Experimental Support for Physically Significant Units of Measure

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Providing for a comprehensive model of physics that describes both the discrete and non-discrete behavior of matter has proved difficult and elusive. Using the principles of Informativity—a model based on counts of the fundamental measures, length, mass and time—we take a new approach to express Heisenberg's Uncertainty Principle. It is shown that the physical significance of measure is an implicit outcome of the principle. Moreover, the bound dividing certainty from uncertainty may be described entirely as counts of these measures. Discrete values for length, mass and time are resolved with physical support demonstrating that they are fundamental and countable. Predictions matching experimental results include: Planck's constant, the fine structure constant, the magnetic constant, the gravitational constant and the gravitational curve.

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1. INTRODUCTION

Discrete behavior is any behavior that cannot exhibit a state with respect to every possible measure. For example, given a bounded system a discrete behavior implies count-ability which in turn implies an observed count of the behavior. To understand discreteness requires that we identify and describe the reference that constrains the behavior.

A discrete approach describing Heisenberg's Uncertainty Principle is presented. This enables us to resolve minimum values for length, mass and time that define the lower bound for physically significant measure. Moreover, properties of measure, such as discreteness, count-ability and how fundamental measures behave as references are quantified in expression form. The model demonstrates not only physical support, but constrains the measurement domain in both the quantum and cosmological regimes eliminating singularities. Building support for discrete units of measure adds to a greater understanding of c=*l/t* and opens the door to *m/t* and *l/m* completing a bounded, quantum picture of nature.

We follow the presentation with several physical predictions such as a quantum description of gravity, a new approach to resolving the value of G as well as Planck's constant $\hbar$, the fine structure constant, the magnetic constant, specific polarization measures necessary for the quantum entanglement of X-rays and a quantum presentation of Heisenberg's Uncertainty Principle.

Importantly, we address the obstacles met by Planck and others in the search for a physically significant connection to Planck's Units. Beginning with a new expression that correlates G and $\hbar$ we unravel how measure affects our understanding of these constants. Applying the principles of discrete measure to Planck's expressions resolves the difference to six significant digits.

Further reading and explanation of each of these phenomena are available in the published record [1][2][3]. The focus of this paper is not to list the predictions to date, but to discuss the foundations of measurement quantization, what is measure and how bounds to measure lead to discrete behavior.

2. METHODS

2.1. Discreteness of Measure

In modern theory, the expressions we use to describe the behavior of matter are presumed to be physically significant to any mathematical precision. But, we also recognize that measure has a lower bound, a measure where the effects described by quantum mechanics constrain the measure of length, mass and elapsed time associated with a phenomenon. Heisenberg's Uncertainty Principle is one tool we may use to identify the scope of a physically significant measurement domain.

We may, for instance, use the Planck Unit expressions for length l_p and mass m_p to start.

$$m_p = \left(\frac{\hbar c}{G}\right)^{1/2}, \tag{1}$$

$$l_p = \left(\frac{\hbar G}{c^3}\right)^{1/2}. \tag{2}$$

Organizing the expression for length, we also resolve that

$$\frac{\hbar}{2}=\frac{l_p^2c^3}{2G}=\frac{l_p^2c^3}{2\left(c^3t_p/m_p\right)}=\frac{l_p^2m_p}{2t_p}. \tag{3}$$

Then using Heisenberg's expression to describe the position and momentum of a particle, we present the expression in terms of Planck Units.

$$\sigma_X\sigma_P\ge\frac{\hbar}{2}, \tag{4}$$

$$(l_p)\left(m_pv\right)\ge\frac{\hbar}{2}. \tag{5}$$

Such that each value is a count of one Planck unit of a respective measure, we generalize the description by incorporating count values for length n_L, mass n_M, time n_T and distance to target n_{Lr},

$$(n_{Lr}l_p)\left(n_Mm_p\frac{n_Ll_p}{n_Tt_p}\right)\ge\frac{l_p^2m_p}{2t_p}, \tag{6}$$

$$2n_{Lr}n_Mn_L\ge n_T. \tag{7}$$

Notably, measure is not a component of the bound. But, this does not mean that the reference measures are physically significant or that the counts are integers. To break down an understanding of what values are permitted, we will need new tools starting with a new fundamental description of G that does not include Planck's constant.

$$\frac{m_p^2}{l_p^2}=\left(\frac{\hbar c}{G}\right)\left(\frac{c^3}{\hbar G}\right)=\left(\frac{c^4}{G^2}\right)=\frac{c^4}{c^6t_p^2m_p^{-2}}=\frac{m_p^2}{l_p^2}, \tag{8}$$

$$G^2=\frac{c^4l_p^2}{m_p^2}=\frac{l_p^4l_p^2}{t_p^4m_p^2}=\frac{l_p^6}{t_p^4m_p^2}=\frac{l_p^6t_p^2}{t_p^6m_p^2}, \tag{9}$$

$$G=\left(\frac{l_p^3}{t_p^3}\right)\left(\frac{t_p}{m_p}\right). \tag{10}$$

Such that c=n_Ll_f/n_Tt_f we recognize that the count relation between length and time is one-to-one. To resolve the relation in Eq. (7) between n_M and n_L or n_T is difficult. For this, we introduce a radian understanding of measure that is also paired with the expression for light. Begin by considering that the arc length of a circle of radius l_p and angle θ in radians is

$$L=r\theta=l_p\left(\frac{\hbar}{2l_p}\right)=\frac{\hbar}{2}. \tag{11}$$

Then, reorganize Planck's unit expression for length $l_p=(\hbar G/c^3)^{1/2}$, reduce with Eq. (10) and solve for $\theta=\hbar/2l_p$ such that

$$\frac{\hbar}{l_p}=\frac{l_pc^3}{G}, \tag{12}$$

$$\theta = \frac{\hbar}{2l_p} = \frac{l_p c^3}{2G} = \frac{l_p c^3}{2\left(c^3 t_p m_p^{-1}\right)} = \frac{l_p m_p}{2t_p} = \frac{1}{2} m_p c \, . \quad (13)$$

We find that the relation between mass and length at the bound c, is specifically a count n_M=1/2, n_L=1 and n_T=1. Returning to Heisenberg's expression as presented in Eq. (7) such that c identifies n_L=n_T=1, then

$$n_{Lr} n_M \geq \frac{1}{2} \quad (14)$$

also provides physical confirmation that n_{Lr}=1 and n_M=1/2 as resolved in Eq. (13). Measure less than n_{Lr}=n_L= n_T=1 and n_M=1/2 is not physically significant.

This does not imply that a phenomenon may not behave non-discretely. But, it does imply that measure is discrete. By example, the mass of an electron 9.11 10^{-31} kg is significantly less than $n_M m_p$=(1/2)·2.18 10^{-8}=1.09 10^{-8} kg. The electron's mass is physically significant, but with respect to the observer that mass may not be measured.

> ***O*₁:** *There are physically significant discrete units of measure.*

Moreover, discrete measure is not just a minimum bound. The discrete measures are the reference units used to compare and count all measures. Specifically, we cannot measure 0.25l_p just as we cannot measure 10.25l_p. To do so would violate Heisenberg's Uncertainty Principle, offering an opportunity through difference to resolve measures less than the bound. All measure is constrained to a whole-unit count of Planck Units.

> **O_2:** *The discrete units of measure are countable.*
> **O_3:** *Each of the discrete units of measure behaves as a reference.*

Finally, is discrete measure also a property of the universe?

No. This would violate the count-ability of measure. Notably, in that the universe expands at the speed of light, information external to the universe is inaccessible. Without external references properties of the universe are conversely non-discrete. Physical evidence for each outcome follows in the results.

> **O_4:** *Measure has no physical significance without a reference.*
> **O_5:** *References give rise to the property of measure.*

2.2. Frames of Reference

Often not stated, references are defined in terms of the observer. Consider now three masses each having a static position that when connected define a line. Observer A and C are at the end-points approximately 8l_p apart. Observer B is at the midpoint.

Observers A and C wish to independently verify their separation using Pythagorean's Theorem. While one may choose any value for sides **a** and **b** of a right-angle triangle, to do so can unnecessarily introduce a mathematical translation with respect to the reference measure. To avoid this, the reference measure is incorporated as a base measure in the triangulation (i.e. side **a=**1).

Observer A proceeds to confirm the distance to observer C. Such that side **a**=1, side **b** is any known count of **a** (in this case 8) and side **c** is the unknown count of the reference, then

$$\left(1^2+8^2\right)^{0.5}=8.06226\ l_p. \tag{15}$$

Observer C considers a different approach, measuring to B at the midpoint and multiplying by 2. In the same fashion as observer A, **a**=1 and **b**=4 such that

$$2\left(\left(1^2+4^2\right)^{0.5}\right)=8.24621\ l_p. \tag{16}$$

The two observers disagree. It would be most straight-forward to argue that the problem is the introduction of discrete counts of the reference l_p. But, as described in the prior section, an observer cannot measure any value other than a whole-unit count of l_p. This gedanken provides additional validation of this principle.

Observer A must round to 8 counts of the reference and observer B must round to 4 multiplying by 2 to resolve a separation of $8l_p$. Notably, the largest fractional result that may be obtained with the Pythagorean Theorem is **a**=1 and **b**=1 (i.e. $(1^2+1^2)^{0.5}$=1.41421) which rounds down. As such, we recognize that all calculations such that **a**=1 round down ensuring the same consistent agreement as above.

Importantly, the example brings to our attention two distinct frames of reference. The first frame is defined with respect to the phenomenon used to ascertain the location of B. The phenomenon light, for example, may have been used. The invariant property of light with respect to all observers is consistent with a property of the universe. And for this reason, we refer to such descriptions as being defined with respect to the universe. The second frame is defined with respect to the observer. That is, what is observed depends on the relative characteristics of the observer. We may then state with this nomenclature that the universe is non-discrete (i.e. having no outside reference) while measure is discrete. The distinction is important when seeking an understanding of a phenomenon.

3. RESULTS

In the presentation to follow we will resolve a quantum description of physics that incorporates and correlates all three frames of reference: the observer, the target and the universe. We will also resolve new quantum expressions for the fundamental measures. We distinguish these from Planck's Units with a subscript *f*. Importantly, we will also correlate Planck's expressions with those derived here to demonstrate their relation and what obstacles have stood in the path to a quantum model of physics.

3.1. Distance

In Figure 1, we present a description of distance using only counts of the reference length l_f as a foundation to understanding the property of measure with respect to any inertial frame. Consider that the long side **c** of a right-angle triangle may be resolved using the Pythagorean Theorem where side **a** is always the reference count 1 and side **b** is some known count of that reference.

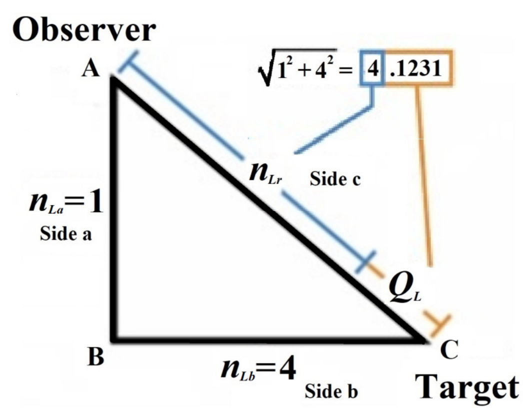

FIG. 1.Count of distance measures between an observer and target where b_L=*4*.

Notably, a count of 1 on side **a** is prerequisite to any definition of unknown factors. By example, if argument was presented that side **a** were arbitrary (i.e. **a**=2) we would find a description that 'assumes' a reference not explicitly incorporated into the definition, a translation of the framework that concealed the discrete properties of measure we were attempting to describe. Thus, side **a**=1 is prerequisite for all considerations of side **b** in any understanding of the unknown distance on side **c**.

$$c = \left(1 + n_{Lb}^2\right)^{1/2} . \tag{17}$$

Any non-whole-unit count relates to a change in distance and may be described by rounding up (repulsion) or down (attraction). The remainder lost to rounding will be denoted by Q_L. For all solutions, Q_L is less than half and thus attractive (see Appendix A for proof). The model provides a count of distance measures that is closer by

$$Q_L = \left(1 + n_{Lb}^2\right)^{1/2} - n_{Lb} \tag{18}$$

at every instant in time t_f. For example, if n_{Lb}=4, then Q_L/n_{Lb}=((√ 17)-4)/4=0.1231/4. Because side **c** always rounds down, we find that n_{Lr} always equals n_{Lb}. In the following, we shall always refer to the 'observed measure count' as n_{Lr}. Moreover, note that the reference measure against which all counts are measured is defined by n_{La}=1. With this we have composed an expression for gravity such that the loss of the remainder relative to the whole-unit count is Q_L/n_{Lr}.

Together Q_L and n_{Lr} are conjectured to represent an important dimensionless ratio that describes gravity. We proceed with that hypothesis by presenting the ratio in meters per second squared (ms^{-2}), where we multiply by l_f for meters and divide by t_f^2 together describing the distance loss at the maximum sampling rate of one sampling every t_f seconds per second,

$$\frac{Q_L l_f}{n_{Lr} t_f^2} . \tag{19}$$

We now note that this quantity is scaled and hence requires a scaling constant; we multiply by the speed of light c and divide by a scaling constant S. Setting $r=n_{Lr}l_f$ and $c=l_f/t_f$, the expression reduces to

$$\frac{Q_L l_f}{n_{Lr} t_f^2}\frac{c}{S} = \frac{Q_L c^2}{n_{Lr} t_f S} = \frac{Q_L l_f c^2}{n_{Lr} l_f t_f S} = \frac{Q_L c^3}{rS} \approx \frac{G}{r^2} . \tag{20}$$

The measure of S is a physically significant value, first measured by Shwartz and Harris and published in their 2011 paper *'Polarization Entangled Photons at X-Ray Energies'* [4]. We refer the reader to the paper, *'Measurement Quantization Unites Classical and Quantum Physics'* [1] for several predictions and corresponding measurement data that collaborate the physical significance of S=3.26239. Notably, the units depend on the frame of reference, radians, momentum or non-dimensional. For brevity, though, you will recognize in Table I that each of the six angular measures reported by Shwartz and Harris correspond to the scaling constant S.

Table I. Angle setting in radians of the **k** vectors of the pump, signal and idler for maximally entangled states at the degenerate frequency with values that correspond with the Shwartz and Harris measurements (Ref. [4]).

Bell's State	θ_p	θ_s	θ_i
$(\|H_s, V_i\rangle + \|V_s, H_i\rangle)/\sqrt{2}$	$(l_f c^3/2G)-\pi$ (0.1208)	$\pi - (l_f c^3/2G)$ (-0.1208)	$\pi - (l_f c^3/2G)$ (-0.1208)
	$2\pi - (l_f c^3/2G)$ (3.02079)	$(l_f c^3/2G)$ (3.26239)	$(l_f c^3/2G)$ (3.26239)

Each expression describes an equidistant angle either side of *0, π* or *2π*. By example, we may use *S* and the 2010 CODATA value for Planck's length [5] (i.e. $b=l_f^{-1}=6.18735\ 10^{34}$) to resolve *G* at *r*=1 m.

$$Q_L = c - b = \sqrt{1+b^2} - b\,, \tag{21}$$

$$Q_L = \sqrt{1+(6.18735\ 10^{34})^2} - 6.18735\ 10^{34} = 8.08100\ 10^{-36}\,, \tag{22}$$

$$\frac{Q_L c^3}{rS} = \frac{8.08100\ 10^{-36} \bullet (299792458)^3}{1 \bullet 3.26239} = 6.67407\ 10^{-11}\ \text{m/kg s}^2. \tag{23}$$

This becomes the first prediction of the model, a calculation of Newton's gravitational constant *G*. Conversely, one may use the measured value of *G* to resolve the value of *S*, but in no situation can a prediction be avoided. A measure of one predicts the other.

More than 20 physical predictions exist [1][2][3]. Notably, the value of *S* is invariant regardless of the frame of reference. To emphasize this property, we have adopted the symbol θ_{si} for all expressions.

3.2. Discrete Measures

Without an understanding of the *fundamental expression*, there is no basis with which to resolve an expression for fundamental mass. Using Eq. (10), $G \approx c^3 t_f / m_f$, and Eq. (20),

$$G \approx r^2 \frac{Q_L c^3}{r\theta_{si}} = n_{Lr} l_f \frac{Q_L c^3}{\theta_{si}} = (Q_L n_{Lr}) \frac{l_f c^3}{\theta_{si}} = \frac{l_f c^3}{2\theta_{si}}\,, \tag{24}$$

then

$$\frac{t_f c^3}{m_f} = \frac{l_f c^3}{2\theta_{si}}\,, \tag{25}$$

$$l_f m_f = 2\theta_{si} t_f\,. \tag{26}$$

The expression is called the *fundamental expression*. The fundamental units are then

$$l_f = \frac{2G\theta_{si}}{c^3} = \frac{2\ 6.67408\ 10^{-11}\ 3.26239}{(299792458)^3} = 1.61620\ 10^{-35}\ \text{m}, \tag{27}$$

$$t_f = \frac{l_f}{c} = \frac{2G\theta_{si}}{c^4} = \frac{2\ 6.67408\ 10^{-11}\ 3.26239}{(299792458)^4} = 5.39106\ 10^{-44}\ \text{s}, \tag{28}$$

$$m_f = \frac{2\theta_{si}}{c} = t_f \frac{c^3}{G} = \frac{2\ 3.26239}{299792458} = 2.17643\ 10^{-8}\ \text{kg}. \tag{29}$$

The difference from Planck's Units is a function of the *informativity differential* $Q_L n_{Lr}$ which is taken in the above expressions at the upper limit where its value equals 1/2. Importantly, though, the effect on the values of G and $\hbar$ are distance sensitive. For instance, the measure of length in expanded form is written as

$$l_f = \frac{G\theta_{si}}{Q_L n_{Lr} c^3} = \left(c^3 \frac{t_f}{m_f} \right) \frac{\theta_{si}}{Q_L n_{Lr} c^3} = \frac{\theta_{si} t_f}{Q_L n_{Lr} m_f} \text{ m.} \tag{30}$$

Likewise, the *fundamental expression* in expanded form is

$$Q_L n_{Lr} m_f l_f = \theta_{si} t_f \, . \tag{31}$$

The conflicts that Planck encountered arose from the missing experimental connection to the effects of distance on G and $\hbar$. Whereas the measure of $\hbar$ is a property of quantum interactions, the measure of G is resolved as a property of macroscopic phenomenon. Mixing the two distance sensitive values in a single expression as Planck defines his units leads to incorrect physical predictions. One example would include the calculation of theta,

$$\theta_{si} = \left(\frac{c^3}{G} \right) Q_{Lf} r_{Lf} l_f = \left(\frac{\hbar}{l_f^2} \right) Q_{Lf} r_{Lf} l_f = \frac{\hbar}{2 l_f} = 3.26250 \text{ radians,} \tag{32}$$

which is known from the Shwartz and Harris measures [4] to be 3.26239 radians. Using c^3/G produces the correct result.

For a second example we must resolve a better understanding of the relation between G and $\hbar$. We begin with the *fundamental expression* $l_f m_f = 2\theta_{si} t_f$ and the expression above.

$$l_f m_f = 2\theta_{si} t_f \, , \tag{33}$$

$$4 t_f \theta_{si}^2 = \left(2 l_f \theta_{si} \right) m_f \, , \tag{34}$$

$$4 t_f \theta_{si}^2 = \hbar m_f \, , \tag{35}$$

$$4 \left(c^3 \frac{t_f}{m_f} \right) \theta_{si}^2 = \hbar c^3 \, , \tag{36}$$

$$4 G \theta_{si}^2 = \hbar c^3 \, . \tag{37}$$

Then, moving terms to the left to match the expression for fundamental length, we find Planck's terms appear on the right matching his expression for length.

$$\frac{4 G \theta_{si}^2}{c^3} = \hbar \, , \tag{38}$$

$$\frac{4 G^2 \theta_{si}^2}{c^6} = \frac{\hbar G}{c^3} \, , \tag{39}$$

$$\frac{2 G \theta_{si}}{c^3} = \left(\frac{\hbar G}{c^3} \right)^{1/2} . \tag{40}$$

The challenge Planck encountered was that there is a quantum variation (an observational skew) in G and $\hbar$ equal in magnitude that cancel as described in Eq. (37). Without expressions and experimental data describing the variation, he lacked a foundation with which to resolve a distance sensitive understanding of $\hbar_f$ (i.e. taken macroscopically, n_{Lr}=∞),

$$\hbar_f = \frac{l_f^2 \theta_{si}}{Q_{Lf} r_{Lf} l_f} = \frac{l_f \theta_{si}}{Q_{Lf} r_{Lf}} = 2\theta_{si} l_f = 1.05454\ 10^{-34}\ \text{Js}. \tag{41}$$

With this distance sensitive value, Planck's expressions now correspond with the macroscopic measure of G as presented in the fundamental expressions for length, mass and time. And of equal importance, the physical conflict with experimental result is now resolved. We may demonstrate this by breaking Planck's length down to resolve an expression for G,

$$l_p = \left(\frac{\hbar G}{c^3}\right)^{1/2}, \tag{42}$$

$$l_p^2 = \frac{\hbar G}{c^3}, \tag{43}$$

$$G = \frac{c^3 l_p^2}{\hbar}. \tag{44}$$

Using the 2010 CODATA for Planck's length l_p and $\hbar$ in comparison with the distance sensitive measures resolved with Informativity, then

$$\text{(Planck)}\ G = \frac{c^3 l_p^2}{\hbar} = \frac{(299792458)^3 \bullet (1.616199\ 10^{-35})^2}{1.05457\ 10^{-34}} = 6.67384\ 10^{-11}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2}, \tag{45}$$

$$\text{(Informativity)}\ G = \frac{c^3 l_f^2}{\hbar_f} = \frac{(299792458)^3 \bullet (1.616200\ 10^{-35})^2}{1.05454\ 10^{-34}} = 6.67407\ 10^{-11}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2}. \tag{46}$$

The Informativity approach conforms to the measurement data (i.e. 6.67408 10^{-11} $m^3kg^{-1}s^{-2}$) [5]. Planck's approach demonstrates a physically significant discrepancy of 0.00023 10^{-11} $m^3kg^{-1}s^{-2}$. When taking into account the *informativity differential*, the discrepancy is resolved.

3.3. Supporting Data

Recognizing the above correlation, we also recognize that gravity is an outcome of discrete measure. There is no way to avoid the phenomenon of gravity and have physically significant reference measures. And with that, we may move forward to look at additional support arising from this model. We may, for instance, equate this description of gravity to that of Newton's

$$\frac{Q_L c^3}{r\theta_{si}} \approx \frac{G}{r^2}. \tag{47}$$

Comparing the difference we see a decrease in distance between the two curves that quickly become physically immeasurable. The difference is a function of $Q_L n_{Lr}$, a function that approaches 1/2 with increasing distance as described in Appendix A. The resulting effect on gravity is described in Table II.

TABLE II. Informativity difference in G/r^2.

	50 l_f	*100 l_f*	*200 l_f*	*300 l_f*	*500 l_f*	*1000 l_f*
Difference	0.00100%	0.00250%	0.00062%	0.00028%	0.00010%	0.00003%

A final physical correlation comes from the well-known relation between the fine-structure constant α and the observed value associated with the energy scale of the electron. Using the principles of Informativity to reinterpret that energy scale with respect to the Bohr radius we can resolve the fine structure constant as a count product, the count of l_f describing the Bohr Radius a_0=5.29177 10^{-11} m times the count of m_f describing the rest mass of an electron m_e=9.10938 10^{-31} kg. I call this the Hudson Interpretation in regards to Hoyt Hudson who brought the relation to my attention.

$$\alpha = \frac{a_0}{l_f}\frac{m_e}{m_f} = 0.007297\ . \tag{48}$$

Using the 2014 CODATA [6] recommended value of the fine structure constant where defined as $\alpha=e^2/4\pi\varepsilon_0\hbar c$, we then set the Informativity relation equal to resolve an understanding of the remaining free variables.

$$\frac{a_0}{l_f}\frac{m_e}{m_f} = \frac{e^2}{4\pi\varepsilon_0\hbar c}\ . \tag{49}$$

In recognition of the new 2019 Standards Organizations worldwide, the electric constant ε_0 will now have a value of $1/\mu_0 c^2$ such that the magnetic constant μ_0 is a measured value. Given $c=l_f/t_f$, Eq. (32) $\hbar\approx 2\theta_{si}l_f$ at a quantum distance and the *fundamental expression* from Eq. (26) $l_f m_f=2\theta_{si}t_f$, then

$$m_e = m_f\frac{\mu_0 c^2}{1}\frac{e^2}{8\pi\theta_{si}a_0 c} = m_f\frac{e^2 c}{8\pi\theta_{si}}\frac{\mu_0}{a_0}\ , \tag{50}$$

$$m_e = m_f\frac{e^2 l_f}{4\pi 2\theta_{si}t_f}\frac{\mu_0}{a_0} \approx m_f\frac{e^2}{4\pi m_f}\frac{\mu_0}{a_0} = \frac{e^2}{4\pi}\frac{\mu_0}{a_0}\ , \tag{51}$$

$$\mu_0 \approx \frac{4\pi}{e^2}m_e a_0\ . \tag{52}$$

In that the electron rest mass is invariant and that the elementary positive charge e in $4\pi/e^2$ is assigned, we find that the magnetic constant and Bohr radius are directly proportional with no additional free variables. The approximation affects only the scaling factor and may be resolved precisely with the expanded form,

$$\mu_0 = \frac{2\pi}{Q_L n_{Lr} e^2}m_e a_0\ . \tag{53}$$

4. DISCUSSION

Organizing the *fundamental expression* such that $c=l_f/t_f=2\theta_{si}/m_f$ allows us to state that any change in the count of discrete units of length must equal the same in a count of discrete units of time for any description of light in a system defined relatively. The relative nature of discrete behavior is crucial because as noted earlier, measure is a function of references in the local inertial frame with respect to a target. But, what if the observed phenomenon is a combination of the effects of relativity on top of characteristics a function of and with respect to the universe?

The question invites again the required understanding as resolved herein, that is, our ability to resolve certain properties of the universe, such as its diameter, mass and age as observed from outside with respect to an external reference. In that the universe expands at the speed of light there is no possibility for a reference external to the universe to play a part in the observer's understanding of a phenomenon. Moreover, we also recognize that discreteness is a property of references. Thus, while matter within the universe may relatively take on discrete behavior, relative to the universe behavior must be non-discrete. The two frameworks express themselves, for one, as a lost difference in distance which we observe as gravity.

Notably, discreteness is not entirely a function of references in the inertial frame. It is a function of the information available. While it may be argued that all observers have a precise understanding of the three measures, some information may be masked. And as such, the missing information will result in a non-discrete, non-deterministic behavior, for example, for any behavior that takes on a measure less than the whole unit values in Heisenberg's reduced particle expression $2n_M n_{Lr} n_L \geq n_T$.

Finally, discrete measure provides bounds that constrain the physical regime and as such resolve the conditions that lead to singularities. By example, l_f/t_f is not the only bound. There is also m_f/t_f and m_f/l_f. Each manifest differently in our universe.

5. APPENDICES

Appendix A: Numerical Limits to $Q_L n_{Lr}$

Throughout the paper, we find the term $Q_L n_{Lr}$ repeatedly. This term is referred to as the *informativity differential* in recognizing the central role it plays in describing how fractional values less than the theoretical limit reflect a distortion effect in distance measurement. Knowing the limits to $Q_L n_{Lr}$ is also essential in resolving the fundamental measures.

The product of $Q_L n_{Lr}$ is Eq. (5) multiplied by b.

$$Q_L n_{Lr} = \left(\sqrt{1+b^2} - b\right) b \qquad \text{(A.1)}$$

Note, what is measured always equals a whole-unit count of a fundamental measure, and with a=1 we find that b=n_{Lr} for all values. This is easily verified in that the highest value for Q_L is obtained for b=1 where $(1+1^2)^{0.5}$– 1= 0.414 and the 'observed' distance of c presented as a count n_{Lr} is always rounded down to the highest integer value equal to the count b with Q_L=0.414 at its highest and quickly approaching 0 with increasing b. Therefore,

$$Q_L n_{Lr} = \left(\sqrt{1+n_{Lr}{}^2} - n_{Lr}\right) n_{Lr} \qquad \text{(A.2)}$$

The lower limit where n_{Lr}=1 is easily produced, $\lim_{r=1} f(Q_L n_{Lr}) = \sqrt{2}-1$. Conversely, if we divide by n_{Lr}, then add n_{Lr}, square, subtract $n_{Lr}{}^2$, and divide by 2, we find that

$$\frac{Q_L{}^2}{2} + Q_L n_{Lr} = \frac{1}{2} \qquad \text{(A.3)}$$

Q_L decreases with increasing n_{Lr} until the left term drops out. Distance does not need to be significant to reduce the *Informativity differential* to 0.5. At just $10^4\ l_f$, $Q_L n_{Lr}$ rounds to 0.5 to nine significant digits.

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