ON OLBERS PARADOX

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Abstract: In this work we discuss a possibility to resolve Olbers paradox that states that if the universe is static, infinite and distributed over with an infinite number of stars then the sky should always be brightly illuminated, and obviously, this contradicts with the observed darkness of the night. Our discussion is based on the observation that matter seems to be distributed mainly on the boundaries of geometric and topological structures in the spatiotemporal continuum, which can be formulated in terms of a CW complex composed of *n*-cells. And in the case of the three-dimensional observable universe the observation can be justified from our work on the spacetime transcription of physical fields that shows that massive matter can only be formed as three-dimensional objects and all four-dimensional physical fields must have the structure of a static electromagnetic field. From the analysis of the possibility of physical creation from spacetime through the process of transcription we may conclude that most of the observable massive matter are formed and distributed on the boundary, which may be composed of 3D balls, of a 3-sphere and therefore the Oblers paradox can be resolved.

To the largest possible scale that we can observe at the present state, the observable universe appears to be isotropic, homogeneous and expanding. And, in particular, it seems to be infinite in extent. This seeming appearance has led to contradiction to our perception and conception of physical existence on the cosmic scale in the so-called Olbers paradox which states that if the universe is static, infinite and distributed over with an infinite number of stars then the sky should always be brightly illuminated. Obviously, this contradicts with the observed darkness of the night [1]. Needless to say, due to our limited physical existence, observations from either a very large scale or a very small scale can be deceptive. For the small scale at the quantum level, perhaps, the most deceptive behaviour of quantum particles manifests in the wave-particle duality. However, we have shown that this delusive behaviour can be explained if quantum particles are represented as differentiable manifolds whose geometric and topological structures can be described in terms of wavefunctions [2-6]. We have also shown that the dynamics of quantum particles can be formulated mathematically as isometric embedding between smooth manifolds, and physically as peristaltic locomotion if the quantum particles are in a fluid state [7-8]. Mathematically, the total spatiotemporal continuum can be described as a fiber bundle which admits different types of fibers for a single base space of spacetime manifold. The apparent geometric and topological structures of the total spatiotemporal manifold are due to the dynamics and the geometric interactions of the decomposed cells from the base space of the total spatiotemporal manifold. The decomposed cells may form different types of fibers which may also geometrically interact with each other. It is assumed that we can only perceive within our physical ability the appearance of the grown intrinsic geometric structures on the base space of the total spatiotemporal manifold and the base space itself may not be observable with the reasonable assumption that a physical object is not observable if it does not have any form of geometric interactions. It could be that the base space of the spatiotemporal manifold at the beginning was only an n-dimensional Euclidean spatiotemporal continuum R^n which had no non-trivial geometric structures therefore contained no physical objects. We could suggest that physical objects could be formed as n-dimensional differentiable manifolds from mass points by contact forces associated with the decomposed 0-cells by the processes of spacetime transcription of physical fields [9]. In physics, physical objects can be assumed to exist in any dimension. In classical and quantum mechanics, an elementary particle is normally assumed to be a mass point without requiring any particular internal substructure. In other physical theories particles can be identified as strings or membranes. And in the most general form, according to Einstein's perception, physics expressed in terms of particles and their mutual interactions should be formulated according to the intrinsic geometric structures of spacetime similar to the geometric formulation of his general relativity [10]. Therefore it is possible to suggest that physical objects can be assumed to be differentiable manifolds which can be endowed with the mathematical structures of a CW complex which is capable of decomposing n-cells. An n-cell can be a closed disk D^n defined as $D^n = \{x \in R^n : |x| \leq r\}$ or an open disk defined as $\mathrm{int}(D^n) = \{x \in R^n : |x| < r\}$. A CW complex is a generalization of a simplicial complex which is a set of simplexes. A simplex is a generalisation of a triangle to higher dimensions [11]. Even though we may consider physical objects of any scale as differentiable manifolds of dimension n which can emit submanifolds of dimension $m \leq n$ by decomposition, however, in order to formulate a physical theory we would need to devise a mathematical framework that allows us to account for the amount of subspaces that are emitted or absorbed by a differentiable manifold, and in order to describe the evolution of a geometric process as a physical interaction the same mathematical framework should also be used to show that an assembly of cells of a specified dimension will give rise to a certain form of physical interactions and the intermediate particles, which are the force carriers of physical fields decomposed during a geometric evolution, may possess the particular geometric structures such as those of the n-spheres and the n-tori. Therefore, for observable physical phenomena, the study of physical dynamics reduces to the study of the geometric evolution of differentiable manifolds. In particular, if a physical object is considered to be a three-dimensional manifold then there are four different types of physical interactions that are resulted from the decomposition of 0-cells, 1-cells, 2-cells and 3-cells and these cells can be associated with the corresponding spatial forces $F_n = k_n r^n$ and temporal forces $F_n = h_n t^n$, respectively. We may assume that a general spatiotemporal force which is a combination of the spatial and temporal forces resulted from the decomposition of spatiotemporal n-cells of all dimensions to take the form

$$F = \sum_{n=-3}^{3} (k_n r^n + h_n t^n) \tag{1}$$

where k_n and h_n are constants which can be determined from physical considerations. Then using equations of motion from both the spatial and temporal Newton's second laws a complete geometric structure of a physical interaction would be a structure that is resulted from the relationship between space and time that satisfies the most general equation in the form

$$m\frac{d^2\mathbf{r}}{d\tau^2} + D\frac{d^2\mathbf{t}}{ds^2} = \sum_{n=-3}^{3} (k_n r^{n-1}\mathbf{r} + h_n t^{n-1}\mathbf{t}) \tag{2}$$

Therefore, in terms of CW complexes in which physical fields exist in the form of *n*-cells, we may propose that physical existence of massive fields can only exist in the forms of single points, closed loops, closed membranes and 3D solid objects. For example, in Bohr model of the hydrogen atom matter is mainly distributed in circles. In fact, as shown in our work on the principle of least action, the Bohr model of the hydrogen atom can also be expressed in a more general form in terms of the fundamental homotopy group. In Schrödinger model of the hydrogen atom matter is mainly distributed on curved surfaces and this can also be expressed in terms of homotopy groups of higher dimensions. It can be seen that the concept of quantisation is equivalent to suggesting that matter can only occupy at positions that allow for them to be formed. This could be true at all level of physical existence in spaces of any dimension. Then 3D physical objects that exist in a four-dimensional Euclidean space may be assumed to be on 3D boundaries, such as the 3D solid boundary of a 3-sphere which is constructed from two 3D solid balls. As an example we may consider the following problem. Since both Bohr and Schrödinger use the same form of the potential for the hydrogen atom then if we use the same form of potential in four-dimensional Euclidean space then we can obtain a model of the hydrogen atom that is formulated as a 3-sphere and it is then expected that matter is distributed in 3D balls. To describe the wave dynamics on a hypersurface embedded or immersed in four-dimensional Euclidean space R^4 we need a four-dimensional time-independent Schrödinger wave equation

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi(\mathbf{r}) - \frac{kq^2}{r}\psi(\mathbf{r}) = E\psi(\mathbf{r}) \tag{3}$$

Consider a d-dimensional hypersphere S_r^d of radius r embedded in the ambient $(d+1)$-dimensional Euclidean space R^{d+1}. If spherical coordinates $(r,\theta,,\theta_1,\dots,\theta_{d-2},\phi)$ are defined in terms of the Cartesian coordinates $(x_1,x_2,\dots,x_{d+1})$ as $x_1 = r\cos\theta$, $x_2 = r\sin\theta\cos\theta_1$,..., $x_{d+1} = r\sin\theta \dots \sin\theta_{d-2}\sin\phi$ then the Laplacian $\nabla_{S^d}^2$ on the hypersphere S_r^d is given as $\nabla_{S^d}^2\psi = (1/r^2)\left(\partial^2\psi/\partial\theta^2 + (d-1)\cot\theta\,\partial\psi/\partial\theta + (1/\sin^2\theta)\nabla_{S^{d-1}}^2\psi\right)$ [12]. For the case of a 3-sphere S^3 embedded in four-dimensional Euclidean space R^4, the four-dimensional time-independent Schrödinger wave equation becomes

$$-\frac{\hbar^2}{2\mu}\left(\frac{1}{r^2}\left(\frac{\partial^2}{\partial\theta^2}+2\cot\theta\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\nabla^2_{S^2}\right)\right)\psi-\frac{kq^2}{r}\psi=E\psi \tag{4}$$

Now, the main topic that we would like to discuss in this work is the Olbers paradox that states that if the universe is static, infinite and distributed over with an infinite number of stars then the sky should always be brightly illuminated. The discussion is based on the idea of spacetime transcription which shows that massive matter that exist in an ambient four-dimensional Euclidean space R^4 can only be formed as three-dimensional physical objects while the physical fields such as the electromagnetic field can be formed as four-dimensional objects. Before giving an outline of spacetime transcription, however, by way of illustration we first give a discussion for the case of two dimensions. The equation of a 2-sphere S^2 in the three-dimensional Euclidean space R^3 is given as $S^2=\{(x,y,z)\in R^3 : x^2+y^2+z^2=R^2\}$

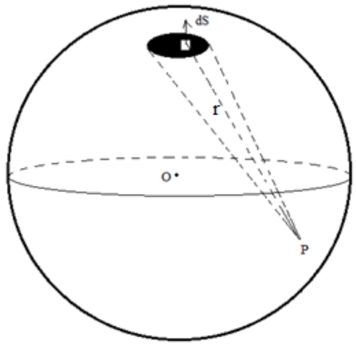

Figure 1: Two-dimensional distribution of matter

We then assume that massive matter can only be formed and distributed on the boundary of the 2-sphere. Even though two-dimensional massive matter can only move on the two-dimensional sphere, radiated light in the form of the electromagnetic field can travel through the third dimension to go from one point to any other point on the surface. As shown in Figure 1, the energy of light that is absorbed at a point P on the surface from a particular source of distance r also on the surface can be calculated using the solid angle Ω defined as $\Omega=\int_S \mathbf{r}.d\mathbf{S}/r^3$. It is observed that two-dimensional physical objects on the surface can only receive a very limited amount of light from sources on the surface because most of light is radiated into the third spatial dimension. Furthermore, as shown in Figure 2, the expansion of a 2-sphere on whose surface massive matter is formed can be interpreted as the result of radiated energy being trapped inside the 2-sphere. We assume that sources of light are formed in the way similar to the formation of stars and galaxies in our observable universe in which two-dimensional galaxies and stars are also formed by the collapse of matter under the action of gravity. When the stars radiate, most of the radiated light will travel into the third dimension, either inside or outside of the 2-sphere. The accumulated amount of light that is trapped inside the 2-sphere will finally make the 2-sphere to expand if the forces of attraction between the massive matter can no longer keep them in their original positions.

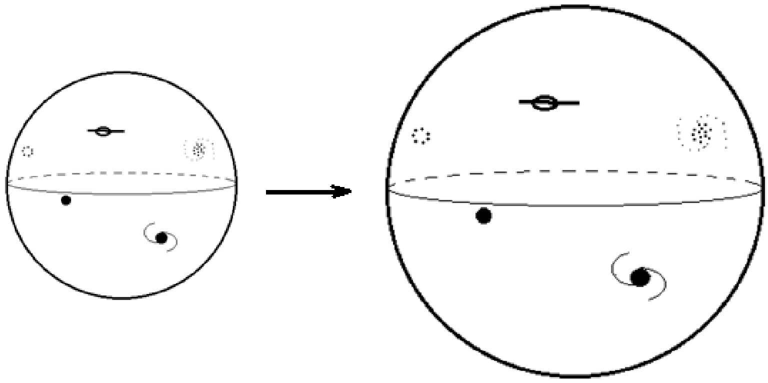

Figure 2: Expansion of two-dimensional universe

The principal features of a spacetime transcription of physical fields are given as follows. Consider $n-1$ vectors $\mathbf{F}_i = f_{i1}\mathbf{i}_1 + f_{i2}\mathbf{i}_2 + \cdots + f_{in}\mathbf{i}_n$, $i = 1, \dots, n-1$, in the Euclidean space R^n. We denote $[\mathbf{F}_1\mathbf{F}_2 \dots \mathbf{F}_{n-1}]$ as the vector product of the $n-1$ vectors that is defined so that whose norm is given as [13-14]

$$|[\mathbf{F}_1\mathbf{F}_2 \dots \mathbf{F}_{n-1}]| = |\mathbf{F}_1||\mathbf{F}_2| \dots |\mathbf{F}_{n-1}|K, \quad K = \begin{vmatrix} 1 & \cos\alpha_{1,2} & \cdots & \cos\alpha_{1,n-1} \\ \cos\alpha_{2,1} & 1 & \cdots & \cos\alpha_{2,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ \cos\alpha_{n-1,1} & \cos\alpha_{n-1,2} & \cdots & 1 \end{vmatrix}^{\frac{1}{2}} \tag{5}$$

where $\cos\alpha_{i,j} = (\mathbf{F}_i . \mathbf{F}_j)/(|\mathbf{F}_i||\mathbf{F}_j|)$. As in the case of the vector product defined in three-dimensional space, the vector $[\mathbf{F}_1\mathbf{F}_2 \dots \mathbf{F}_{n-1}]$ is perpendicular to all vectors $\mathbf{F}_i$. For the four-dimensional Euclidean space R^4, the vector product of three vectors $\mathbf{F}_i = f_{i1}\mathbf{i}_1 + f_{i2}\mathbf{i}_2 + f_{i3}\mathbf{i}_3 + f_{i4}\mathbf{i}_4$, $i = 1, 2, 3$, can be written in the form

$$[\mathbf{F}_1\mathbf{F}_2\mathbf{F}_3] = \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \end{vmatrix} \tag{6}$$

whose norm is

$$|[\mathbf{F}_1\mathbf{F}_2\mathbf{F}_3]| = |\mathbf{F}_1||\mathbf{F}_2||\mathbf{F}_3|K, \quad K = \begin{vmatrix} 1 & \cos\alpha_{1,2} & \cos\alpha_{1,3} \\ \cos\alpha_{2,1} & 1 & \cos\alpha_{2,3} \\ \cos\alpha_{3,1} & \cos\alpha_{3,2} & 1 \end{vmatrix}^{\frac{1}{2}} \tag{7}$$

Using the above definition of the vector product in the four-dimensional Euclidean space, the curl of two vectors $(\mathbf{F}_1, \mathbf{F}_2)$ can be defined as follows

$$\nabla \times (\mathbf{F}_1, \mathbf{F}_2) = [\nabla \mathbf{F}_1 \mathbf{F}_2] = \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \dfrac{\partial}{x_1} & \dfrac{\partial}{x_2} & \dfrac{\partial}{x_3} & \dfrac{\partial}{x_4} \\ f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \end{vmatrix} \tag{8}$$

For any three-dimensional vector $\mathbf{T} = T_1\mathbf{i}_1 + T_2\mathbf{i}_2 + T_3\mathbf{i}_3$ and if we use the unit vector in the fourth dimension $\mathbf{S} = \mathbf{i}_4$ then the normal three-dimensional curl of the vector $\mathbf{T}$ can be expressed as

$$\nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) = \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \dfrac{\partial}{x_1} & \dfrac{\partial}{x_2} & \dfrac{\partial}{x_3} & \dfrac{\partial}{x_4} \\ T_1 & T_2 & T_3 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$
$$= \left(\frac{\partial T_3}{\partial x_2} - \frac{\partial T_2}{\partial x_3}\right)\mathbf{i}_1 - \left(\frac{\partial T_3}{\partial x_1} - \frac{\partial T_1}{\partial x_3}\right)\mathbf{i}_2 + \left(\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2}\right)\mathbf{i}_3 \tag{9}$$

It is seen that in order for the curl of the three-dimensional vector $\mathbf{T} = T_1\mathbf{i}_1 + T_2\mathbf{i}_2 + T_3\mathbf{i}_3$ to be expressed in a four-dimensional form we would need to choose a particular form for the four-dimensional vector $\mathbf{S}$. We may then ask the question about the physical nature of the vector $\mathbf{S}$ and what role it would play in the physical formulation of natural phenomena that appear in the three-dimensional space if the whole spatiotemporal continuum has more than three spatial dimensions. We now show that physical fields such as the electromagnetic field and Dirac quantum fields may in fact be the product of a spacetime transcription from designed patterns in a spatiotemporal manifold with four spatial dimensions.

For the case of the electromagnetic field, we identify the vector $\mathbf{T}$ either as the electric field $\mathbf{E}$ or as the magnetic field $\mathbf{B}$. It is seen that to be able to associate the electric and magnetic field with possible spacetime structures in four dimensions, instead of the unit vector in the fourth dimension $\mathbf{S} = \mathbf{i}_4$ we would need to extend the vector $\mathbf{S}$ to include the three-dimensional components in the form $\mathbf{S} = S_1\mathbf{i}_1 + S_2\mathbf{i}_2 + S_3\mathbf{i}_3 + \mathbf{i}_4$. Then the four-dimensional curl for the three-dimensional vector $\mathbf{T} = T_1\mathbf{i}_1 + T_2\mathbf{i}_2 + T_3\mathbf{i}_3$ can be written as

$$\nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) = \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \dfrac{\partial}{x_1} & \dfrac{\partial}{x_2} & \dfrac{\partial}{x_3} & \dfrac{\partial}{x_4} \\ T_1 & T_2 & T_3 & 0 \\ S_1 & S_2 & S_3 & 1 \end{vmatrix}$$

$$= \left(\frac{\partial T_3}{\partial x_2} - \frac{\partial T_2}{\partial x_3} + \frac{\partial}{\partial x_4}(S_3 T_2 - S_2 T_3)\right)\mathbf{i}_1$$

$$- \left(\frac{\partial T_3}{\partial x_1} - \frac{\partial T_1}{\partial x_3} + \frac{\partial}{\partial x_4}(S_3 T_1 - S_1 T_3)\right)\mathbf{i}_2$$

$$+ \left(\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2} + \frac{\partial}{\partial x_4}(S_2 T_1 - S_1 T_2)\right)\mathbf{i}_3$$

$$- \left(\frac{\partial}{\partial x_1}(S_3 T_2 - S_2 T_3) - \frac{\partial}{\partial x_2}(S_3 T_1 - S_1 T_3) + \frac{\partial}{\partial x_3}(S_2 T_1 - S_1 T_2)\right)\mathbf{i}_4 \qquad (10)$$

It is observed from Equation (10) that the four-dimensional curl $\nabla \times \mathbf{T}$ can be reduced to a three-dimensional vector if the following condition is satisfied

$$\left(\frac{\partial}{\partial x_1}(S_3 T_2 - S_2 T_3) - \frac{\partial}{\partial x_2}(S_3 T_1 - S_1 T_3) + \frac{\partial}{\partial x_3}(S_2 T_1 - S_1 T_2)\right) = 0 \qquad (11)$$

The equation given in Equation (11) can be satisfied by either imposing various conditions on the four-dimensional vector **S** or establishing different relationships between the vector **S** and the vector **T**. In this work we are interested in the following relationship between the vector **S** and the vector **T**

$$T_1 = kS_1, \quad T_2 = kS_2 \quad \text{and} \quad T_3 = kS_3 \qquad (12)$$

The condition given in Equation (12) is called a spacetime transcription, which is a spacetime process from which the vector **T** is formed using the four-dimensional spatiotemporal vector **S** as a designed pattern. From the spacetime transcription, the curl of a three-dimensional vector expressed in a four-dimensional Euclidean space is reduced to the normal three-dimensional curl

$$\nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) = \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \dfrac{\partial}{x_1} & \dfrac{\partial}{x_2} & \dfrac{\partial}{x_3} & \dfrac{\partial}{x_4} \\ T_1 & T_2 & T_3 & 0 \\ S_1 & S_2 & S_3 & 1 \end{vmatrix}$$

$$= \left(\frac{\partial T_3}{\partial x_2} - \frac{\partial T_2}{\partial x_3}\right)\mathbf{i}_1 - \left(\frac{\partial T_3}{\partial x_1} - \frac{\partial T_1}{\partial x_3}\right)\mathbf{i}_2 + \left(\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2}\right)\mathbf{i}_3 \qquad (13)$$

Now, if we identify the vector **T** with the electric field **E** and the magnetic field **B** then we may say that the three-dimensional electromagnetic field is a product obtained from a process that can be identified with a spacetime transcription. As shown in our work on the fluid state of a steady electromagnetic field only $\nabla \times \mathbf{E} = 0$ and $\nabla \times \mathbf{B} = 0$ are required to construct possible structure of the quantum particle of the electromagnetic field, namely the photon. However, we have only discussed the process of a spatial transcription, and this process alone does not establish a complete three-dimensional electromagnetic field that would require other relationships between $\nabla \times \mathbf{E}$ and $\nabla \times \mathbf{B}$ and the temporal rates, as well as different physical entities, such as charge, as described by Maxwell field equations of electromagnetism $\nabla \times \mathbf{E} + \partial \mathbf{B}/\partial t = 0$, $\nabla \times \mathbf{B} - \epsilon\mu\, \partial \mathbf{E}/\partial t = \mu \mathbf{j}_e$. Such an anticipated formulation requires further investigation into the temporal transcription of physical fields in the temporal submanifold of the spatiotemporal continuum.

In order to formulate a similar spacetime transcription for Dirac quantum fields, we consider a two-dimensional vector $\mathbf{T} = T_1\mathbf{i}_1 + T_2\mathbf{i}_2$ and a four-dimensional spacetime vector $\mathbf{S}$ of the form $\mathbf{S} = S_1\mathbf{i}_1 + S_2\mathbf{i}_2 + \mathbf{i}_3 + \mathbf{i}_4$. Then the four-dimensional curl $\nabla \times \mathbf{T}$ of the vector $\mathbf{T}$ takes the form

$$\begin{aligned} \nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) &= \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \dfrac{\partial}{x_1} & \dfrac{\partial}{x_2} & \dfrac{\partial}{x_3} & \dfrac{\partial}{x_4} \\ T_1 & T_2 & 0 & 0 \\ S_1 & S_2 & 1 & 1 \end{vmatrix} \\ &= \left(-\frac{\partial T_2}{\partial x_3} + \frac{\partial T_2}{\partial x_4}\right)\mathbf{i}_1 - \left(-\frac{\partial T_1}{\partial x_3} + \frac{\partial T_1}{\partial x_4}\right)\mathbf{i}_2 \\ &+ \left(\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2} + \frac{\partial}{\partial x_4}(S_2T_1 - S_1T_2)\right)\mathbf{i}_3 \\ &- \left(\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2} + \frac{\partial}{\partial x_3}(S_2T_1 - S_1T_2)\right)\mathbf{i}_4 \end{aligned} \tag{14}$$

If we impose the spacetime transcription conditions $T_1 = kS_1$, $T_2 = kS_2$ then we have

$$\begin{aligned} \nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) &= \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \dfrac{\partial}{x_1} & \dfrac{\partial}{x_2} & \dfrac{\partial}{x_3} & \dfrac{\partial}{x_4} \\ T_1 & T_2 & 0 & 0 \\ S_1 & S_2 & 1 & 1 \end{vmatrix} \\ &= \left(-\frac{\partial T_2}{\partial x_3} + \frac{\partial T_2}{\partial x_4}\right)\mathbf{i}_1 - \left(-\frac{\partial T_1}{\partial x_3} + \frac{\partial T_1}{\partial x_4}\right)\mathbf{i}_2 + \left(\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2}\right)\mathbf{i}_3 \\ &- \left(\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2}\right)\mathbf{i}_4 \end{aligned} \tag{15}$$

It is seen from Equation (15) that the required condition for the curl $\nabla \times \mathbf{T}$ to be a two-dimensional vector is

$$\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2} = 0 \tag{16}$$

To form a complete system in which the quantities T_1 and T_2 can be identified as the velocity potential and the stream function of a two-dimensional fluid flow for Dirac quantum fields as we have presented in our previous works, we need to consider a second two-dimensional vector field of the form $\mathbf{T} = T_1\mathbf{i}_1 - T_2\mathbf{i}_2$. The four-dimensional curl of the vector $\mathbf{T}$ takes the form

$$\nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) = \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \frac{\partial}{x_1} & \frac{\partial}{x_2} & \frac{\partial}{x_3} & \frac{\partial}{x_4} \\ T_1 & -T_2 & 0 & 0 \\ S_1 & S_2 & 1 & 1 \end{vmatrix}$$
$$= \left(\frac{\partial T_2}{\partial x_3} - \frac{\partial T_2}{\partial x_4}\right)\mathbf{i}_1 - \left(-\frac{\partial T_1}{\partial x_3} + \frac{\partial T_1}{\partial x_4}\right)\mathbf{i}_2$$
$$+ \left(-\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2} + \frac{\partial}{\partial x_4}(S_2T_1 + S_1T_2)\right)\mathbf{i}_3$$
$$- \left(-\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2} + \frac{\partial}{\partial x_3}(S_2T_1 + S_1T_2)\right)\mathbf{i}_4 \quad (17)$$

If we also impose the spacetime transcription conditions $T_1 = kS_1$, $T_2 = -kS_2$ then we have

$$\nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) = \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \frac{\partial}{x_1} & \frac{\partial}{x_2} & \frac{\partial}{x_3} & \frac{\partial}{x_4} \\ T_1 & -T_2 & 0 & 0 \\ S_1 & S_2 & 1 & 1 \end{vmatrix}$$
$$= \left(\frac{\partial T_2}{\partial x_3} - \frac{\partial T_2}{\partial x_4}\right)\mathbf{i}_1 - \left(-\frac{\partial T_1}{\partial x_3} + \frac{\partial T_1}{\partial x_4}\right)\mathbf{i}_2 + \left(-\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2}\right)\mathbf{i}_3$$
$$- \left(-\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2}\right)\mathbf{i}_4 \quad (18)$$

The condition for the four-dimensional curl $\nabla \times \mathbf{T}$ to be reduced to a two-dimensional vector is

$$\frac{\partial T_2}{\partial x_1} + \frac{\partial T_1}{\partial x_2} = 0 \quad (19)$$

Equations (16) and (19) together form the Cauchy-Riemann equations in the (x_1, x_2)-plane

$$\frac{\partial T_2}{\partial x_1} - \frac{\partial T_1}{\partial x_2} = 0, \qquad \frac{\partial T_2}{\partial x_1} + \frac{\partial T_1}{\partial x_2} = 0 \quad (20)$$

From the system of Cauchy-Riemann equations given in Equation (20) the quantities T_1 and T_2 can be identified as the velocity potential and the stream function of a two-dimensional fluid flow. For the case of Dirac fields, if we want to form a standing wave that can be used to represent a quantum particle then we would need another two vector fields of the forms $\mathbf{T} = T_3\mathbf{i}_1 + T_4\mathbf{i}_2$ and $\mathbf{T} = T_3\mathbf{i}_1 - T_4\mathbf{i}_2$. By using the four-dimensional spacetime vector $\mathbf{S}$ also of the form $\mathbf{S} = S_1\mathbf{i}_1 + S_2\mathbf{i}_2 + \mathbf{i}_3 + \mathbf{i}_4$, then we arrive at similar conditions for the four-dimensional curl of the vectors $\mathbf{T} = T_3\mathbf{i}_1 + T_4\mathbf{i}_2$ and $\mathbf{T} = T_3\mathbf{i}_1 - T_4\mathbf{i}_2$ to be two-dimensional vectors

$$\frac{\partial T_3}{\partial x_1} + \frac{\partial T_4}{\partial x_2} = 0, \qquad \frac{\partial T_3}{\partial x_1} - \frac{\partial T_4}{\partial x_2} = 0 \tag{21}$$

The conditions given in Equation (21) shows that T_3 and T_4 can also be identified as a vector potential and stream function of a two-dimensional fluid flow, which are complementary to the quantities T_1 and T_2 with similar identification.

We have shown that both the electromagnetic field and Dirac field of quantum particles can be regarded as being formed from the processes of spacetime transcription that convert the four-dimensional patterns into three and two-dimensional physical objects, we may ask whether it is possible to form four-dimensional physical objects in similar processes of spacetime transcription. For example, if we consider a general four-dimensional vector $\mathbf{T} = T_1\mathbf{i}_1 + T_2\mathbf{i}_2 + T_3\mathbf{i}_3 + T_4\mathbf{i}_4$ and $\mathbf{S} = S_1\mathbf{i}_1 + S_2\mathbf{i}_2 + S_3\mathbf{i}_3 + S_4\mathbf{i}_4$ then the curl of the vector $\mathbf{T}$ with regard to the vector $\mathbf{S}$ takes the more general form

$$\begin{aligned}
\nabla \times \mathbf{T} = \nabla \times (\mathbf{T}, \mathbf{S}) &= \begin{vmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 & \mathbf{i}_4 \\ \frac{\partial}{x_1} & \frac{\partial}{x_2} & \frac{\partial}{x_3} & \frac{\partial}{x_4} \\ T_1 & T_2 & T_3 & T_4 \\ S_1 & S_2 & S_3 & S_4 \end{vmatrix} \\
&= \left(\frac{\partial}{\partial x_2}(S_4T_3 - S_3T_4) - \frac{\partial}{\partial x_3}(S_4T_2 - S_2T_4) + \frac{\partial}{\partial x_4}(S_3T_2 - S_2T_3)\right)\mathbf{i}_1 \\
&- \left(\frac{\partial}{\partial x_1}(S_4T_3 - S_3T_4) - \frac{\partial}{\partial x_3}(S_4T_1 - S_1T_4) + \frac{\partial}{\partial x_4}(S_3T_1 - S_1T_3)\right)\mathbf{i}_2 \\
&+ \left(\frac{\partial}{\partial x_1}(S_4T_2 - S_2T_4) - \frac{\partial}{\partial x_2}(S_4T_1 - S_1T_4) + \frac{\partial}{\partial x_4}(S_2T_1 - S_1T_2)\right)\mathbf{i}_3 \\
&- \left(\frac{\partial}{\partial x_1}(S_3T_2 - S_2T_3) - \frac{\partial}{\partial x_2}(S_3T_1 - S_1T_3) + \frac{\partial}{\partial x_3}(S_2T_1 - S_1T_2)\right)\mathbf{i}_4
\end{aligned} \tag{22}$$

Again, if we impose the conditions $T_1 = kS_1$, $T_2 = kS_2$, $T_3 = kS_3$ and $T_4 = kS_4$ then the curl $\nabla \times \mathbf{T} \equiv 0$, which indicates that there are no unified four-dimensional electromagnetic fields that exist in the four-dimensional Euclidean space R^4, similar to the existence of the three-dimensional electromagnetic field in the three-dimensional Euclidean space R^3. However, it can be seen that a steady four-dimensional photon can be constructed in the four-dimensional Euclidean space. From the above analysis of the possibility of physical creation from spacetime through the process of transcription we may conclude that most of the observable massive matter is formed and distributed on the boundary in the form of 3D balls of a 3-sphere and therefore the Oblers paradox can be resolved.

As described for the case of 2-sphere above, the expansion of the observable universe which exists as a 3D ball can also be assumed to be caused by the radiated energy that is trapped inside the 3-sphere. This expansion can also be interpreted as the bending of the observable universe into the fourth spatial dimension. The amount of the trapped energy inside the 3-sphere can be calculated using the observed energy in the observable universe. In a coordinate system with the coordinates (x_1, x_2, x_3, x_4) the equation of a 3-sphere with radius r centred at the origin is given as $S^3 = \{(x_1, x_2, x_3, x_4) \in R^4 : \; x_1^2 + x_2^2 + x_3^2 + x_4^2 = r^2\}$, and the volume V of the four-dimensional region bounded by a 3-sphere of radius r is given as $V = \pi^2 r^4/2$. Even though the trapped energy inside the 3-sphere cannot be observed and measured because it is associated with the fourth spatial dimension, the amount of trapped energy can be calculated using the observed energy. If ρ_D is a four-dimensional volume energy density then the trapped energy E_D stored in the 3-sphere is given by $E_D = \rho_D V$. Let E is the observable energy that is responsible for the supposed rate of expansion of the observable 3D universe. If we let σ is the ratio between the trapped energy E_D and the observed energy E, i.e., $E_D = \sigma E$, then the total energy would be $E_T = 2(E + E_D) = 2(1 + \sigma)E$, in which we have taken into account the fact that a 3-sphere may be formed from two 3D solid balls. If R is the radius of the 3-sphere then the four-dimensional energy density ρ_D is given as

$$\rho_D = \frac{4(1+\sigma)E}{\pi^2 R^4} \tag{23}$$

The ratio σ can be established from the missing energy that is required for the rate of expansion of the observable universe. It should also be mentioned here that the radius R of the cosmological 3-sphere S^3 can be determined from a physical theory such as Einstein theory of general relativity and a model for the cosmological evolution such as the Robertson-Walker metric. If we apply Einstein field equations of general relativity

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R - \Lambda g_{\alpha\beta} = -\frac{8\pi G}{c^4} T_{\alpha\beta} \tag{24}$$

and the line element for the cosmological evolution by the Robertson-Walker metric

$$ds^2 = c^2 dt^2 - R^2(t)\left(\frac{1}{1-kr^2} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) \tag{25}$$

and by assuming an energy-momentum tensor of the form $T_{\alpha\beta} = diag(-g_{ii}p, c^2\rho)$ for the Einstein field equations, then the radius R can be shown to satisfy the following system of equations

$$\frac{2}{R}\frac{d^2R}{dt^2} + \frac{1}{R^2}\left(\frac{dR}{dt}\right)^2 + \frac{kc^2}{R^2} - \Lambda c^2 = -\frac{8\pi G\rho}{c^4} \tag{26}$$

$$\frac{1}{R^2}\left(\frac{dR}{dt}\right)^2 + \frac{kc^2}{R^2} - \frac{\Lambda c^2}{3} = -\frac{8\pi G\rho}{3} \tag{27}$$

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