecture deals with even prime gap 2 and is a subset of Polignac's conjecture which deals with all even prime gaps 2, 4, 6, 8, 10,.... Treated as Incompletely Predictable problems, we solve these conjectures as Plus Gap 2 Composite Number Continuous Law and Plus-Minus Gap 2 Composite Number Alternating Law obtained using novel research method Information-Complexity conservation.

Keywords Dimensional analysis · Incompletely Predictable problems · Information-Complexity conservation · Plus Gap 2 Composite Number Continuous Law · Plus-Minus Gap 2 Composite Number Alternating Law · Polignac's and Twin prime conjectures

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1 Introduction

Uncountable complex numbers ($\mathbb{C}$) include uncountable real numbers (**R**). Uncountable **R** = countable rational numbers (**Q**) + uncountable irrational numbers (**R – Q**). Uncountable **R – Q** = countable algebraic numbers + uncountable transcendental numbers. Countable **Q** include countable integers (**Z**) which include countable whole numbers (**W**) which in turn include countable natural numbers (**N**). Countable **N** is constituted by either countable even numbers (**E**) and countable odd numbers (**O**) or countable prime numbers (**P**), countable composite numbers (**C**) and Number '1'. Then (i) Set **N** = Set **E** + Set **O**, (ii) Set **N** = Set **P** + Set **C** + Number '1', and (iii) Set **N** $\subset$ Set **W** $\subset$ Set **Z** $\subset$ Set **Q** $\subset$ Set **R** $\subset$ Set $\mathbb{C}$.

In order of increasing magnitude, arbitrary Set **X** belongs to countable finite set (CFS), countable infinite set (CIS) or uncountable infinite set (UIS). Cardinality of Set **X**, |**X**|, measures the "number of elements" in Set **X**. E.g. Set **even P** has CFS of even **P** with |**even P**| = 1, Set **N** has CIS of **N** with |**N**| = $\aleph_0$, and Set **R** has UIS of **R** with |**R**| = $\mathfrak{c}$ (cardinality of the continuum). Respectively, CIS of **P** and **C** are *Incompletely Predictable*

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numbers dependently calculated directly and indirectly from *complex algorithm* Sieve of Eratosthenes. Involving proposals that prime gaps and associated sets of prime numbers are infinite in magnitude, Twin prime conjecture deals with even prime gap 2 and is a subset of Polignac's conjecture dealing with all even prime gaps 2, 4, 6, 8, 10,.... Activities to prove these open problems in number theory equate to solving *Incompletely Predictable problems*.

All claims arising from these activities are made meaningful by providing definitions on above mentioned terms. Respectively, an Incompletely (Completely) Predictable number is locationally defined as a number whose position is *dependently* (*independently*) determined by complex (simple) calculations using complex (simple) equation or algorithm with (without) needing to know related positions of all preceding numbers in its neighborhood. Simple properties are inferred from a phrase such as: "...simple equation or algorithm by itself will intrinsically incorporate actual location [and actual positions] of all Completely Predictable numbers". Solving Completely Predictable problems with simple properties amendable to *simple* treatments using *usual* mathematical tools such as Calculus will result in their 'Simple Elementary Fundamental Laws'-based solutions. Complex properties are inferred from a phrase such as: "...complex equation or algorithm by itself will intrinsically incorporate actual location [but not actual positions] of all Incompletely Predictable numbers". Solving Incompletely Predictable problems with complex properties amendable to *complex* treatments using *unusual* mathematical tools such as our novel research method Information-Complexity conservation as well as using *usual* mathematical tools such as Calculus will result in their 'Complex Elementary Fundamental Laws'-based solutions.

1.1 Dimensional analysis on Cardinality and "Dimensions"

For 'base quantities' such as *length*, *mass* and *time*; their fundamental SI 'units of measurement' are [respectively] given by meter (m), kilogram (kg) and second (s). The word 'dimension' is commonly used to denote 'units of measurement' in well-defined equations. Dimensional analysis (DA) is an analytic tool with resulting DA homogeneity and non-homogeneity (respectively) denoting valid and invalid equation when 'units of measurements' are "balanced" and "unbalanced" across both sides of the equation. E.g. 2 m + 3 m = 5 m is a valid equation but 2 m + 3 kg = 5 mkg is an invalid equation.

We use the word "Dimensions" to denote well-defined Incompletely Predictable entities "Dimensions" obtained from Information-Complexity conservation. Relevant "Dimensions" *dependently* represent Number '1', **P** and **C**. Then *by default* any (sub)sets of **P** and **C** in well-defined equations can also be represented by their corresponding "Dimensions".

Remark 1.1. We can apply Dimensional analysis to "Dimensions" from Information-Complexity conservation and cardinality of relevant sets in certain well-defined equations.

Let **X** denote **E**, **O**, **N** [which are classified as Completely Predictable numbers], **P** and **C** [which are classified as Incompletely Predictable numbers]. For x = 1, 2, 3, 4, 5,..., ∞; consider all **X** $\leq$ x. Then this "all **X** $\leq$ x" is the definition for **X**-$\pi(x)$ [denoting "**X** counting function"] resulting in the following two types of equations coined as (I) 'Exact' equation **N**-$\pi(x)$ = **E**-$\pi(x)$ + **O**-$\pi(x)$ with "non-varying" relationships **E**-$\pi(x)$ = **O**-$\pi(x)$ for all x = **E** and **E**-$\pi(x)$ = **O**-$\pi(x)$ - 1 for all x = **O**, and (II) 'Inexact' equation **N**-$\pi(x)$ = 1 + **P**-$\pi(x)$ + **C**-$\pi(x)$ with "varying" relationships **P**-$\pi(x) >$ **C**-$\pi(x)$ for all x $\leq$ 8; **P**-$\pi(x)$ = **C**-$\pi(x)$ for x = 9, 11, and 13; and **P**-$\pi(x) <$ **C**-$\pi(x)$ for x = 10, 12, and all x $\geq$ 14.

Let "Dimensions" and different (sub)sets of **E**, **O**, **N**, **P** and **C** be 'base quantities'. Then exponent '1' of "Dimensions" and cardinality of these (sub)sets in well-defined equations are corresponding 'units of measurement'. Performing DA on "Dimensions" for **PC** pairing are depicted in later parts of this paper. Performing DA on cardinality are depicted next.

For Set **N** = Set **E** + Set **O**, then |**N**| = |**E**| + |**O**| $\implies \aleph_0 = \aleph_0 + \aleph_0$ thus conforming with DA homogeneity.

For Set **N** = Set **P** + Set **C** + Number '1', then Set **N** - Number '1' = Set **P** + Set **C** and |**N** - Number '1'| = |**P**| + |**C**| $\implies \aleph_0 = \aleph_0 + \aleph_0$ thus conforming with DA homogeneity.

For Set **N** - Set **even P** - Number '1'= Set **odd P** + Set **even C** + Set **odd C**, then |**N** - **even P** - Number '1'| = |**odd P**| + |**even C**| + |**odd C**| $\implies \aleph_0 = \aleph_0 + \aleph_0 + \aleph_0$ thus conforming with DA homogeneity. Symbolically represented by all available **O** prime gap = 1 and **E** prime gaps = 2, 4, 6, 8, 10,...; **O** composite gap = 1 and **E** composite gap = 2; and **O** natural gap = 1; then |**Gap 1 N** - **Gap 1 P** - Number '1'| = |**Gap 2 P**| + |**Gap 4 P**| + |**Gap 6 P**| + |**Gap 8 P**| + |**Gap 10 P**| + ... + |**Gap 1 C**| + |**Gap 2 C**| $\implies \aleph_0 = \aleph_0 + \aleph_0 + \aleph_0 + \aleph_0 + \aleph_0 + ... \aleph_0 + \aleph_0$ thus conforming with DA homogeneity. It is known that |**Gap 1 P**| = |Number '1'| = 1 and |**Gap 1 N**| = |**Gap 1 C**| = |**Gap 2 C**| = $\aleph_0$. Then solving Polignac's and Twin prime conjectures translate to successfully proving |**Gap 2 P**| = |**Gap 4 P**| = |**Gap 6 P**| = |**Gap 8 P**| = |**Gap 10 P**| = ... = $\aleph_0$ with |**E prime gaps**| = $\aleph_0$.

Outline of proof for Polignac's and Twin prime conjectures. Requires simultaneously satisfying two mutually inclusive conditions: I. *With rigid manifestation of DA homogeneity*, quantitive[1] fulfillment by considering i $\in$ **E** for each Subset **odd P**$_i$ generated by **E** prime gap = i from Set **E prime gaps** occurs only if solitary cardinality value is present in equation Set **odd P** = $\sum_{i=2}^{\infty}$Subset **odd P**$_i$ with |**odd P**| = |**odd P**$_i$| = |**E prime gaps**| = $\aleph_0$, and II. *With rigid manifestation of DA non-homogeneity*, quantitive[1] fulfillment by considering i $\in$ **E** for each Subset **odd P**$_i$ generated by **E** prime gap = i from Set **E prime gaps** does not occur if more than one cardinality values are present in equation Set **odd P** $> \sum_{i=2}^{\infty}$Subset **odd P**$_i$ with |**E prime gaps**| = $\aleph_0$ having incorrect |Subset(s) **odd P**| = N (finite value) and/or Set **odd P** $> \sum_{i=2}^{N}$Subset **odd P**$_i$ with |**odd P**$_i$| = $\aleph_0$ having incorrect |**E prime gaps**| = N (finite value).

Footnote 1: Qualitative fulfillment of |**odd P**| = |**odd P**$_i$| = |**all E prime gaps**| = $\aleph_0$ equates to Plus-Minus Gap 2 Composite Number Alternating Law being precisely obeyed by all **E** prime gaps apart from first **E** prime gap precisely obeying Plus Gap 2 Composite Number Continuous Law. Derived using Information-Complexity conservation, these Laws symbolize "end-result" proof on Polignac's and Twin prime conjectures. *Law of Continuity* is a heuristic principle *whatever succeed for the finite, also succeed for the infinite*. Then these Laws which inherently manifest 'Gap 2 Composite Number' on finite and infinite time scale should in principle "succeed for the finite, also succeed for the infinite".

Polignac's and Twin prime conjectures mathematical foot-prints. Six identifiable steps to prove these conjectures: *Step 1* Considering x $\in$ **N**, obtain Dimensions $(2x - 2)^1$, $(2x - 4)^1$, $(2x - 5)^1$, $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, ..., $(2x - \infty)^1$ with specific groupings to constitute all elements of Set **P** [culminating in obtaining all prime gaps (= **E** prime gaps + Solitary **O** prime gap) with |**all prime gaps**| = $\aleph_0$]. Note Dimension $(2x - 2)^1$ represents x = 1 (Number '1') which is neither **P** nor **C**. *Step 2* Considering i $\in$ **E**, confirm perpetual recurrences of individual **E** prime gap = i (associated with its unique odd **P**$_i$) occur only when depicted as specific groupings of these Dimensions endowed with exponent '1' for all ranges of x. *Step 3* Perform DA on exponent '1' in these Dimensions. *Step 4* Perform DA on equation Set **odd P** = $\sum_{i=2}^{\infty}$Subset **odd P**$_i$ to obtain |**odd P**| = |**odd P**$_i$| = $\aleph_0$ whereby Subset **odd P**$_i$

is derived from its associated unique **E** prime gap = i with |**E prime gaps**| = $\aleph_0$. *Step 5* Confirm 'Prime number' variable and 'Prime gap' variable complex algorithm "containing" all **P** with knowing their overall actual location [but not actual positions][2]. *Step 6* Derive Plus-Minus Gap 2 Composite Number Alternating Law and Plus Gap 2 Composite Number Continuous Law from formal arguments based on Information-Complexity conservation.

Footnote 2: This phrase implies all **P** (and **C**) are treated as Incompletely Predictable numbers. Actual positions will require using complex algorithm Sieve of Eratosthenes to *dependently* calculate positions of all preceding **P** (and **C**) in the neighborhood.

'Complex Elementary Fundamental Laws'-based solutions of Plus-Minus Gap 2 Composite Number Alternating Law and Plus Gap 2 Composite Number Continuous Law are obtained by undertaking certain non-negotiable mathematical steps outlined above. These Laws are literally Completely Predictable meta-properties ('overall' *complex properties*) arising from "interactions" between **P** and **C** producing relevant patterns of Gap 2 Composite Number perpetual appearances [albeit with Incompletely Predictable timing]. We logically deduce that explicit mathematical explanation for this meta-property requires "complex" mathematical arguments. Attempts to give explicit mathematical explanation with "simple" mathematical arguments would intuitively mean the Incompletely Predictable numbers **P** and **C** be (incorrectly and impossibly) treated as Completely Predictable numbers.

1.2 Brief overview of Polignac's and Twin prime conjectures

Occurring over 2000 years ago (c. 300 BC), ancient Euclid's proof on infinitude of **P** in totality [viz. |**P**| = $\aleph_0$ for Set **P**] predominantly by *reductio ad absurdum* (proof by contradiction) is earliest known but not the only proof for this simple problem in number theory. Since then dozens of proofs have been devised such as three chronologically listed: Goldbach's Proof using Fermat numbers (written in a letter to Swiss mathematician Leonhard Euler, July 1730), Furstenberg's Topological Proof in 1955[1], and Filip Saidak's Proof in 2006[2]. The strangest candidate is likely to be Furstenberg's Topological Proof.

In 2013, Yitang Zhang proved a landmark result showing some unknown even number 'N' $<$ 70 million such that there are infinitely many pairs of **P** that differ by 'N'[3]. By optimizing Zhang's bound, subsequent Polymath Project collaborative efforts using a new refinement of GPY sieve in 2013 lowered 'N' to 246; and assuming Elliott-Halberstam conjecture and its generalized form have further lower 'N' to 12 and 6, respectively. Then 'N' has intuitively more than one valid values such that there are infinitely many pairs of **P** that differ by each of those 'N' values [thus proving existence of more than one Subset **odd** $\mathbf{P}_i$ with |**odd** $\mathbf{P}_i$| = $\aleph_0$]. We can only theoretically lower 'N' to 2 (in regards to **P** with 'small gaps') but there are still an infinite number of **E** prime gaps (in regards to **P** with 'large gaps') that will require "the proof that each will generate its unique set of infinite **P**".

Remark 1.2. Existence of maximal and non-maximal prime gaps supply crucial indirect evidence to intuitively support but does not prove proposition "Each even prime gap will generate an infinite magnitude of odd prime numbers on its own accord".

Comments relevant to Remark 1.2 are given in Section 2 below.

2 Supportive role of maximal and non-maximal prime gaps

We analyze data of all **P** obtained when extrapolated out over a wide range of x $\geq$ 2 integer values. As sequence of **P** carries on, **P** with ever larger prime gaps will appear. For given

Table 1 First 17 prime gaps depicted in the format utilizing maximal prime gaps [depicted with asterisk symbol (*)] and non-maximal prime gaps [depicted without this asterisk symbol].

Prime gap	Following the prime number
1*	2
2*	3
4*	7
6*	23
8*	89
10	139
12	199
14*	113
16	1831

Prime gap	Following the prime number
18*	523
20*	887
22*	1129
24	1669
26	2477
28	2971
30	4297
32	5591

range of x integer values, prime gap = n_2 is a 'maximal prime gap' if prime gap = $n_1 <$ prime gap = n_2 for all $n_1 < n_2$. In other words, the largest such prime gaps in this range are called maximal prime gaps. The term 'first occurrence prime gaps' refers to first occurrences of maximal prime gaps whereby maximal prime gaps are prime gaps of "at least of this length".

We use maximal prime gaps to denote 'first occurrence prime gaps'. CIS non-maximal prime gaps (endorsed with nickname 'slow jumpers') will always lag behind CIS maximal prime gaps for onset appearances in **P** sequence. These are shown for first 17 prime gaps in Table 1. Apart from **O** prime gap = 1 representing solitary even **P** '2', remaining **P** depicted in Table 1 consist of representative single odd **P** for each **E** prime gap. These odd **P** will individually make one-off appearance in **P** sequence in a *perpetual albeit Incompletely Predictable manner*. Initial seven of [majority] "missing" odd **P** are 5, 11, 13, 17, 19, 29, 31,... belonging to Subset **P** with 'residual' prime gaps are potential source of odd **P** in relation to proposal that each **E** prime gap from Set **E prime gaps** will generate its specific Subset **odd P**. Set **all P** from all prime gaps = Subset **P** from maximal prime gaps + Subset **P** from non-maximal prime gaps + Subset **P** from 'residual' prime gaps. Subset **P** from 'residual' prime gaps with representation from all **E** prime gaps must include all correctly selected "missing" odd **P**. These observations support but does not prove proposition that each **E** prime gap will generate its own Subset **odd P** with |**odd P**| = $\aleph_0$.

For i ∈ **N**; primordial $P_i\#$ is analog of usual factorial for **P** = 2, 3, 5, 7, 11, 13,.... Then $P_1\#$ = 2, $P_2\#$ = 2 X 3 = 6, $P_3\#$ = 2 X 3 X 5 = 30, $P_4\#$ = 2 X 3 X 5 X 7 = 210, $P_5\#$ = 2 X 3 X 5 X 7 X 11 = 2310, $P_6\#$ = 2 X 3 X 5 X 7 X 11 X 13 = 30030, etc. English mathematician John Horton Conway coined the term 'jumping champion' in 1993. An integer n is a 'jumping champion' if n is the most frequently occurring difference (prime gap) between consecutive **P**<x for some x integer values. Example: for any x with 7<x<131, n = 2 (indicating twin **P**) is the 'jumping champion'. It has been conjectured that (i) the only 'jumping champions' are 1, 4 and primorials 2, 6, 30, 210, 2310, 30030,... and (ii) 'jumping champions' tend to infinity. Their required proofs will likely need proof of k-tuple conjecture. **P** from 'jumping champion' prime gaps have their onset appearances in **P** sequence in a *perpetual albeit Incompletely Predictable manner* [as another example to that outlined in previous paragraph].

3 Information-Complexity conservation

A formula, as equation or algorithm, is simply a Black Box generating necessary Output (with qualitative structural 'Complexity') when supplied with given Input (with quantitative data 'Information'). This 'Information' and 'Complexity' are what is referred to in the term 'Information-Complexity conservation'.

N (CIS): 1, 2, 3,..., +∞. Let x be from Set **X** such that x ∈ **N**. Consider x for the upper boundary of interest in Set **X** whereby**X** is chosen from **N**, **E**, **O**, **P** or **C**.

Lemma 3.1. The natural counting function **N**-$\pi(x)$, defined as |**N** ≤ x|, is Completely Predictable by independently using simple algorithm to be equal to x.

Proof Formula to generate **N** with 100% certainty is $\mathbf{N}_i$ = i whereby $\mathbf{N}_i$ is the i^{th} **N** and i = 1, 2, 3,..., ∞. For a given $\mathbf{N}_i$, its i^{th} position is simply i. Natural gap (G_{Ni}) = $\mathbf{N}_{i+1}$ - $\mathbf{N}_i$, with G_{Ni} always = 1. There are x **N** ≤ x. Thus **N**-$\pi(x)$ = |**N** ≤ x| = x. *The proof is now complete for Lemma 3.1*□.

Lemma 3.2. The even counting function **E**-$\pi(x)$, defined as |**E** ≤ x|, is Completely Predictable by independently using simple algorithm to be equal to floor(x/2).

Proof. Formula to generate **E** with 100% certainty is $\mathbf{E}_i$ = iX2 whereby $\mathbf{E}_i$ is the i^{th} **E** and i = 1, 2, 3,..., ∞ abiding to mathematical label "All **N** always ending with a digit 0, 2, 4, 6 or 8". For a given $\mathbf{E}_i$, its i^{th} position is calculated as i = $\mathbf{E}_i$/2. Even gap (G_{Ei}) = $\mathbf{E}_{i+1}$ - $\mathbf{E}_i$, with G_{Ei} always = 2. There are $\lfloor \frac{x}{2} \rfloor$ **E** ≤ x. Thus **E**-$\pi(x)$ = |**E** ≤ x| = floor(x/2). *The proof is now complete for Lemma 3.2*□.

Lemma 3.3. The odd counting function **O**-$\pi(x)$, defined as |**O** ≤ x|, is Completely Predictable by independently using simple algorithm to be equal to ceiling(x/2).

Proof. Formula to generate **O** with 100% certainty is $\mathbf{O}_i$ = (iX2) - 1 whereby $\mathbf{O}_i$ is the i^{th} odd number and i = 1, 2, 3,..., ∞ abiding to mathematical label "All **N** always ending with a digit 1, 3, 5, 7, or 9". For a given $\mathbf{O}_i$ number, its i^{th} position is calculated as i = ($\mathbf{O}_i$ + 1)/2. Odd gap (G_{Oi}) = $\mathbf{O}_{i+1}$ - $\mathbf{O}_i$, with G_{Oi} always = 2. There are $\lceil \frac{x}{2} \rceil$ **O** ≤ x. Thus **O**-$\pi(x)$ = |**O** ≤ x| = ceiling(x/2). *The proof is now complete for Lemma 3.3*□.

Lemma 3.4. The prime counting function **P**-$\pi(x)$, defined as |**P** ≤ x|, is Incompletely Predictable with Set **P** dependently obtained using complex algorithm Sieve of Eratosthenes.

Proof. Algorithm to generate $\mathbf{P}_i$ whereby $\mathbf{P}_1$ (= 2), $\mathbf{P}_2$ (= 3), $\mathbf{P}_3$ (= 5), $\mathbf{P}_4$ (= 7),..., ∞ with 100% certainty is based on Sieve of Eratosthenes abiding to mathematical label "All **N** apart from 1 that are evenly divisible by itself and by 1". Although we can check primality of a given **O** by trial division, we can never determine its position without knowing positions of preceding **P**. Prime gap (G_{Pi}) = $\mathbf{P}_{i+1}$ - $\mathbf{P}_i$, with G_{Pi} constituted by all **E** except 1^{st} G_{P1} = 3 - 2 = 1. **P**-$\pi(x)$ = |**P** ≤ x|. This is Incompletely Predictable and always need to be calculated via mentioned algorithm. Using definition of prime gap, every **P** [represented here with aid of 'n' notation instead of usual 'i' notation] can be written as $\mathbf{P}_{n+1} = 2 + \sum_{i=1}^{n} G_{Pi}$ with '2' denoting $\mathbf{P}_1$. Here i & n = 1, 2, 3, 4, 5, ..., ∞. *The proof is now complete for Lemma 3.4*□.

Lemma 3.5. The composite counting function **C**-$\pi(x)$, defined as |**C** ≤ x|, is Incompletely Predictable with Set **C** derived as Set **N** - Set **P** [dependently obtained using complex algorithm Sieve of Eratosthenes] - Number '1'.

Proof. Composite numbers abide to mathematical label "All **N** apart from 1 that are evenly divisible by numbers other than itself and 1". Algorithm to generate $\mathbf{C}_i$ whereby $\mathbf{C}_1$ (= 4), $\mathbf{C}_2$ (= 6), $\mathbf{C}_3$ (= 8), $\mathbf{C}_4$ (= 9),..., ∞ with 100% certainty is based [indirectly] on Sieve of Eratosthenes via selecting non-prime **N** to be **C**. We define Composite gap G_{Ci} as $\mathbf{C}_{i+1}$ - $\mathbf{C}_i$ with G_{Ci} constituted by 1 & 2. **C**-$\pi(x)$ = **C** ≤ x. This is Incompletely Predictable and always need to be calculated indirectly via mentioned algorithm. Using definition of composite gap, every **C** [represented here with aid of 'n' notation instead usual 'i' notation] can be written as $\mathbf{C}_{n+1} = 4 + \sum_{i=1}^{n} G_{Ci}$ with '4' denoting $\mathbf{C}_1$. Here i & n = 1, 2, 3, 4, 5, ..., ∞. *The proof is now complete for Lemma 3.5*□.

Denote **X** to be **N**, **E**, **O**, **P** or **C**. X-$\pi(x)$ = |**X** ≤ x| with x ∈ **N**. We define and compute entity 'Grand-Total Gaps for **X** at x' (Grand-Total ΣX_x-Gaps).

Proposition 3.6. For any given x ≥ 1 values in Set **N**, designated Complexity is represented by $\Sigma\mathbf{N}_x$-Gaps = x - N with N = 1 being maximal.

Proof. Set **N** (for x = 1 to 12): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. **N**-$\pi(x)$ = 12. There are x - 1 = 11 **N**-Gaps each of '1' magnitude: 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1. $\Sigma\mathbf{N}_x$-Gaps = 11 X 1 = 11. This equates to "x - 1" – regarded as Complexity for **N**. *The proof is now complete for Proposition 3.6*□.

Proposition 3.7. For any given x ≥ 1 values in constituent Set **E** and Set **O**, designated Complexity is represented by $\Sigma\mathbf{EO}_x$-Gaps = 2x - N with N = 4 being maximal.

Proof. Set **E** and Set **O** (for x = 1 to 12): 2, 4, 6, 8, 10, 12 and 1, 3, 5, 7, 9, 11. **E**-$\pi(x)$ = 6 and **O**-$\pi(x)$ = 6. There are $\lfloor \frac{x}{2} \rfloor$ - 1 = 5 **E**-Gaps each of '2' magnitude: 2, 2, 2, 2, 2. $\Sigma\mathbf{E}_x$-Gaps = 5 X 2 = 10, and $\lceil \frac{x}{2} \rceil$ - 1 = 5 **O**-Gaps each of '2' magnitude: 2, 2, 2, 2, 2. $\Sigma\mathbf{O}_x$-Gaps = 5 X 2 = 10. Grand-Total $\Sigma\mathbf{EO}_x$-Gaps = 10 + 10 = 20. Depicted by Table 3 and Figure 2 in Appendix I, this 2x - N = "2x - 4" [perpetual persistent appearances of "N = 4 being maximal"] is Complexity for **E** and **O**. *The proof is now complete for Proposition 3.7*□.

Proposition 3.8. For selected x ≥ 2 values in constituent Set **P** and Set **C**, designated Complexity is cyclically represented by $\Sigma\mathbf{PC}_x$-Gaps = 2x - N with N = 7 being minimal.

Proof. Set **P** and Set **C** (for x = 2 to 12): 2, 3, 5, 7, 11 and 4, 6, 8, 9, 10, 12. **P**-$\pi(x)$ = 5 and **C**-$\pi(x)$ = 6. There are four **P**-Gaps of 1, 2, 2, 4 magnitude and five **C**-Gaps of 2, 2, 1, 1, 2 magnitude. $\Sigma\mathbf{P}_x$-Gaps = 1 + 2 + 2 + 4 = 9. $\Sigma\mathbf{C}_x$-Gaps = 2 + 2 + 1 + 1 + 2 = 8. Grand-Total $\Sigma\mathbf{PC}_x$-Gaps = 9 + 8 = 17. Depicted by Table 2 and Figure 1, 2x - N = "2x - 7" [perpetual intermittent and cyclical appearances of "N = 7 being minimal"] is Complexity for **P** and **C**. *The proof is now complete for Proposition 3.8*□.

Designated Complexity is (i) x - N with N = 1 (maximal) for Completely Predictable **N**, (ii) 2x - N with N = 7 (minimal) for Incompletely Predictable **P** & **C**, and (iii) 2x - N with N = 4 (maximal) for Completely Predictable **E** & **O**. Interpretations: **N** has minimal Complexity, **E** & **O** have intermediate Complexity, and **P** & **C** have maximal [varying] Complexity. Defacto baseline "2x - 4" Grand-Total Gaps [with minus 4 value] in **E-O** pairing > Defacto baseline "2x - ≥7" Grand-Total Gaps [with minus ≥7 values] in **P-C** pairing.

Let both x & N ∈ **N**. We tabulate in Table 2 and graph in Figure 1 [Incompletely Predictable] **P-C** mathematical landscape for a relatively larger x = 2 to 64 here (and ditto for [Completely Predictable] **E-O** mathematical landscape for relatively larger x = 1 to 64 in Appendix I). The term "mathematical landscape" denotes specific mathematical patterns in tabulated and graphed data. "Dimension" contextually denotes relevant Dimension 2x - N whereby (i) allocated [infinite] N values result in Dimensions 2x - 7, 2x - 8, 2x - 9, ..., 2x - ∞ for **P-C** finite scale mathematical landscape and (ii) allocated [finite] N values for **E-O** finite scale mathematical landscape result in Dimension 2x - 4. For **P-C** pairing, initial one-off Dimensions 2x - 2, 2x - 4 and 2x - 5 (in consecutive order) are exceptions [with Dimension 2x - 2 validly representing the Number '1' which is neither **P** nor **C**]. For **E-O** pairing, initial one-off Dimension 2x - 2 is an exception. **P-C** mathematical landscape consisting of relevant Dimensions will intrinsically incorporate **P** and **C** in an integrated manner and there will be infinite times whereby relevant Dimensions will deviate away from 'baseline' Dimension 2x - 7 simply because **P** [and, by default, **C**] in totality are rigorously proven to be infinite in magnitude. In contrast, there is complete lack of deviation away from 'baseline' Dimension 2x - 4 apart from one-off deviation by initial Dimension 2x - 2 in Appendix I.

Table 2 Prime-Composite finite scale mathematical (tabulated) landscape using data obtained for x = 2 to 64. The Number '1' is neither prime nor composite. Legend: C = composite, P = prime, Y = Dimension 2x - 7 (for visual clarity), N/A = Not Applicable.

x	**P_i or C_i, Gaps**	**ΣPC_x-Gaps**	Dimension	**x**	**P_i or C_i, Gaps**	**ΣPC_x-Gaps**	Dimension
1	N/A	0	2x-2	33	C21, 1	58	2x-8
2	P1, 1	0	2x-4	34	C22, 1	59	2x-9
3	P2, 2	1	2x-5	35	C23, 1	60	2x-10
4	C1, 2	1	Y	36	C24, 2	61	2x-11
5	P3, 2	3	Y	37	P12, 4	67	Y
6	C2, 2	5	Y	38	C25, 1	69	Y
7	P4, 4	7	Y	39	C26, 1	70	2x-8
8	C3, 1	9	Y	40	C27, 1	71	2x-9
9	C4, 1	10	2x-8	41	P13, 2	75	Y
10	C5, 2	11	2x-9	42	C28, 2	77	Y
11	P5, 2	15	Y	43	P14, 4	79	Y
12	C6, 2	17	Y	44	C29, 1	81	Y
13	P6, 4	19	Y	45	C30, 1	82	2x-8
14	C7, 1	21	Y	46	C31, 2	83	2x-9
15	C8, 1	22	2x-8	47	P15, 6	87	Y
16	C9, 1	23	2x-9	48	C32, 1	89	Y
17	P7, 2	27	Y	49	C33, 1	90	2x-8
18	C10, 2	29	Y	50	C34, 1	91	2x-9
19	P8, 4	31	Y	51	C35, 1	92	2x-10
20	C11, 1	33	Y	52	C36, 1	93	2x-11
21	C12, 1	34	2x-8	53	P16, 6	99	Y
22	C13, 2	35	2x-9	54	C37, 1	101	Y
23	P9, 6	39	Y	55	C38, 1	102	2x-8
24	C14, 1	41	Y	56	C39, 1	103	2x-9
25	C15, 1	42	2x-8	57	C40, 1	104	2x-10
26	C16, 1	43	2x-9	58	C41, 1	105	2x-11
27	C17, 1	44	2x-10	59	P17, 2	111	Y
28	C18, 2	45	2x-11	60	C42, 2	113	Y
29	P10, 2	51	Y	61	P18, 6	115	Y
30	C19, 2	53	Y	62	C43, 1	117	Y
31	P11, 6	55	Y	63	C44, 1	118	2x-8
32	C20, 1	57	Y	64	C45, 1	119	2x-9

In Figure 1, Dimensions 2x - 7, 2x - 8, 2x - 9, ..., 2x - ∞ are symbolically represented by -7, -8, -9, ..., ∞ with 2x - 7 displayed as 'baseline' Dimension whereby Dimension trend (Cumulative Sum Gaps) must repeatedly reset itself onto this 'baseline' Dimension on a perpetual basis. Dimensions symbolically represented by ever larger negative integers will correspond to **P** associated with ever larger prime gaps and this phenomenon will generally happen at ever larger x values (with complete presence of Chaos and Fractals being manifested in our graph). At ever larger x values, **P**-$\pi(x)$ will overall become larger but with a *decelerating* trend whereas **C**-$\pi(x)$ will overall become larger but with an *accelerating* trend. This support ever larger prime gaps appearing at ever larger x values.

Definitive derivation of data in Table 2 is given and illustrated by two examples for position x = 31 & 32. For i & x ∈ **N**; ΣPC_x-Gap = ΣPC_{x-1}-Gap + Gap value at $\mathbf{P}_{i-1}$ or Gap value at $\mathbf{C}_{i-1}$ whereby (i) $\mathbf{P}_i$ or $\mathbf{C}_i$ at position x is determined by whether relevant x value belongs to a **P** or **C**, and (ii) both ΣPC_1-Gap and ΣPC_2-Gap = 0. Example, for position x = 31: 31 is **P** (**P11**). Desired Gap value at **P10** = 2. Thus ΣPC_{31}-Gap (55) = ΣPC_{30}-Gap (53) + Gap value at **P10** (2). Example, for position x = 32: 32 is **C** (**C20**). Desired Gap value at **C19** = 2. Thus ΣPC_{32}-Gap (57) = ΣPC_{31}-Gap (55) + Gap value at **C20** (2). 'Overall magnitude

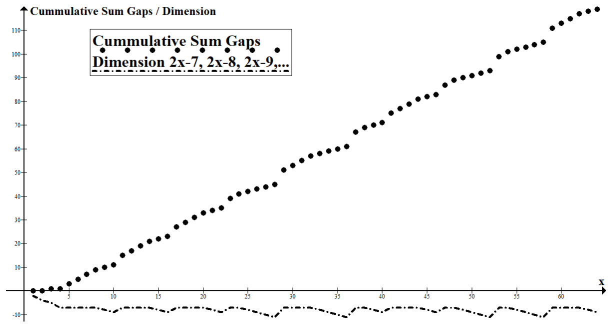


Fig. 1 Prime-Composite finite scale mathematical (graphed) landscape using data obtained for x = 2 to 64. Bottom graph symbolically represent "Dimensions" using ever larger negative integers.

of **C** will always be greater than that of **P**' will hold true from x = 14 onwards. For instance, position x = 61 corresponds to **P** 61 which is 18^{th} **P**, whereas [the one lower] position x = 60 corresponding to **C** 60 is the [much higher] 42^{nd} **C**.

4 Polignac's and Twin prime conjectures

Previous section alludes to **P-C** finite scale mathematical landscape. This section alludes to **P-C** infinite scale mathematical landscape. Let 'Y' symbolizes (baseline) Dimension 2x - 7. Let prime gap at $\mathbf{P}_i$ = $\mathbf{P}_{i+1}$ - $\mathbf{P}_i$ with $\mathbf{P}_i$ & $\mathbf{P}_{i+1}$ respectively symbolizing consecutive "first" & "second" **P** in any $\mathbf{P}_i$-$\mathbf{P}_{i+1}$ pairings. We denote (i) Dimensions YY grouping [depicted by 2x - 7 initially appearing twice in (iii)] as representing signal for appearances of **P** pairings other than twin **P** such as cousin **P**, sexy **P**, etc; (ii) Dimension YYYY grouping as representing signal for appearances of **P** pairings as twin **P**; and (iii) Dimension (2x - $\geq$7)-Progressive-Grouping allocated to 2x - 7, 2x - 7, 2x - 8, 2x - 9, 2x - 10, 2x - 11,..., 2x - ∞ as elements of *precise* and *proportionate* CFS Dimensions representation of an individual $\mathbf{P}_i$ with its associated prime gap namely, Dimensions 2x - 7 & 2x - 7 pairing = twin **P** (with both of its prime gap & CFS cardinality = 2); 2x - 7, 2x - 7, 2x - 8 & 2x - 9 pairing = cousin **P** (with both of its prime gap & CFS cardinality = 4); 2x - 7, 2x - 7, 2x - 8, 2x - 9, 2x - 10 & 2x - 11 pairing = sexy **P** (with both of its prime gap & CFS cardinality = 6); and so on. The higher order [which is traditionally defined as closest possible] prime groupings of three **P** as **P** triplets, of four **P** numbers as prime quadruplets, of five **P** numbers as prime quintuplets, etc will consist of relevant serendipitous groupings abiding to mathematical rule: With exception of three 'outlier' **P** 3, 5, & 7; groupings of any three **P** as **P**, **P**+2, **P**+4 combination (viz. manifesting two consecutive twin **P**) is a mathematical impossibility. The 'anomaly' that one of every three consecutive **O** is a multiple of three, and hence this particular number cannot be **P**, explains this mathematical impossibility. Then closest possible **P** grouping [viz. for prime triplet] must be either **P**, **P**+2, **P**+6 format or **P**, **P**+4, **P**+6 format.

P groupings not respecting traditional closest-possible-prime groupings are also the norm occurring infinitely often, indicating continual presence of prime gaps $\geq$ 6. As **P** become sparser at larger range, perpetual presence of (i) prime gaps $\geq$ 6 [which we propose to wider scientific community to arbitrarily represent 'large gaps'] and (ii) prime gaps 2 & 4 [which we propose to wider scientific community to arbitrarily represent 'small gaps'] with progressive greater magnitude will cummulatively occur for each prime gap but always in a decelerating manner. With permanent requirement at larger range of intermittently resetting to baseline Dimension 2x - 7 occurring [either two or] four times in a row, nature seems to dictate, at the very least, perpetual twin **P** or one other non-twin **P** occurrences is inevitable.

We dissect Dimension YYYY unique signal for twin **P** appearances. Initial two CFS Dimensions YY components of YYYY represent "first" **P** component of twin **P** pairing. Last two Dimensions YY components of YYYY signifying appearance of "second" **P** component of twin **P** pairing is also the initial first-two-element component of full CFS Dimensions representation for "first" **P** component of following non-twin **P** pairing. Twin **P** are uniquely represented by repeating *single* type Dimension 2x - 7. In all other 'higher order' **P** pairings (with prime gaps $\geq$ 4), they require *multiple* types Dimension representation. There is qualitative aspect association of *single* type Dimension representation for twin **P** resulting in "less colorful" Plus Gap 2 Composite Number *Continuous Law* as opposed to *multiple* types Dimension representation for all other 'higher order' **P** pairings resulting in "more colorful" Plus-Minus Gap 2 Composite Number *Alternating Law*. 'Gap 2 Composite Number' occurrences in both Laws on finite scale are (directly) observed in Figure 1 & Table 2 for x = 2 to 64, and on infinite scale are (indirectly) deduced using logical arguments for all x values.

We endow all "Dimensions" with exponent of '1' for perusal in on-going mathematical arguments. $\mathbf{P}_1$ = 2 is represented by CFS as Dimension $(2x - 4)^1$ (with both prime gap & CFS cardinality = 1); $\mathbf{P}_2$ = 3 is represented by CFS as Dimensions $(2x - 5)^1$ & $(2x - 7)^1$ (with both prime gap & CFS cardinality = 2); $\mathbf{P}_3$ = 5 is represented by CFS Dimension $(2x - 7)^1$ & $(2x - 7)^1$ (with both prime gap & CFS cardinality = 2), etc.

Proposition 4.1. Let Case 1 be Completely Predictable **E** & **O** pairing and let Case 2 be Incompletely Predictable **P** & **C** pairing. Furthermore, let Case 1 and Case 2 be independent of each other. Then for any given x value, there exist grand total number of Dimensions [Complexity] such that it exactly equal to either two combined subtotal number of Dimensions [Complexity] to precisely represent **E** & **O** in Case 1, or combined subtotal number of Dimensions [Complexity] to precisely represent **P** & **C** & Number '1' in Case 2.

Proof. **N** is directly constituted from either combined **E** & **O** in Case 1 or combined **P** & **C** & Number '1' in Case 2 – Number '1' is neither **P** nor **C**. Correctly designated infinitely many CFS of Dimensions used to represent combined **E** & **O** in Case 1 and combined **P** & **C** & Number '1' in Case 2 must also directly and proportionately be representative of relevant **N** arising from combined subtotal of **E** & **O** in Case 1 and from combined subtotal of **P** & **C** & Number '1' in Case 2. *The proof is now complete for Proposition 4.1*□.

Proposition 4.2. Let Case 1 be Completely Predictable **E** & **O** pairing and let Case 2 be Incompletely Predictable **P** & **C** pairing. Furthermore, let Case 1 and Case 2 be independent of each other. Part I: For any given x value apart from x = 1 value in Case 1 and x = 1, 2, and 3 values in Case 2; Dimension $(2x - N)^1$ [Complexity] representations of all Completely Predictable **E** & **O** in Case 1 and all Incompletely Predictable **P** & **C** & Number '1' in Case 2 are such that they are given by N = 4 in Case 1 and by N $\geq$ 7 in Case 2. Part II: Odd **P** obeys 'Plus-Minus Composite Gap 2 Number Alternating Law' for prime gaps $\geq$ 4 and 'Plus Composite Gap 2 Number Continuous Law' for prime gap = 2.

Proof. Apart from first Dimension $(2x - 2)^1$ representation in **E** & **O** pairing in Case 1 and first three Dimension $(2x - 2)^1$, Dimension $(2x - 4)^1$ and Dimension $(2x - 5)^1$ representations in **P** & **C** pairing in Case 2; possible N value in Dimension $(2x - N)^1$ representation has been shown to be (constantly) maximal 4 for Case 1 and (variably) minimal 7 for Case 2. For Case 2, we again note Dimension $(2x - 2)^1$ to (validly) represent Number '1' which is neither **P** nor **C**. These nominated Dimensions simply represent possible (constant) baseline "2x - 4" Grand-Total Gaps as per Proposition 3.7 for Case 1 & (variable) baseline "2x - 7" Grand-Total Gaps as per Proposition 3.8 for Case 2. Note that all CFS of Dimensions that can be used to precisely represent combined **E** & **O** in Case 1 will persistently consist of same [solitary] Dimension $(2x - 4)^1$ after first Dimension $(2x - 2)^1$. Perpetual repeated deviation of N values away from N = 7 (minimum) in Case 2 is simply representing infinite magnitude of **P** & **C**. *The proof is now complete for Part I of Proposition 4.2*□.

Derived Dimensions will comply with Incompletely Predictable property as explained using **P** '61'. At Position x = 61 equating to $\mathbf{P}_{18}$ = 61, it is represented by CFS Dimensions $(2x - 7)^1$, $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$ & $(2x - 11)^1$ (with both prime gap & CFS cardinality = 6). This representation indicates an "unknown but correct" **P** with prime gap = 6 when we intentionally conceal full information '61' = 31^{st} **O** = 18^{th} **P** with prime gap = 6. But to arrive at this representation requires calculations of all preceding CFS Dimensions thus manifesting hallmark Incompletely Predictable property of CFS Dimensions.

Overall sum total of individual CFS Dimensions required to represent every **P** is infinite in magnitude because |**all P**| = $\aleph_0$. Standalone Dimensions YY groupings [representing signals for "higher order" non-twin **P** appearances] &/or as front Dimensions YY (sub)groupings [which by itself is fully representative of twin **P** as Dimensions YYYY appearances] need to recur on an indefinite basis. Then twin **P** and "higher order" cousin **P**, sexy **P**, etc should aesthetically all be infinite in magnitude because (respectively) they regularly and universally arise as part of Dimension YYYY and Dimension YY appearances. An isolated **P** is defined as a **P** such that neither **P** - 2 nor **P** + 2 is **P**. In other words, isolated **P** is not part of a twin **P** pair. Example 23 is an isolated **P** since 21 and 25 are both **C**. Then repeated inevitable presence of Dimension YY grouping is nothing more than indicating repeated occurrences of isolated **P**. This constitutes another view on Dimension YY.

CIS of Gap 1 Composite Numbers are fully associated with non-twin **P** as they eternally occur in between any two consecutive non-twin **P**. CIS of Gap 2 Composite Numbers are (i) fully associated with twin **P** as they are eternally present in between any twin **P** pair, and (ii) partially associated with non-twin **P** as they are eternally present alternatingly or intermittently in between any two consecutive non-twin **P**. Then (i) Gap 1 Composite Numbers do not have valid representation by **E** prime gap = 2, and (ii) Gap 2 Composite Numbers have valid representations by all **E** prime gaps = ["consistently" only for] 2, ["inconsistently" for each of] 4, 6, 8, 10,.... This provide an alternative view on **P** from perspective of CFS composite gaps [instead of CIS prime gaps] with observable intrinsic patterns involving *alternating presence* and *absence* of Gap 2 Composite Numbers associated with every CFS Dimensions representations of **P** with prime gaps ≥ 4, viz. 'Plus-Minus Gap 2 Composite Number Alternating Law'. CFS Dimensions representations of Twin **P** are always associated with Gap 2 Composite Numbers, viz. 'Plus Gap 2 Composite Number Continuous Law'.

Examples for both Laws: A twin **P** (with prime gap = 2) in its unique CFS Dimensions format always has Gap 2 Composite Numbers in a [constant] pattern. A cousin **P** (with prime gap = 4) in its unique CFS Dimensions format always has two Gap 1 Composite Numbers & then one Gap 2 Composite Number [combined] pattern *alternating* with three consecutive Gap 1 Composite Numbers [non-combined] pattern. From this simple observation alone, one deduce that we can generate an infinite magnitude of **C** from each of composite gaps 1 & 2. We see that Gap 2 Composite Numbers *alternating* pattern behavior in cousin **P** will not hold true unless twin **P** & all other non-cousin **P** are infinite in magnitude and integratedly supplying essential "driving mechanism" to eternally sustain this Gap 2 Composite Numbers *alternating* pattern behavior in cousin **P**. Thus we establish that twin **P** and cousin **P** in their CFS Dimensions formats are CIS intertwined together when depicted using **C** with composite gaps = 1 & 2 with each supplying their own peculiar (infinite) share of associated Gap 2 Composite Numbers [thus contributing to overall pool of Gap 2 Composite Numbers].

An inevitable statement in relation to "Gap 2 Composite Numbers pool contribution" based on above reasoning: At the bare minimum, *either* twin **P** *or* at least one of non-twin

P must be infinite in magnitude. An inevitable impression: All generated subsets of **P** from 'small gaps' [of 2 & 4] and 'large gaps' [of $\geq$ 6] alike should each be CIS thus allowing true uniformity in **P** distribution. Again we see in Table 2 above depicting **P-C** data for x = 2 to 64 that, for instance, **P** with prime gap = 6 must also persistently have this 'last-place' Gap 2 Composite Numbers intermittently appearing in certain rhythmic *alternating* patterns, thus complying with Plus-Minus Gap 2 Composite Number Alternating Law. This CFS Dimensions representation for **P** with prime gaps = 6 will again generate their infinite share of associated Gap 2 Composite Numbers to contribute to this pool. The presence of this last-place Gap 2 Composite Numbers in various alternating pattern in appearances & non-appearances must *self-generatingly* be similarly extended in a mathematically consistent fashion *ad infinitum* to all other remaining infinite number of prime gaps [which were not discussed in details above]. *The proof is now complete for Part II of Proposition 4.2*□.

5 Rigorous Proofs now named as Polignac's and Twin prime hypotheses

The proofs on lemmas and propositions from previous section supply all necessary evidences to fully support Theorem Polignac-Twin prime I to IV below thus depicting proofs for Polignac's and Twin prime conjectures in a rigorous manner. Gap 1 Composite Numbers do not have valid representation by **E** prime gap = 2, and Gap 2 Composite Numbers have valid representations by all **E** prime gaps = ["consistently" only for] 2, ["inconsistently" for each of] 4, 6, 8, 10,.... Plus-Minus Gap 2 Composite Number Alternating Law confirms that Gap 2 Composite Numbers present in each **P** with prime gaps $\geq$ 4 situation must appear as some sort of "rhythmic patterns of alternating presence and absence" for Gap 2 Composite Numbers. Twin **P** with prime gap = 2 obeying Plus Gap 2 Composite Number Continuous Law can be understood as special situation of "(non-)rhythmic patterns with continual presence" for relevant Gap 2 Composite Numbers.

In 1849 when French mathematician Alphonse de Polignac (1826 - 1863) was admitted to Polytechnique, he made what is known as Polignac's conjecture which relates complete set of odd **P** to all **E** prime gaps. Twin prime conjecture, which relates twin prime numbers to prime gap = 2, is nothing more than a subset of Polignac's conjecture.

Theorem Polignac-Twin prime I. Incompletely Predictable prime numbers $\mathbf{P}_n$ = 2, 3, 5, 7, 11, ..., ∞ or composite numbers $\mathbf{C}_n$ = 4, 6, 8, 9, 10, ..., ∞ are CIS with overall actual location [but not actual positions] of all prime or composite numbers accurately represented by complex algorithm involving prime gaps G_{Pi} viz. $\mathbf{P}_{n+1} = 2 + \sum_{i=1}^{n} G_{Pi}$ or involving composite gaps G_{Ci} viz. $\mathbf{C}_{n+1} = 4 + \sum_{i=1}^{n} G_{Ci}$ whereby prime & composite numbers are symbolically represented here with aid of 'n' notation instead of usual 'i' notation; and i & n = 1, 2, 3, 4, 5, ..., ∞. The number '2' in first algorithm represents $\mathbf{P}_1$, the very first (and only even) **P**. The number '4' in second algorithm represent $\mathbf{C}_1$, the very first (and even) **C**.

Proof. We treat above algorithms as unique mathematical objects looking for key intrinsic properties and behaviors. Each **P** or **C** is assigned a unique prime or composite gap. Absolute number of **P** or **C** and (thus) prime or composite gaps are infinite in magnitude. As original formulae containing all possible **P** or **C** by themselves (viz. without supplying prime or composite gaps as "input information" to generate **P** or **C** as "output complexity"), these algorithms intrinsically incorporate overall actual location [but not actual positions] of all **P** or **C**. *The proof is now complete for Theorem Polignac-Twin prime I*□.

Theorem Polignac-Twin prime II. Set of prime gaps G_{Pi} = 2, 4, 6, 8, 10, ..., ∞ is infinite in magnitude whereby these prime gaps accurately and completely represented by Dimensions $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, ..., $(2x - \infty)^1$ must satisfy Information-Complexity conservation in a consistent manner.

Proof. Part I of Proposition 4.2 proved all **P** are represented by Dimension $(2x - N)^1$ with N ≥ 7 for any given x value (except for x = 2 & 3 values). Note that although x = 1 is neither **P** nor **C**, it is validly represented by Dimension $(2x - 2)^1$. If each **P** is endowed with a specific prime gap value, then each such prime gap must [via logical mathematical deduction] be represented by Dimension $(2x - N)^1$. We advocate this nominated method of prime gap representation using Dimensions be [purportedly] the only way to achieve Information-Complexity conservation. The preceding mathematical statements are correct as there is a unique prime gap value associated with each **P**. Proposition 5.1 below based on principles from Set theory provides further supporting materials that prime gaps are infinite in magnitude. *The proof is now complete for Theorem Polignac-Twin prime II*□.

Theorem Polignac-Twin prime III. To maintain Dimensional analysis (DA) homogeneity, those Dimensions $(2x - N)^1$ from Theorem Polignac-Twin prime II must contain eternal repetitions of well-ordered sets constituted by Dimensions $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$, ..., $(2x - \infty)^1$.

Proof. This Theorem is stated in greater details as "To maintain DA homogeneity, those aforementioned [endowed with exponent 1] Dimensions $(2x - N)^1$ from Theorem Polignac-Twin prime II must repeat themselves indefinitely in following specific combinations – (i) Dimension $(2x - 7)^1$ only appearing as twin [two-times-in-a-row] and quadruplet [four-times-in-a-row] sequences, and (ii) Dimensions $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$,..., $(2x - \infty)^1$ appearing as progressive groupings of **E** 2, 4, 6, 8, 10,..., ∞." To accommodate the only even **P** '2', exceptions to this DA homogeneity compliance will expectedly occur right at beginning of **P** sequence – (i) one-off appearance of Dimensions $(2x - 2)^1$, $(2x - 4)^1$ and $(2x - 5)^1$ and (ii) one-off appearance of Dimension $(2x - 7)^1$ as a quintuplet [five-times-in-a-row] sequence which is equivalent to (eternal) non-appearance of Dimension $(2x - 6)^1$ at x = 4. [We again note Dimension $(2x - 2)^1$ used to validly represent Number '1' which is neither **P** nor **C**.] These sequentially arranged sets are CFS whereby from x = 11 onwards, each set will always commence initially as 'baseline' Dimension $(2x - 7)^1$ at x = **O** values and will always end with its last Dimension at x = **E** values. Each set will also have varying cardinality with values derived from all **E**; and the correctly combined sets will always intrinsically generate two infinite sets of **P** and, by default, **C** in an integrated manner. Our Theorem Polignac-Twin prime III simply represent a mathematical summary derived from Section 3 & 4 of all expressed characteristics of Dimension $(2x - N)^1$ when used to represent **P** with intrinsic display of DA homogeneity. See Proposition 5.2 below for further details on DA aspect. *The proof is now complete for Theorem Polignac-Twin prime III*□.

Theorem Polignac-Twin prime IV. Aspect 1. The "quantitive" aspect to existence of both prime gaps and their associated prime numbers as sets of infinite magnitude will be shown to be correct by utilizing principles from Set theory. Aspect 2. The "qualitative" aspect to existence of both prime gaps and their associated prime numbers as sets of infinite magnitude will be shown to be correct by 'Plus-Minus Gap 2 Composite Number Alternating Law' and 'Plus Gap 2 Composite Number Continuous Law'.

Proof. Required concepts from Set theory involve cardinality of a set with its 'well-ordering principle' application. Supporting materials for these concepts based on 'pigeonhole principle' in relation to Aspect 1 are outlined in Proposition 5.1 below. 'Plus-Minus Gap 2 Composite Number Alternating Law' is applicable to all **E** prime gaps [apart from first **E** prime gap = 2 for twin primes]. The prime gap = 2 situation will obey 'Plus Gap 2

Composite Number Continuous Law'. These Laws are essentially Laws of Continuity inferring underlying intrinsic driving mechanisms that enables infinity magnitude association for both prime gaps & prime numbers to co-exist. By the same token, these Laws have the important implication that they must be applicable to those relevant prime gaps on an perpetual time scale. Supporting materials in relation to Aspect 2 are found in Proposition 4.2 above. *The proof is now complete for Theorem Polignac-Twin prime IV*□.

We note two mutually inclusive conditions: Condition 1. Presence of all Dimensions that repeat themselves on an indefinite basis and with exponent of '1' will give rise to complete sets of **P** & **C** ["DA-wise one & only one mathematical possibility argument" associated with inevitable *de novo* DA homogeneity], and Condition 2. Presence of any Dimension(s) that do not repeat itself (themselves) on an indefinite basis or with exponent other than '1' will give rise to incomplete set of **P** & **C** or incorrect set of non-**P** & non-**C** ["DA-wise mathematical impossibility argument" associated with inevitable *de novo* DA non-homogeneity]. When met, these two conditions will fully support the point that CFS Dimensions representations of **P** & **C** [with respective prime & composite gaps] are totally accurate. Condition 1 reflect proof from Theorem Polignac-Twin prime III above as all **P** & **C** are associated with DA homogeneity when their Dimensions are endowed with exponent of '1'. Condition 2 invoke corollary on inevitable appearance of incomplete **P** or **C** or non-**P** or non-**C** [associated with DA non-homogeneity] being tightly incorporated into this mathematical framework. See Propositions 5.1 and 5.2, and Corollary 5.3 below for supporting materials on DA homogeneity & non-homogeneity.

We analyze **P** (& **C**) in terms of (i) measurements based on cardinality of CIS and (ii) pigeonhole principle which states that if n items are put into m containers, with $n>m$, then at least one container must contain more than one item. We note that ordinality of all infinite **P** (& **C**) is "fixed" implying that each one of the infinite well-ordered Dimension sets conforming to CFS type as constituted by Dimensions $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$, ..., $(2x - \infty)^1$ on respective gaps for **P** (& **C**) must also be "fixed".

Proposition 5.1. "Even number prime gaps are infinite in magnitude with each even number prime gap generating odd prime numbers which are again infinite in magnitude" is supported by principles from Set theory and two Laws based on Gap 2 Composite Number.

Proof. We validly exclude even **P** '2' in our arguments. Let (i) cardinality $T = \aleph_0$ for Set **all odd P** derived from **E** prime gaps 2, 4, 6,..., ∞, (ii) cardinality $T_2 = \aleph_0$ for Subset **odd P** derived from **E** prime gap 2, cardinality $T_4 = \aleph_0$ for Subset **odd P** derived from **E** prime gap 4, cardinality $T_6 = \aleph_0$ for Subset **odd P** derived from **E** prime gap 6, etc. Paradoxically $T = T_2 + T_4 + T_6 + ... + T_\infty$ equation is valid despite $T = T_2 = T_4 = T_6 = ... = T_\infty$ [when defined in terms of 'well-ordering principle' applied to cardinality of each (sub)set]. But if Subset **odd P** derived from one or more **E** prime gap(s) are finite in magnitude, this will breach $\aleph_0$ 'uniformity' resulting in (i) DA non-homogeneity and (ii) inequality $T > T_2 + T_4 + T_6 + ... + T_\infty$. In language of pigeonhole principle, residual **odd P** (still CIS in magnitude) not accounted for by CFS-type **E** prime gap(s) will have to be [incorrectly] contained in one (or more) of composite gap(s). These arguments using cardinality constitute proof that (i) **E** prime gaps and (ii) odd **P** generated from each **E** prime gap, must all be CIS. *The proof [on "quantitative" aspect] is now complete for Proposition 5.1*□.

Complete set of **P** is represented by Dimensions $(2x - N)^1$. Table 2 & Figure 1 on **PC** finite scale mathematical landscape depict perpetual repeating features used in "qualitative" statements supporting (i) Plus-Minus Gap 2 Composite Number Alternating Law (stated as **C** with composite gaps = 2 present in each of **P** with prime gaps ≥ 4 situation must be observed to appear as some sort of rhythmic patterns of alternating presence and absence of this type of **C**), and (ii) Plus Gap 2 Composite Number Continuous Law (stated as **C** with composite gaps = 2 continual appearances in each of (twin) **P** with prime gap = 2 situation). Plus-Minus Gap 2 Composite Number Alternating Law has intrinsic mechanism to automatically generate all prime gaps ≥ 4 in a mathematically consistent *ad infinitum* manner. Plus Gap 2 Composite Number Continuous Law has built-in intrinsic mechanism to further generate prime gap = 2 appearances in a mathematically consistent *ad infinitum* manner. *The proof [on "qualitative" aspect] is now complete for Proposition 5.1*□.

Proposition 5.2. The presence of Dimensional analysis homogeneity will always result in correct and complete set of prime (and composite) numbers.

Proof. DA homogeneity is completely dependent on all Dimensions being consistently endowed with exponent '1'. As all **P** (& **C**) are "fixed", we can deduce from Figure 1 & Table 2 that there is one (& only one) way to represent Information-Complexity conservation using our defined Dimensions. Thus, there is one (& only one) way to depict all **P** (& **C**) using these Dimensions in a self-consistent manner and this can only be achieved with the one (& only one) DA homogeneity possibility. *The proof is now complete for Proposition 5.2*□.

Corollary 5.3. The presence of Dimensional analysis non-homogeneity will always result in incorrect and/or incomplete set of prime (and composite) numbers.

Proof. For optimal clarity, we endow all Dimensions with exponent '1' depicted as $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$,..., $(2x - \infty)^1$. Proposition 5.2 equates DA homogeneity with correct & complete set of **P** (& **C**). There are "more than one" DA non-homogeneity possibilities. For instance, if a particular $(2x - 7)^1$ Dimension derived from $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$,..., $(2x - \infty)^1$ terminates prematurely and does not perpetually repeat [with loss of continuity and depicting one DA non-homogeneity possibility]; then there are intuitively two 'broad' DA possibilities here; namely, (one) DA homogeneity possibility and "all others" endowed with DA non-homogeneity possibilities. Mathematical consistency of Dimensions $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$,..., $(2x - \infty)^1$ appearing as progressive groupings of [**E**] 2, 4, 6, 8, 10,..., ∞ will be halted without justification. Then a particular Dimension, using $(2x - 7)^1$ example, that stop recurring at some point in **P** (or **C**) sequence would have DA non-homogeneity and be depicted against-all-trends as $(2x - 7)^0$ when endowed with a different exponent – arbitrarily set as '0' in this case. A Dimension that stop recurring will cause well-ordered CFS sets from progressive groupings of [**E**] 2, 4, 6, 8, 10,..., ∞ for Dimensions $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$,..., $(2x - \infty)^1$ to stop existing (and ultimately for sequential **P** (or **C**) to stop appearing) at that point using this grouping method with ensuing outcome that **P** (or **C**) may overall be incorrectly finite or incomplete in magnitude. Finally, a Dimension with fractional exponent values other than '1' such as '$\frac{2}{5}$' or '$\frac{3}{5}$' will result in non-**P** (or non-**C**) [fractional] numbers. *The proof is now complete for Corollary 5.3*□.

Each [fixed] finite scale mathematical landscape "page" as part of [fixed] infinite scale mathematical landscape "pages" for **P** & **C** display Chaos [sensitivity to initial conditions viz. positions of subsequent **P** & **C** are "sensitive" to positions of initial **P** & **C**] and Fractals [manifesting fractal dimensions with self-similarity viz. those aforementioned Dimensions for **P** & **C** must always be present, albeit in non-identical manner, for all ranges of $x \geq 2$]. Advocated in another manner, Chaos and Fractals phenomena of those Dimensions for **P** & **C** must always be correctly present signifying accurate composition of **P** & **C** in different [predetermined] finite scale mathematical landscape "(snapshot) pages" for **P** & **C** that are self-similar but never identical – and there are an infinite number of these finite scale mathematical landscape "(snapshot) pages". The crucial mathematical step in representing all **P** (& **C**) and prime (& composite) gaps with "Dimensions" based on Information-Complexity conservation allows us to obtain the two Laws based on Gap 2 Composite Numbers and perform DA on these entities. The 'strong' principle argument is DA homogeneity equates to complete set of **P** (& **C**) whereas DA non-homogeneity does not equate to complete set of **P** (& **C**). We could also advocate for a 'weak' principle argument supporting DA homogeneity for **P** (& **C**) in that nature should not "favor" any particular Dimension(s) to terminate and therefore DA non-homogeneity does not, and cannot, exist for **P** (& **C**). Abiding to the convention that 'conjecture' be termed 'hypothesis' once proven; we now advocate Polignac's & Twin prime conjectures to be Polignac's & Twin prime hypotheses.

6 Conclusions

CIS of [Completely Predictable] natural numbers 1, 2, 3, 4, 5, 6, 7,... having CIS of [Completely Predictable] natural gaps 1, 1, 1, 1, 1, 1,... are constituted by three dependent sets of numbers: (i) CIS of [Incompletely Predictable] odd prime numbers 3, 5, 7, 11, 13, 17,... having CIS of [Incompletely Predictable] prime gaps 2, 2, 4, 2, 4,... plus CFS of solitary [Incompletely Predictable] even prime number 2 having CFS of [Incompletely Predictable] prime gap 1 (ii) CIS of [Incompletely Predictable] even and odd composite numbers 4, 6, 8, 9, 10, 12,... having CIS of [Incompletely Predictable] composite gaps 2, 2, 1, 1, 2, 2,.... and (iii) CFS of solitary odd number '1' [neither prime nor composite]. We gave relatively elementary proofs on Polignac's and Twin prime conjectures utilizing this harnessed property.

Prime number theorem describes asymptotic distribution of prime numbers among positive integers by formalizing intuitive idea that prime numbers become less common as they become larger through precisely quantifying rate at which this occurs using probability. Nontrivial zeros [part of 'Axes intercept relationship interface' relevant to Riemann hypothesis] and prime numbers [part of 'Numerical relationship interface' relevant to prime number theorem] are Incompletely Predictable entities and numbers. Deep-seated connections exist between successfully solving Riemann hypothesis and prime number theorem. We can now fully delineate prime number theorem by prime counting function [denoted here with $\pi(x)$]. In mathematics, logarithmic integral function or integral logarithm li(x) is a special function. Relevant to problems of physics and with number theoretic significance, it occurs in prime number theorem as an estimate of $\pi(x)$ whereby the form of this special function is defined so that li(2) = 0; viz. li(x) $\equiv \int_2^x \frac{du}{\ln u}$ = li(x) - li(2). Solving Riemann hypothesis is instrumental in proving efficacy of techniques that estimate $\pi(x)$ efficiently. This should now confirm "best possible" bound for error ("smallest possible" error) of prime number theorem.

There are less accurate ways of estimating $\pi(x)$ such as conjectured by Gauss and Legendre at end of 18th century. This is approximately x/ln$x$ in the sense $\lim_{x\to\infty}\frac{\pi(x)}{x/\ln x} = 1$. Skewes' number is any of several extremely large numbers used by South African mathematician Stanley Skewes as upper bounds for smallest natural number x for which li(x)$<\pi(x)$. These bounds have since been improved by others: there is a crossing near $e^{727.95133}$ but it is not known whether this is the smallest. John Edensor Littlewood, who was Skewes' research supervisor, proved in 1914[4] that there is such a [first] number; and found that sign of difference $\pi(x)$ - li(x) changes infinitely often. This refute all prior numerical evidence that seem to suggest li(x) was always more than $\pi(x)$. The key point here is that [100% accurate] $\pi(x)$ mathematical tool being "wrapped around" by [less-than-100% accurate] approximate mathematical tool li(x) infinitely often via this 'sign of difference' changes meant that li(x) must be the most efficient approximate mathematical tool. Contrast this with the "crude" x/ln$x$ approximate mathematical tool where values obtained diverge away from $\pi(x)$ at in-

Table 3 Even-Odd mathematical (tabulated) landscape using data obtained for x = 1 to 64. Legend: **E** = even, **O** = odd, Y = Dimension 2x - 4.

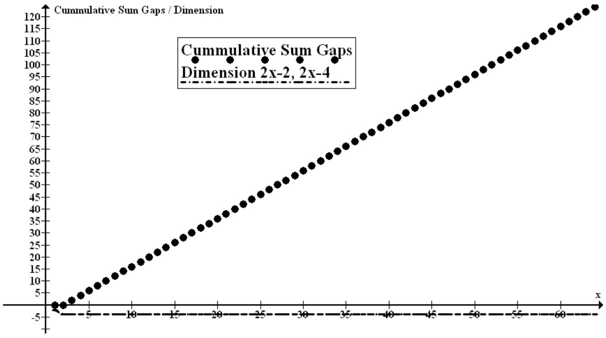
x	**E_i or O_i, Gaps**	**ΣEO_x-Gaps**	Dimension	**x**	**E_i or O_i, Gaps**	**ΣEO_x-Gaps**	Dimension
1	O1, 2	0	2x-2	33	O17, 2	62	Y
2	E1, 2	0	Y	34	O17, 2	64	Y
3	O2, 2	2	Y	35	O17, 2	66	Y
4	E2, 2	4	Y	36	O17, 2	68	Y
5	O3, 2	6	Y	37	O17, 2	70	Y
6	E3, 2	8	Y	38	O17, 2	72	Y
7	O4, 2	10	Y	39	O17, 2	74	Y
8	E4, 2	12	Y	40	O17, 2	76	Y
9	O5, 2	14	Y	41	O17, 2	78	Y
10	E5, 2	16	Y	42	O17, 2	80	Y
11	O6, 2	18	Y	43	O17, 2	82	Y
12	E6, 2	20	Y	44	O17, 2	84	Y
13	O7, 2	22	Y	45	O17, 2	86	Y
14	E7, 2	24	Y	46	O17, 2	88	Y
15	O8, 2	26	Y	47	O17, 2	90	Y
16	E8, 2	28	Y	48	O17, 2	92	Y
17	O9, 2	30	Y	49	O17, 2	94	Y
18	E9, 2	32	Y	50	O17, 2	96	Y
19	O10, 2	34	Y	51	O17, 2	98	Y
20	E10, 2	36	Y	52	O17, 2	100	Y
21	O11, 2	38	Y	53	O17, 2	102	Y
22	E11, 2	40	Y	54	O17, 2	104	Y
23	O12, 2	42	Y	55	O17, 2	106	Y
24	E12, 2	44	Y	56	O17, 2	108	Y
25	O13, 2	46	Y	57	O17, 2	110	Y
26	E13, 2	48	Y	58	O17, 2	112	Y
27	O14, 2	50	Y	59	O17, 2	114	Y
28	E14, 2	52	Y	60	O17, 2	116	Y
29	O15, 2	54	Y	61	O17, 2	118	Y
30	E15, 2	56	Y	62	O17, 2	120	Y
31	O16, 2	58	Y	63	O17, 2	122	Y
32	E16, 2	60	Y	64	O17, 2	124	Y

creasingly greater rate when larger range of prime numbers are being studied.

Appendix I: Tabulated and graphical depictions on Even-Odd mathematical landscape for x = 1 to 64

We tabulate (in Table 3) and graph (in Figure 2) [Completely Predictable] **E-O** mathematical landscape for x = 1 to 64. Involved Dimensions are 2x - 2 & 2x - 4 with Y denoting Dimension 2x - 4 for visual clarity. This mathematical landscape of Dimension 2x - 4 (except for first and only Dimension 2x - 2) will intrinsically incorporate **E** & **O** in an integrated manner. Except for first **O**, all Completely Predictable **E** & **O** and all their associated gaps are represented by countable finite set of [single] Dimension 2x - 4.

In Figure 2, Dimensions 2x - 2 & 2x - 4 are symbolically represented by -2 & -4 with 2x - 4 displayed as 'baseline' Dimension whereby the Dimension trend (Cumulative Sum Gaps) must reset itself onto this (Grand-Total Gaps) 'baseline' Dimension after the initial Dimension 2x - 2 on a permanent basis. Graphical appearances of Dimensions symbolically represented by the two negative integers are Completely Predictable with both Even-$\pi(x)$ and Odd-$\pi(x)$ becoming larger at a constant rate. There is a complete absence of Chaos and Fractals phenomena being manifested in our graph.



Even Odd.png

Fig. 2 Even-Odd mathematical (graphed) landscape using data obtained for x = 1 to 64.

Definitive derivation of data in Table 3 is given & illustrated by two examples for position x = 31 & 32. For i & x $\in$ 1, 2, 3, ..., ∞; ΣEO_x-Gap = ΣEO_{x-1}-Gap + Gap value at E_{i-1} or Gap value at O_{i-1} whereby (i) E_i or O_i at position x is determined by whether relevant x value belongs to **E** or **O**, and (ii) both ΣEO_1-Gap and ΣEO_2-Gap = 0. Example, for position x = 31: 31 is **O** (O16). Our desired Gap value at O15 = 2. Thus ΣEO_{31}-Gap (58) = ΣEO_{30}-Gap (56) + Gap value at O15 (2). Example, for position x = 32: 32 is **E** (E16). Our desired Gap value at E15 = 2. Thus ΣEO_{32}-Gap (60) = ΣEO_{31}-Gap (58) + Gap value at E15 (2).

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References

1. Furstenberg, H. (1955). On the infinitude of primes. *Amer. Math. Monthly, 62*, (5) 353, http://dx.doi.org/10.2307/2307043
2. Saidak, F. (2006), A New Proof of Euclid's theorem, *Amer. Math. Monthly, 113*, (10) 937, http://dx.doi.org/10.2307/27642094
3. Zhang, Y. (2014), Bounded gaps between primes, *Ann. Math. 179*(3) (2014) 1121 – 1174, http://dx.doi.org/10.4007/annals.2014.179.3.7
4. Littlewood, J. E. (1914), Sur la distribution des nombres premiers. Comptes Rendus de l'Acad. Sci. Paris, 158, 1869 – 1872