Denial of Summers' Malice and Alice as a logical puzzle

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Abstract: The clauses of the Malice and Alice logical puzzle of Summers are mapped for evaluation of the conjecture. It should return tautology for all pairwise answers, before removing pairs of the antecedent to discover the correct pair of murder and victim. However, the conjecture is *not* tautologous, albeit one value shy. This means the puzzle as rendered is not well formed, thereby denying the status of a puzzle. Therefore the conjecture is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Avigad, J.; Lewis,Y.; van Doorn, F. (2004). Logic and proof.01.
leanprover.github.io/logic_and_proof/logic_and_proof.pdf

Propositional logic

2.1 A Puzzle The following puzzle, titled "Malice and Alice," is from George J. Summers' *Logical Deduction Puzzles*.

Alice, Alice's husband, their son, their daughter, and Alice's brother were involved in a murder.

One of the five killed one of the other four. The following facts refer to the five people mentioned:

(D1.1)

LET	p,	q,	r,	s,	t,	w,	z:
	Alice;	Alice's husband;	their daughter;	their son;	Alice's brother;	bar;	beach.

((((p=u)>(v=((q+r)+(s+t)))) + ((q=u)>(v=((p+r)+(s+t))))) + (((r=u)>(v=((p+q)+(s+t)))) + ((s=u)>(v=((p+q)+(r+t)))))) + ((t=u)>(v=((p+q)+(r+s)))) ; (D1.2)

Remark D1.2: While Eq. D1.2 is trivial, it defines combinations of murder and victim, and always returns tautology. This serves as the antecedent of the conjecture in our strategy, which could be selectively pared down to find the pair of the murderer and victim, should the conjecture return as tautologous.

We list our assumptions. Alice and her husband are assumed to be the *natural* parents of their children, so Alice and husband are older than either child, to avoid step children older than step parents or step relatives. The gender of players is not separate variables because it is enumerated once only.

1. A man and a woman were together in a bar at the time of the murder. (1.1)

w>((q+(s+t))&(p+r)) ; (1.2)

2. The victim and the killer were together on a beach at the time of the murder. (2.1)

z>(u&v) ; (2.2)

3. One of Alice's two children was alone at the time of the murder. (3.1)

(~(z>r)+~(w>s))&(~(w>r)+~(r>s)) ; (3.2)

Remark 3.1: Summer and Avigad apparently assume a child alone does not imply location other than bar or beach. Hence it is unclear if one alone could refer to before *or* after the murder on the beach.

4. Alice and her husband were not together at the time of the murder. (4.1)

((w>~p)&(w>~q))&((r>~p)&(r>~q)) ; (4.2)

Remark 4.1: This mapping is expanded individually for clarity.

5. The victim's twin was not the killer. (5.1)

Remark 5.1: A twin is Alice or her brother and the daughter or the son: (p+t), (r+s)
as: If p=v then t@ u; If t=v then p@u; If r=v then s@u; If s=v then r@u.
(((p=v)>(t@ u))+((t=v)>(p@ u)))+(((r=v)>(s@u))+((s=v)>(r@u))) ; (5.2)

6. The killer was younger than the victim. (6.1)

u<v ; (6.2)

Which one of the five was the victim?
The conjectured equation is: (D1) > ((((1)&(2))&((3)&(4))) > ((5)&(6))) ; (7.1)

(((((p=u)>(v=((q+r)+(s+t)))) + ((q=u)>(v=((p+r)+(s+t))))) + (((r=u)>(v=((p+q)+(s+t)))) +
((s=u)>(v=((p+q)+(r+t)))))) + ((t=u)>(v=((p+q)+(r+s)))))
> ((((w>((q+(s+t))&(p+r)))&(z>(u&v)))&(((~(z>r)+~(w>s))&(~(w>r)
+~(r>s)))&(((w>~p)&(w>~q))&((r>~p)&(r>~q))))) > (((((p=v)>(t@u))+((t=v)>(p@u)))+
(((r=v)> (s@u))+((s=v)>(r@u))))&(u<v))) ; (7)

TTTT TTTT TTTT TTTT(15),
TTTT **F**TTT TTTT TTTT(1) (7.2)

(You should assume that the victim's twin is one of the five people mentioned.)

Summers' book offers the following hint: "First find the locations of two pairs of people at the time of the murder, and then determine who the killer and the victim were so that no condition is contradicted."

Eq. 7 should be tautologous before removing pairs of the antecedent to discover the pair of murder and victim. However, 7 is *not* tautologous, albeit one value shy. This means the puzzle as rendered in bold in Summer's book is not well formed, thereby denying the puzzle.