Refutation of temporal type theory (TTT), temporal landscapes, and Scott’s topology

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Abstract: From a footnote, and then a three part definition, temporal type theory (TTT), and then temporal landscapes for open sets are not tautologous and hence refuted. That further refutes Scott’s topology. These results therefore form a non tautologous fragment of the universal logic $VŁ4$.

We assume the method and apparatus of Meth8/$VŁ4$ with Tautology as the designated proof value, $\mathbf{F}$ as contradiction, $\mathbf{N}$ as truthity (non-contingency), and $\mathbf{C}$ as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

Temporal landscapes: a graphical temporal logic for reasoning. (2019). bfo@mit.edu

\footnote{Note that the constant reals can be considered as a subtype $R \subseteq \mathbb{R}$ of the varying real numbers.}

\begin{align*}
\text{LET} & \quad \sim \text{Not, } \neg; + \text{ Or, } \lor, \cup; - \text{ Not Or; } \& \text{ And, } \land, \cap, \sqcap; \setminus \text{ Not And;} \\
& \quad > \text{ Imply, greater than, } \to, \Rightarrow, >, \supset, \ast; \\
& \quad < \text{ Not Imply, less than, } \in, <, \subset, \prec, \neq, \preceq, \leq; \\
& \quad = \text{ Equivalent, } \equiv, \iff, \leftrightarrow, \approx, \simeq; \quad \not\equiv \text{ Not Equivalent, } \#;
\end{align*}

\begin{align*}
\text{LET} & \quad p, q, r, s, t, u: \\
& \quad t_1, t_2, R, t'_2, t'_1, L
\end{align*}

\begin{align*}
\neg(\neg r < r) & = (p = p) ; & & \text{FFFF TTTT FFFF TTTT } \quad (1.1.1)
\end{align*}

\begin{align*}
\text{Remark 1.2.2:} & \quad \text{The constant reals considered as a subtype of varying real numbers is not tautologous. This refutes temporal type theory (TTT) at its outset. However, we press on assuming that difficulty may be overcome by simply avoiding it.}
\end{align*}

\begin{align*}
\text{Definition 2.1.} & \quad \text{A temporal landscape on } R \text{ is a set } L \text{ of time intervals } [t_1,t_2] \subseteq R, \text{ where } t_1 \leq t_2, \text{ such that}
\end{align*}

\begin{align*}
& \text{(a) if } [t_1,t_2] \in L, \text{ and } t'_1 \leq t_1 \leq t_2 \leq t'_2, \text{ then } [t'_1,t'_2] \in L. \quad (2.1.a.1)
\end{align*}

\begin{align*}
& \iff((p \& q) < u) \& \sim(\sim(s < q) < \sim(p < t)) \iff((t \& s) < u) ; \\
& \text{TTT TTT TTT TTT TTT TTT (1),} \\
& \text{TTF TTTF TTTT TTTT (1),} \\
& \text{TTTT TTTT TTTT TTTT (2) } \quad (2.1.a.2)
\end{align*}
(b) if \([t_1, t_2] \in L\) then there exists \(t_1' < t_1 \leq t_2 < t_2'\) such that \([t_1', t_2'] \in L\). \hspace{1cm} (2.1.b.1)

\[
((p \& q) < u) > (\neg ((q < s) < (t < p)) = (p = p)) > ((t \& s) < u);
\]

\begin{align*}
\text{FFFF} & \text{FFFF} \text{FFFF FFFF} (1), \\
\text{FFFF} & \text{FFFF} \text{TTTT TTTT} (1), \\
\text{FFFF} & \text{FFFF} \text{FFFF FFFF} (2) \hspace{1cm} (2.1.b.2)
\end{align*}

We write Prop for the set of temporal landscapes. Together, requirements (a) and (b) state that temporal landscapes form the open sets of the Scott topology on the interval domain IR, a well-studied topological space in domain theory. \hspace{1cm} (2.1.1.1)

\[
((u < r) > (r < (p \& q)) \& (q < (s < t))) > (((((p \& q) < u) \& (\neg (s < q) \& \neg (s < t))) > ((t \& s) < u)) \& \>
(((p \& q) < u) > (\neg (q < s) < (t < p)) = (p = p)) > ((t \& s) < u));
\]

\begin{align*}
\text{FFFF} & \text{FFFF} \text{FFFF FFFF} (1), \\
\text{FFFF} & \text{FFFF} \text{TTTT TTTT} (1), \\
\text{FFFF} & \text{FFFF} \text{TTTT FFFF} (1), \\
\text{FFFF} & \text{FFFF} \text{FFFF FFFF} (1) \hspace{1cm} (2.1.1.2)
\end{align*}

Eqs. 2.1.1.2, .b.2, and .1.2 are not tautologous. This refutes Def. 2.1 and temporal landscapes for open sets, and hence denies Scott’s topology.